532.58

Paper No. 141-11

Flow Past Cone Placed on Flat Plate*

By Tetsushi OKAMOTO** Miki YAGITA***
and Shin-ichi KATAOKA****

This paper presents an experimental investigation on the flow past a cone placed on a flat plate and its wake. The vertex angle of the cone was varied 60°, 90°, 120° and 150°. The surface-pressure distributions on the cone and the flat plate, the velocity and the static pressure in the wake were measured, and the separation and the vortex formation were observed by visual means. The main results are as follows: (1) There occur neck-lace-like vortices around the cone of vertex angle 60°, 120°, and horse-shoe vortices generated behind the cone are shed downstream periodically. (2) The lift acting on the cone and the drag decrease with an increase in the vertex angle. (3) There exists a region of negative wake, which becomes maximum at a downstream distance of about twelve times the height of cone.

Nomenclature

1. Introduction

The flow past a large number of cones arranged in a regular fashion on a flat plate has been studied experimentally by Schlichting⁽¹⁾ in order to obtain the relation between the roughness of a wall and the resistance coefficient. Furthermore, he examined the flows past a large number of spheres, spherical segments and flat plates (short angles). However, there have been few studies⁽²⁾ on the flow past a single cone on a flat plate, and it has been difficult to deal with the single cone as the roughness element of a wall.

Recently the arrangements of flat plates or circular cylinders of finite length have been used as shear flow production elements in order to simulate an atmospheric boundary-layer in a wind tunnel. In such simulations, use of the cone as a shear flow production element is also possible. Furthermore, the cone can be regarded as a conical mountain like Mt. Fuji. Therefore, it is necessary to examine the flow past a single cone, particularly the wake behind it for the sake of aviation.

For this reason, this paper deals with the flow past a single cone on a flat plate and its wake. The vertex angle of the cone was varied 60°, 90°, 120° and 150°. The surface-pressure distributions on the cone and flat plate having the cone on it, the velocity and the static pressure and the flow direction in the wake were measured. Also, the separation of the flow and the vortex formation were observed visually.

x, y, z : Cartesian co-ordinates with origin at leading edge of flat plate (see Fig.1)
 X, Y, Z : Cartesian co-ordinates with origin at leading edge of cone (see Fig.1)
 D : diameter of cone base
 H : height of cone
 p : surface-pressure
 p_s : static pressure of free stream
 r : radius of cone at an arbitrary position of cone surface
 u : velocity of wake flow
 u_τ : frictional velocity = $\sqrt{\tau_0/\rho}$
 U : velocity of uniform free stream
 α : semi-vertex angle of cone
 δ : boundary-layer thickness of flat plate
 δ^* : displacement thickness of flat plate
= $\int_0^\infty (1 - \frac{u}{U}) dz$
 θ : angular distance from front stagnation point
 τ_0 : frictional shearing stress at flat plate
 ν : kinematic viscosity

C_{Dz} : local drag coefficient = $\int_0^\pi C_p \cos\theta d\theta$

C_D : drag coefficient

$$= 2 \int_0^\pi \int_0^\pi C_p (1 - \frac{z}{H}) \cos\theta d\theta d(z/H)$$

C_{Lz} : local lift coefficient = $-\frac{1}{\pi} \int_0^\pi C_p d\theta$

Fig.1 working section of wind tunnel and notations

* Received 17th February, 1975.

** Professor, College of Science and Engineering, Aoyama Gakuin University, Setagaya-ku, Tokyo.

*** Research Associate, Tokyo Institute of Technology.

**** Nippon Steel Corporation.

C_L : lift coefficient=

$$-\frac{2}{\pi} \int_0^{\pi} \int_0^{\frac{z}{H}} C_p (1 - \frac{z}{H}) d\theta d(z/H)$$

C_f : local skin friction coefficient of flat plate $= \tau_w / \frac{1}{2} \rho U^2$

C_p : pressure coefficient $= (p - p_\infty) / \frac{1}{2} \rho U^2$

Re : Reynolds number $= UD/\nu$

Table 1 Dimensions of cones

	60°	90°	120°	150°
H mm	86.43	49.94	28.67	13.46
2α°	60.10	90.07	120.3	149.9
D mm	100.0			

Table 2

2α	60°	90°	120°	150°
δ/H	0.090	0.157	0.273	0.579

Table 3 Experimental conditions

Free stream velocity U	10 m/s			
Vertex angle α	60°	90°	120°	150°
$Re_H = UH/\nu$	5.8×10	3.3×10	1.9×10	0.89×10
$Re = UH/\nu$	6.7×10

a) Velocity profiles in boundary-layer

(b) Boundary-layer thickness

(c) Local skin friction coefficient

fig.2 Boundary-layer on flat plate ($U=10\text{m/s}$)

2. Apparatus and procedure of measurement

The wind tunnel used in this experiment is Eiffel type having a circular working section of 800mm diameter and 1450mm long (see Fig.1). The free stream velocity in the working section containing a flat plate is uniform within 0.5% over the region from the leading edge of the flat plate to 1000mm downstream. The intensity of turbulence $\sqrt{u'}/U$ of the free stream is about 0.5% at $U=10\text{m/s}$. An aluminium board of 3mm thick and 1000mm×1400mm long is used as the flat plate, and is carefully set on the frame keeping parallel to the air stream. The flat plate used has a number of pressure holes, and the cone is set on it at a position of 256mm from the leading edge.

Four cones were made of aluminium and their vertex angles were 60°, 90°, 120° and 150°, respectively. The diameter of the cone base was 100mm and constant for all cones. The dimensions of the cones are shown in Table 1. The cones had a number of pressure holes of 0.5mm diameter on the surface along their generating lines (twenty holes for vertex angles of 60° and 90°, ten holes for 120° and 150°), and the surface-pressure was transmitted through the cone base to a manometer. The cone was set on a turntable. Therefore, the pressure distribution over the cone surface could be measured by rotating each hole about the cone axis.

The flow direction in the wake of the cone was measured by the hot-wire anemometer of constant-temperature type with an X-wire of 0.005mm diameter. The velocity and the static pressure were measured by the total and the static pressure tubes made of brass tubing of 1mm diameter.

(a) $2\alpha=90^\circ$

(b) $2\alpha=150^\circ$

Fig.3 Surface-pressure of cones

Table 2 shows the ratio δ/H for each cone, where δ is the boundary-layer thickness at the position ($x=265\text{mm}$) of the cone and is equal to 7.6mm . Table 3 shows the Reynolds numbers of measurements.

Measurements of the flow direction were performed only in the center plane (XZ -plane), and those of velocity and static pressure were performed in the Z -direction in the region of $-1 \leq X/D \leq 8$ for the center plane and in detail in the Y - and Z -directions at the sections of $X/D=4$ and 8 .

Observations of the separation of flow and vortex formation were made as follows: (1) The flow was made visible by black ink in a water tank having a working section of $250\text{mm} \times 400\text{mm} \times 1500\text{mm}$ long. (2) The flow on the cone surface and the flat plate was made

visible by the oil-film method in the wind tunnel.

3. Results and discussion

3.1. Boundary-layer flow on flat plate

Fig.2 shows the boundary-layer flow on the flat plate without a cone. There is no laminar boundary-layer due to the fact that the flow separates at the leading edge of the flat plate. The velocity profiles nearly agree with that of $1/5$ -th-power law in the turbulent boundary-layer downstream from the position of the cone as shown in Fig.2(a). Fig.2(b) shows the boundary-layer thickness δ , and Fig.2(c) shows the local skin friction coefficient C_f of the flat plate obtained by Clauser's method⁽⁹⁾. Also, the static pres-

Fig.4 Contours of constant surface-pressure of cones

Broken lines indicate outer edge of boundary-layer

sure of the flow along the flat plate remains constant in the whole field of flow measurements.

3.2. Pressure distributions on cone surface and flat plate

Fig.3 shows the surface-pressure distributions on the cones along the circumference for various values of Z/H . The pressure distributions on cones of vertex angles 60° , 90° and 120° are similar to that on a circular cylinder (see Fig.3(a)), but the distribution on the cone of vertex angle 150° is different from those of the other cones and resembles a cosine-curve (see Fig.3(b)). This is due to the fact that in the case of vertex angle 150° the vortex region caused by separation only exists in the neighborhood of the cone vertex. This fact will be shown later in the experiments by the water tank and the oil-film methods. Fig.4 shows the contours of constant pressure obtained from the pressure distributions on each cone. It is found from the contours that the front stagnation point is found on the front surface ($\theta=0^\circ$) of the cone for cones of vertex angles 60° and 120° , and a pair of depressions of the pressure appear on the rear surface of the cones (in the neighborhood of $\theta=\pm 150^\circ$). For the cone of vertex angle 150° , the front stagnation point seems to be at the front point of the cone base and the depressions of the pressure do not occur. It is found from observation of the flow in the water tank that the positions of the depressions correspond to the end points of a horse-shoe vortex formed by the separated flow.

Fig.5 shows the pressure distributions

Table 4 Separation point θ°

r mm	2α	60°	90°	120°	150°
5		66.5	62.5	59.5	67.5
10		67.5	65.0	64.0	71.0
15		68.0	67.0	67.5	82.0
20		69.0	70.0	72.0	90.0
25		69.5	73.0	75.5	95.5
30		70.5	76.0	79.5	
35		72.5	79.0	83.5	
40		75.5	82.0	87.5	
45		79.5	86.0	92.0	
50		84.0	90.0	95.5

Fig.5 Pressure distribution along z -axis at $\theta=0^\circ$

along the Z -axis at $\theta=0^\circ$ for each cone. The pressure distributions have a peak in the proximity of the cone base for cones of vertex angles 60° , 90° and 120° . But there is no peak for the cone of vertex angle 150° . The pressure of the peak shows a negative pressure gradient toward the flat plate and therefore it indicates the presence of a necklace-vortex around the cone. The results for a semi-infinite circular cylinder⁽⁴⁾ are also shown in this figure in comparison with those of cones.

The separation point of the flow on the cone surface could not be obtained from the surface-pressure distribution. Figs.6 and 7 show the separation point obtained by the oil-film method. Its position is shown in Table 4 by r and θ . Also, the results are shown in Fig.4 by broken lines. θ of the separation point decreases in the neighborhood of the cone vertex as the vertex angle increases, and in the proximity to the cone base θ increases with the vertex angle. In the case of the cone vertex angle 150° , the separation point is not clear except in the neighborhood of the cone vertex. The flow pattern on the cone surface can also be inferred from these figures.

By way of example, Fig.8 shows the pressure distribution on the flat plate.

Fig.6 Oil-film method ($2\alpha=60^\circ$)

Fig.7 Oil-film method ($2\alpha=150^\circ$)

The pressure gradient is positive toward the cone in front of it, and the pressure gradient decreases as the vertex angle increases. For the cones of vertex angles 60° , 90° and 120° , there are a pair of depressions of the surface-pressure on the flat plate in rear of the cone. The depression is reduced as the vertex angle increases. In the case of the cone of vertex angle 150° , there is no depression on the flat plate nor on the cone surface. In experiments with the water tank, the flow that had separated on the cone surface formed a horse-shoe vortex on the cone surface. (At this stage, the positions of the ends of the vortex correspond to those of the depressions on the cone surface.) When the horse-shoe vortices shed periodically downstream, the ends of the vortices move from the cone surface to the flat plate behind the cone. Therefore, it is found that the positions of the depressions on the flat plate correspond to the ends of the shed horse-shoe vortices. This is also confirmed by the presence of a pair of vortices on the flat plate behind the cone as shown in Fig.6.

As seen in Fig.8, there is a peak of positive pressure on the X-axis behind the cone for cones of vertex angles 60° , 90° and 120° . The peak increases with the vertex angle. The peak is caused by the downwash of the free stream across the cone, and its

Fig.8 Contours of constant surface-pressure of flat plate ($2\alpha=90^\circ$)

Fig.9 Velocity vectors ($2\alpha=90^\circ$, $Y/D=0$)

position indicates the scale of the vortex region behind the cone. The position shifts downstream as the vertex angle decreases. In the case of the cone of vertex angle 150° , the peak is found on the cone surface, not on the flat plate, and the pressure decreases gradually in the X-direction from the peak.

3.3. Separation of flow and vortex formation

In the previous section, the necklace-vortices formed in front of the cone and the horse-shoe vortices behind it were examined using the surface-pressure distributions on the cone and the flat plate. In this section, these vortices are discussed using the results of the flow visualization in the water tank.

Before proceeding, it is necessary to refer to the results of the wind tunnel ex-

Fig.10 Vortex region

Fig.11 Separation lines observed in water tank ($2\alpha=60^\circ$)

Fig.12 Separation lines observed in water tank ($2\alpha=150^\circ$)

periment. Fig.9 shows the velocity vectors which combine the results of flow direction with the resultant velocity in the wind tunnel. From the results of the surface-pressure, the flow in XZ-plane can be indicated as in Fig.10. The positive pressure gradient occurs in front of the cone. Therefore, the boundary-layer flow separates from the flat plate in front of the cone and forms necklace-vortices. These have axes of rotation parallel to the flow and they move downstream. Also, it is found from Fig.9 that, on a time-average basis, a closed vortex region is formed behind the cone.

Figs.11 and 12 show the flow pattern on the cone surface and the flat plate in the water tank. The separation lines on the cone surface and on the flat plate in front of the cone are clearly shown in these figures. Although the Reynolds number in the water tank experiment ($Re = 9.7 \times 10^3$) is different from that in the wind tunnel ($Re = 6.7 \times 10^4$), the behavior of the vortex formation in the

Fig.13 Two necklace-vortices observed in water tank ($2\alpha = 60^\circ$)

Fig.14 Schematic flow pattern around cone

Fig.15 Local drag coefficient

wind tunnel is considered to be practically the same as that in the water tank. Fig.13 shows the presence of two necklace-vortices for the cone of vertex angle 60° . The two necklace-vortices have axes which rotate in the same direction. Therefore, the two necklace-vortices begin to coil around each other in the vicinity of the cone sides. The vortices decay gradually and increase velocity in the wake of the cone as described in the later section.

From the results examined up to now, the flow pattern around the cone for vertex angles 60° and 90° can be drawn schematically as shown in Fig.14. In the case of the cone for vertex angle 120° , the necklace-vortex is single and weak, and dissipates behind the cone. Also, there are no necklace-vortices for the cone of vertex angle 150° .

When the vertex angle of the cone becomes small, the vortices, which separate from the cone surface and shed downstream, begin to shed alternately from the both sides of the cone.⁽⁵⁾⁽⁶⁾ But, in the present experiment for $2\alpha \geq 60^\circ$, the vortices separate simultaneously from the both sides of the cone and form horse-shoe vortices. The precise degree of the vertex angle from which vortices begin to shed alternately is a subject for future research.

3.4. Drag and lift coefficients

Fig.16 Local lift coefficient

Fig.17 Drag and lift coefficients

Fig. 22 Velocity defect of wake flow of cone

Fig. 23 Variation of velocity increase by "negative wake effect" with x -axis

defect is smaller than that of Clauser shows the acceleration of the flow by the "negative wake effect". Also, it is found from these results that the region of the increased velocity is limited to a region in which the universal velocity-distribution law holds.

In order to find the variation of the velocity increase in the flow direction for each cone, the difference between $(U-u)/u_\tau$ of velocity defect at the center plane (XZ -plane) of the flow and that of the two-dimensional turbulent boundary-layer was considered. Fig. 23 shows the difference of velocity defect plotted against a non-dimensional abscissa $X/D / \cot \alpha (=X/2H)$. The variation of the velocity increase is expressed by a single curve independent of the vertex angle of the cone and it has a maximum value at $X/2H \approx 6$. Also, the boundary-layer in the center of the wake returns very gradually to the two-dimensional turbulent boundary-layer with an increasing $X/2H$.

4. Conclusions

The results of this study are summarized as follows:

- (1) The surface-pressure distributions on the cone and the flat plate are similar for cones of vertex angles $60^\circ \sim 120^\circ$, but are different from those for vertex angle 150° . This is because the flow separation and vortices are different.
- (2) For the cones of vertex angles $60^\circ \sim 120^\circ$, there are necklace-vortices and horse-shoe vortices formed by the separated flow on the cone surface shed periodically. The ends of the shed horse-shoe vortices occur on the flat plate behind the cone. For the cone of vertex angle 150° , there is no necklace-vortex, and the horse-shoe vortices shed from the cone top.
- (3) The separation points of the cones are obtained by the oil-film method.
- (4) The drag and lift coefficients are obtained from the surface-pressure distributions. The drag coefficient decreases as the vertex angle increases and approaches that of a circular cylinder of finite length as the vertex angle decreases. Also, the lift force acts on the cone upward.
- (5) In the wake of the cone on a flat plate, there is a "negative wake effect": that is, the velocity of the wake is accelerated by the downwash of the free stream and by the secondary flow. In estimating the velocity defect of the wake by Clauser's method, the velocity increase of the wake is maximum at $X/2H \approx 6$ independently of the vertex angle of the cone.

References

- (1) Schlichting, H., Ing. Arch., Vol. 7 (1936), p. 1.
- (2) Sato, H. and Onda, Y., J. Japan Soc. aeron. space Sci. (in Japanese) Vol. 22, No. 245 (1974-6), p. 277.
- (3) Clauser, F., J. Aero. Sci., Vol. 21 (1954), p. 91.
- (4) Okamoto, T., et al., Preprint of Japan Soc. Mech. Engrs. (in Japanese) (Lecture Meeting of Hokuriku-Shinetsu branch of JSME), (1972-10), p. 113.
- (5) Taneda, S., J. Phys. Soc. Japan, Vol. 20 (1965), p. 1714.
- (6) Gaster, M., J. Fluid Mech., Vol. 38 (1969), p. 565.
- (7) Okamoto, T. and Yagita, M., Bull. JSME, Vol. 16, No. 95 (1973), p. 805.
- (8) Schlichting, H., Boundary-Layer Theory, Pergamon Press, (1968), p. 614.