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Dynamic Analysis of a Plate Heat Exchanger System*

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This paper presents a systematic analysis of static and dynamic characteristics of plate heat exchangers having a number of heat transfer plates. There are various flow types in a plate heat exchanger system. The results are numerically compared under the condition that each flow rate of the hot- and cold-side fluids remains constant, respectively in any flow type. The significant results are : 1) In almost every flow type, the temperature effectiveness increases as the number of passages increases. 2) Dynamic responses may greatly depend on the relation between the inlet and outlet passages where each fluid flows in a counter- or a parallel current. 3) Series flow types where the inlet and outlet passages are adjacent and the flow is a countercurrent have the best statics and dynamics among all types when the number of passages is the same.

this heat exchanger system.

1. Introduction

Dynamic analysis and control of various kinds of heat exchangers have been discussed so far in the literature.^{(1)~(4)} And plate heat exchangers are those which exchange heat energy in the hot and cold fluids flowing through every two passages formed by heat transfer plates, and are used extensively in such industries as mechanical, chemical and food industries since they have the following principal advantages. 1) The film coefficient of heat transfer is very high and the heat exchanger itself can be made compact. 2) Heat transfer plates can be standardized and plate heat exchangers have an advantage in mass production. 3) Both sides of the plates are fully accessible for cleaning and inspection. However, for these plate heat exchangers only the researches on the heat transfer characteristics,^{(5)~(8)} on designs of the new type plate heat exchangers,⁽⁹⁾ and on steady state characteristics^{(5), (10)} have been published and there have been few on dynamic analysis. Recently the authors⁽¹¹⁾ derived the basic equations for the plate heat exchangers, and analysed systematically the statics and dynamics on the most fundamental case of four passage types and also made experiments with a favorable agreement between the theoretical and experimental results on the two typical flow types.

This paper presents a systematic analysis of the statics and dynamics of a plate heat exchanger system having a number of heat transfer plates, and suggests available design and control schemes for

2. Flow pattern

Plate heat exchangers are formed by a stack of heat transfer plates as shown in Fig.1. According to the combination and the number of heat transfer plates with a hole or a blank suitably placed at each corner, there may be various flow types constructed in a plate heat exchanger system. These flow types can be classified into three categories depending on the condition whether fluids split or not. They are Series flow pattern, Parallel flow pattern, and Complex flow pattern as shown in Fig.2.

3. Derivation of basic equation

The notations used in this paper are as follows. α ($\propto v^v$, v : exponent) : film coefficient of heat transfer [$\text{kcal/m}^2\text{hr}^\circ\text{C}$], θ : temperature or a vector representing temperatures of fluids or plates [$^\circ\text{C}$], t : time [hr], v : fluid velocity [m/hr], w : heat capacity per unit length along the flow = $C_p \gamma S$ [$\text{kcal/m}^\circ\text{C}$] (C_p : specific heat [$\text{kcal/kg}^\circ\text{C}$], γ : density [kg/m^3]), x : distance [m], A : heat transfer area= LW [m^2], F : flow rate [m^3/hr], L : length of passage [m], S : cross-sectional area [m^2], T : residence time of the hot-side fluid in Series flow pattern = L/v_1 [hr], W : heat transfer area per unit length along the flow [m^2/m] Subscripts c: cold-side fluid, h: hot-side fluid, i: inlet, m: m-th passage or plate, o: outlet, p: heat transfer plate, 1: the first fluid, 2: the second fluid Nondimensional terms $\mu = w_h/w_c$, $\mu_p = w_h/w_p$, $\tau =$

Fig.1 Heat transfer plate (with a hole at each corner)

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t/T , $\xi=x/L$, $a'=\alpha' A/wv$, $b'=\alpha' A/w_p v_1$, $r=v_1/v_2=F_1/F_2$ ($v_1=v_3=\dots$, $v_2=v_4=\dots$), S : Laplace transform operator for τ

The following assumptions usually used and justified in most of the researches⁽¹⁾,⁽⁵⁾,⁽⁸⁾,⁽¹⁰⁾,⁽¹¹⁾ are made. 1) The heat loss to the surroundings is negligible. This assumes that the two end plates serve as insulating walls. 2) There is no heat conduction in the direction of fluid flow in the plates and the fluids themselves. 3) The temperatures and flow rates are uniform across the flow passage.⁽⁸⁾ 4) The film coefficient of heat transfer is greatly dependent on the fluid velocity and is proportional to an exponential function of the fluid velocity or the flow rate.^{(5)~(8)},⁽¹¹⁾ 5) Except in the case of series flow pattern, the fluids will split equally between passages.

Under the above assumptions, an application of the energy equation to the control volume shown in Fig. 3 yields Eq. (1).

$$\left. \begin{aligned} \frac{\partial \theta_m}{\partial t} \pm v_m \frac{\partial \theta_m}{\partial \xi} &= \frac{\alpha_m W}{w_m} (\theta_{pm} - \theta_m) + \frac{\alpha_m W}{w_m} (\theta_{p,m+1} - \theta_m) \\ \frac{\partial \theta_{pm}}{\partial t} &= \frac{\alpha_{m-1} W}{w_p} (\theta_{m-1} - \theta_{pm}) + \frac{\alpha_m W}{w_p} (\theta_m - \theta_{pm}) \end{aligned} \right\} \dots (1)$$

'+' in the double sign means the fluid flow in the direction shown in Fig. 3, while '-' does that in the opposite direction. The hot- and the cold-side fluid flow through odd- and even-numbered passages, respectively and for v_m and w_m , $v_1=v_3=\dots$, $v_2=v_4=\dots$, $w_1=w_3=\dots$, $w_2=w_4=\dots$.

To generalize the expression (1), the nondimensional terms should be substituted and subsequent equations are as follows.

$$\left. \begin{aligned} \frac{v_1}{v_m} \frac{\partial \theta_m}{\partial \tau} \pm \frac{\partial \theta_m}{\partial \xi} &= a_m (\theta_{pm} - \theta_m) + a_m (\theta_{p,m+1} - \theta_m) \\ \frac{\partial \theta_{pm}}{\partial \tau} &= b_{m-1} (\theta_{m-1} - \theta_{pm}) + b_m (\theta_m - \theta_{pm}) \end{aligned} \right\} \dots (2)$$

where $a_m = \alpha_m A / w_m v_m$, $b_m = \alpha_m A / w_p v_1$ and

$$\frac{v_1}{v_m} = \begin{cases} 1 & m:\text{odd} \\ r & m:\text{even} \end{cases}$$

The basic equations of any flow type can be expressed by Eq. (2) though various flow types exist as shown in Fig. 2. For example, the basic equations of a flow type S4a are given with the boundary conditions. For fluid temperatures:

$$\left. \begin{aligned} \frac{\partial \theta_{11}}{\partial \tau} + \frac{\partial \theta_{11}}{\partial \xi} &= a_1 (\theta_{p1} - \theta_{11}) + a_1 (\theta_{p2} - \theta_{11}) \\ r \frac{\partial \theta_{12}}{\partial \tau} + \frac{\partial \theta_{12}}{\partial \xi} &= a_2 (\theta_{p2} - \theta_{12}) + a_2 (\theta_{p3} - \theta_{12}) \\ \frac{\partial \theta_{21}}{\partial \tau} - \frac{\partial \theta_{21}}{\partial \xi} &= a_1 (\theta_{p3} - \theta_{21}) + a_1 (\theta_{p4} - \theta_{21}) \\ r \frac{\partial \theta_{22}}{\partial \tau} - \frac{\partial \theta_{22}}{\partial \xi} &= a_2 (\theta_{p4} - \theta_{22}) + a_2 (\theta_{p5} - \theta_{22}) \end{aligned} \right\} \dots (3)$$

For plate temperatures:

Fig. 3 Control volume used to derive the energy equation

Fig. 2 Flow patterns of a plate heat exchanger system

$$\left. \begin{aligned} \frac{\partial \theta_{p1}}{\partial \tau} &= b_1(\theta_1 - \theta_{p1}) \\ \frac{\partial \theta_{p2}}{\partial \tau} &= b_1(\theta_1 - \theta_{p2}) + b_2(\theta_2 - \theta_{p2}) \\ \frac{\partial \theta_{p3}}{\partial \tau} &= b_2(\theta_2 - \theta_{p3}) + b_1(\theta_3 - \theta_{p3}) \\ \frac{\partial \theta_{p4}}{\partial \tau} &= b_1(\theta_3 - \theta_{p4}) + b_2(\theta_4 - \theta_{p4}) \\ \frac{\partial \theta_{p5}}{\partial \tau} &= b_2(\theta_4 - \theta_{p5}) \end{aligned} \right\} \dots\dots\dots (4)$$

For boundary conditions :

$$\left. \begin{aligned} \theta_1(0, \tau) &= \theta_{s1}(\tau), & \theta_2(0, \tau) &= \theta_{s2}(\tau) \\ \theta_1(1, \tau) &= \theta_{s1}(1, \tau), & \theta_2(1, \tau) &= \theta_{s2}(1, \tau) \end{aligned} \right\} \dots\dots\dots (5)$$

where

$$\left. \begin{aligned} a_1 &= \alpha_1 A / w_1 v_1, & a_2 &= \alpha_2 A / w_2 v_2, & b_1 &= \alpha_1 A / w_p v_1 \\ b_2 &= \alpha_2 A / w_p v_1, & r &= v_1 / v_2 \end{aligned} \right\} \dots\dots\dots (6)$$

4. Dynamic analysis

4.1 Dynamic analysis approach

The basic equations and boundary conditions of any flow type can be generally expressed as Eqs. (7) and (8) using vectors and matrices.

For basic equation :

$$V \frac{\partial \theta}{\partial \xi} + C \frac{\partial \theta}{\partial \tau} + H \theta = 0 \dots\dots\dots (7)$$

where θ : a vector representing temperatures of fluids and/or plates, V : a matrix representing axial transport, C : a matrix representing the heat capacity, and H : a matrix representing the heat transfer between various components of a vector θ .

For boundary condition :

$$P_1 \theta(0, \tau) + P_2 \theta(1, \tau) = \theta_i(\tau) \dots\dots\dots (8)^*$$

where $\theta_i(\tau)$ is a vector whose components are inlet fluid temperatures.

Now a dynamic analysis approach is shown to give frequency responses for the change of inlet fluid temperatures. Since the basic equations are linear with respect to temperature variables, initial temperature distributions can be put zero without loss of generality when small deviations from an equilibrium state are considered. Laplace-transforming Eqs. (7) and (8) with respect to τ with $\theta(\xi, 0) = 0$ yields Eqs. (9) and (10).

$$V \frac{d}{d\xi} \theta(\xi, s) + (sC + H) \theta(\xi, s) = 0 \dots\dots\dots (9)$$

$$P_1 \theta(0, s) + P_2 \theta(1, s) = \theta_i(s) \dots\dots\dots (10)$$

And therefore,

$$\frac{d\theta}{d\xi} = F\theta, \quad \theta(0, s) = ? \dots\dots\dots (11)^{**}$$

where $F = -V^{-1}(sC + H)$.

* The components of matrices P_1 and P_2 are in general arranged arbitrarily but should be determined in such a way as the inverse in Eq. (13) exists.

** The notation F is defined as flow rate in Chpt.3 but the usage is always clear from context. And also Eq. (9) should be reduced to the expression without heat transfer plate temperatures by describing them as linear combinations of fluid temperatures.

Solving Eq. (11) with Eq. (10) leads in general to a two-point boundary-value problem.

As the system considered is fortunately linear, a fundamental solution matrix of Eq. (11) may be formally written as $e^{F\xi}$, which can be easily obtained with a fourth-order Runge-Kutta-Gill integration routine. Therefore, Eq. (11) can be formally solved as follows.

$$\theta(\xi, s) = e^{F\xi} \theta(0, s) \dots\dots\dots (12)$$

Substituting Eq. (12) into Eq. (10) and solving Eq. (10) with respect to $\theta(0, s)$ yield Eq. (13) as follows.

$$\left. \begin{aligned} P_1 \theta(0, s) + P_2 e^{F} \theta(0, s) &= \theta_i(s) \\ \theta(0, s) &= (P_1 + P_2 e^F)^{-1} \theta_i(s) \end{aligned} \right\} \dots\dots\dots (13)$$

Substituting Eq. (13) into Eq. (12) yields a complete solution of Eq. (11) as follows.

$$\theta(\xi, s) = e^{F\xi} (P_1 + P_2 e^F)^{-1} \theta_i(s) \dots\dots\dots (14)$$

When a transfer function matrix for the inlet temperatures of hot- and cold-side fluids is referred to $G(\xi, s)$, Eq. (14) can also be written as the following expression.

$$\theta(\xi, s) = G(\xi, s) \begin{bmatrix} \theta_{s1}(s) \\ \theta_{s2}(s) \end{bmatrix} \dots\dots\dots (15)$$

Therefore, from Eqs. (14) and (15) a column of the transfer function matrix $G(\xi, s)$ can be obtained as a column or a sum of the columns in the matrix $e^{F\xi} (P_1 + P_2 e^F)^{-1}$ corresponding to one or some of the inlet passages.***

As the basic equation is linear with respect to the temperatures, the above Eq. (15) also gives nondimensional steady state temperature profiles when $s=0$. Namely, since it is clear that

$$G_{cc} = I - G_{ss} \dots\dots\dots (16)$$

with only $s=0$,

$$\begin{aligned} \theta_c &= G_{cs}(\xi, 0) \theta_{s1} + G_{cc}(\xi, 0) \theta_{c1} \\ &= G_{cs}(\xi, 0) \theta_{s1} + (I - G_{ss}(\xi, 0)) \theta_{c1} \\ \theta_c - \theta_{c1} &= G_{cs}(\xi, 0) (\theta_{s1} - \theta_{c1}) \end{aligned}$$

Therefore,

$$\frac{\theta_c(\xi, 0) - \theta_{c1}}{\theta_{s1} - \theta_{c1}} = G_{cs}(\xi, 0) \dots\dots\dots (17)$$

Furthermore, from the definition a temperature effectiveness with respect to the cold-side fluid is obtained as follows.

$$\phi = \frac{\theta_{cs} - \theta_{c1}}{\theta_{s1} - \theta_{c1}} = G_{cs}(0, 0), \text{ or } G_{cs}(1, 0) \dots\dots\dots (18)$$

As mentioned above the characteristics such as the statics (steady state temperature profiles, temperature effectiveness, and so on) and the dynamics is always derived from the transfer function matrix. Therefore, any flow type can be formulated as Eqs. (9) and (10) in a plate heat exchanger system and can be solved as Eq. (14). And then a transfer

*** In Series flow pattern a column of the transfer function matrix is a column of the matrix $e^{F\xi} (P_1 + P_2 e^F)^{-1}$ corresponding to an inlet passage, while in Parallel and Complex flow patterns a column is a sum of the columns in $e^{F\xi} (P_1 + P_2 e^F)^{-1}$ corresponding to the inlet passages.

function matrix can be obtained, which may lead to the heat transfer characteristics.

4.2 Series flow pattern

As an example of this flow pattern, S4a in Fig.2(a) will be analysed. First, Laplace-transforming Eqs.(3) - (5) with respect to τ and subtracting all the plate temperatures yield the following equation corresponding to Eq.(11).

$$\frac{d}{d\xi} \begin{bmatrix} \theta_1(\xi, s) \\ \theta_2(\xi, s) \\ \theta_3(\xi, s) \\ \theta_4(\xi, s) \end{bmatrix} = \begin{bmatrix} -f_{11} & g_{12} & 0 & 0 \\ g_{21} & -f_{22} & g_{23} & 0 \\ 0 & -g_{12} & f_{33} & -g_{12} \\ 0 & 0 & -g_{21} & f_{44} \end{bmatrix} \begin{bmatrix} \theta_1(\xi, s) \\ \theta_2(\xi, s) \\ \theta_3(\xi, s) \\ \theta_4(\xi, s) \end{bmatrix}$$

$$\theta(0, s) = \begin{bmatrix} \theta_{h1}(s) \\ \theta_{c1}(s) \\ \theta_3(0, s) \\ \theta_4(0, s) \end{bmatrix} \dots \dots \dots (19)$$

where

$$f_{11} = s + \frac{a_1 s}{s+b_1} + \frac{a_1(s+b_2)}{s+b_1+b_2}$$

$$f_{22} = rs + \frac{2a_2(s+b_1)}{s+b_1+b_2}, \quad f_{33} = s + \frac{2a_1(s+b_2)}{s+b_1+b_2}$$

$$f_{44} = rs + \frac{a_2(s+b_1)}{s+b_1+b_2} + \frac{a_2 s}{s+b_2}$$

$$g_{12} = \frac{a_1 b_2}{s+b_1+b_2}, \quad g_{21} = \frac{a_2 b_1}{s+b_1+b_2}$$

And boundary conditions can be written as follows corresponding to Eq.(10).

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1(0, s) \\ \theta_2(0, s) \\ \theta_3(0, s) \\ \theta_4(0, s) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1(1, s) \\ \theta_2(1, s) \\ \theta_3(1, s) \\ \theta_4(1, s) \end{bmatrix} = \begin{bmatrix} \theta_{h1}(s) \\ \theta_{c1}(s) \\ 0 \\ 0 \end{bmatrix} \dots \dots \dots (20)$$

The components of two matrices P_1 and P_2 in Eq.(20) may be arranged arbitrarily but will be determined by the following rules. These rules can be applied to Parallel and Complex flow patterns. 1) First, a boundary condition with an inlet fluid temperature is arranged as a diagonal component which has a positive sign. 2) When a row of one matrix is determined by a boundary condition, the corresponding row of the other is put as a zero vector. According to these rules, it is clearly avoided from Eq.(13) that arbitrary rows of P_1 and P_2 may be zero vectors at the same time.

Next, in consideration of Eq. (20) a transfer function matrix can be obtained as the first column and the second one of the matrix in Eq. (14). Since dynamic responses will be obtained as frequency responses in this paper, putting $s=j\omega$ ($j=\sqrt{-1}$) and solving Eq. (19) with 1 as the i -th component of initial values and with zero as the remaining ones yield the i -th column of a fundamental solution matrix ϵ^{F_i} and all the columns of ϵ^{F_i} are obtained with i varied. The first column and the second one of this matrix will be the frequency transfer function matrix to be obtained.

4.3 Parallel flow pattern

The additional assumptions are made to compare the characteristics among the flow

patterns. 1) Total flow rates of the hot- and cold-side fluids are equal to those of Series flow pattern, respectively and the fluids will split equally between passages. 2) The value of film coefficient of heat transfer will be one corresponding to the flow rate of each passage, unlike that of Series flow pattern and may be distinguished by a prime.

The basic equations are obtained as Eq. (21) and then Eq. (22).

$$\left. \begin{aligned} \frac{\partial \theta_m}{\partial t} \pm \frac{v_m}{N} \frac{\partial \theta_m}{\partial x} &= \frac{\alpha_m' W}{w_m} (\theta_{pm} - \theta_m) + \frac{\alpha_m' W}{w_m} (\theta_{pm+1} - \theta_m) \\ \frac{\partial \theta_{pm}}{\partial t} &= \frac{\alpha_{m-1}' W}{w_p} (\theta_{m-1} - \theta_{pm}) + \frac{\alpha_m' W}{w_p} (\theta_m - \theta_{pm}) \end{aligned} \right\} \dots \dots (21)$$

$$\left. \begin{aligned} N \frac{v_m}{v_m} \frac{\partial \theta_m}{\partial \tau} \pm \frac{\partial \theta_m}{\partial \xi} &= Na_m' (\theta_{pm} - \theta_m) + Na_m' (\theta_{pm+1} - \theta_m) \\ \frac{\partial \theta_{pm}}{\partial \tau} &= b_{m-1}' (\theta_{m-1} - \theta_{pm}) + b_m' (\theta_m - \theta_{pm}) \end{aligned} \right\} \dots \dots (22)$$

where v_m is the value in the case of Series flow pattern, and N is the number of split streams (if the number of passages is odd, the number of cold-side split streams is by one less than that of hot-side split streams), and the nondimensional term with respect to time is the same as Series flow pattern.

As an example of this flow pattern, P4b in Fig.2(b) will be explained.

For fluid temperatures :

$$\left. \begin{aligned} 2 \frac{\partial \theta_1}{\partial \tau} + \frac{\partial \theta_1}{\partial \xi} &= 2a_1' (\theta_{p1} - \theta_1) + 2a_1' (\theta_{p2} - \theta_1) \\ 2r \frac{\partial \theta_2}{\partial \tau} - \frac{\partial \theta_2}{\partial \xi} &= 2a_2' (\theta_{p2} - \theta_2) + 2a_2' (\theta_{p3} - \theta_2) \\ 2 \frac{\partial \theta_3}{\partial \tau} + \frac{\partial \theta_3}{\partial \xi} &= 2a_1' (\theta_{p3} - \theta_3) + 2a_1' (\theta_{p4} - \theta_3) \\ 2r \frac{\partial \theta_4}{\partial \tau} - \frac{\partial \theta_4}{\partial \xi} &= 2a_2' (\theta_{p4} - \theta_4) + 2a_2' (\theta_{p5} - \theta_4) \end{aligned} \right\} \dots \dots (23)$$

For plate temperatures :

$$\left. \begin{aligned} \frac{\partial \theta_{p1}}{\partial \tau} &= b_1' (\theta_1 - \theta_{p1}) \\ \frac{\partial \theta_{p2}}{\partial \tau} &= b_1' (\theta_1 - \theta_{p2}) + b_2' (\theta_2 - \theta_{p2}) \\ \frac{\partial \theta_{p3}}{\partial \tau} &= b_2' (\theta_2 - \theta_{p3}) + b_1' (\theta_3 - \theta_{p3}) \\ \frac{\partial \theta_{p4}}{\partial \tau} &= b_1' (\theta_3 - \theta_{p4}) + b_2' (\theta_4 - \theta_{p4}) \\ \frac{\partial \theta_{p5}}{\partial \tau} &= b_2' (\theta_4 - \theta_{p5}) \end{aligned} \right\} \dots \dots (24)$$

For boundary conditions :

$$\left. \begin{aligned} \theta_1(0, \tau) &= \theta_{h1}(\tau), \quad \theta_3(0, \tau) = \theta_{c1}(\tau) \\ \theta_2(1, \tau) &= \theta_{c1}(\tau), \quad \theta_4(1, \tau) = \theta_{c1}(\tau) \end{aligned} \right\} \dots \dots (25)$$

The dynamic analysis approach will be also applied to this flow type. The temperature at a confluence may be considered to be equal to the average of fluid temperatures.

The equations corresponding to Eqs.(11) and (10) are as follows.

$$\frac{d}{d\xi} \begin{bmatrix} \theta_1(\xi, s) \\ \theta_2(\xi, s) \\ \theta_3(\xi, s) \\ \theta_4(\xi, s) \end{bmatrix} = \begin{bmatrix} -f_{11} & g_{12} & 0 & 0 \\ -g_{21} & f_{22} & -g_{21} & 0 \\ 0 & g_{12} & -f_{33} & g_{12} \\ 0 & 0 & -g_{21} & f_{44} \end{bmatrix} \begin{bmatrix} \theta_1(\xi, s) \\ \theta_2(\xi, s) \\ \theta_3(\xi, s) \\ \theta_4(\xi, s) \end{bmatrix}$$

$$\theta(0, s) = \begin{bmatrix} \theta_{h1}(s) \\ \theta_2(0, s) \\ \theta_{h1}(s) \\ \theta_4(0, s) \end{bmatrix} \dots \dots \dots (26)$$

where

$$\begin{aligned}
 f_{11} &= 2 \left(s + \frac{a_1' s}{s+b_1'} + \frac{a_1' (s+b_2')}{s+b_1'+b_2'} \right) \\
 f_{22} &= 2 \left(rs + \frac{2a_2' (s+b_1')}{s+b_1'+b_2'} \right), \quad f_{33} = 2 \left(s + \frac{2a_1' (s+b_2')}{s+b_1'+b_2'} \right) \\
 f_{44} &= 2 \left(rs + \frac{a_2' (s+b_1')}{s+b_1'+b_2'} + \frac{a_2' s}{s+b_2'} \right) \\
 g_{12} &= \frac{2a_1' b_2'}{s+b_1'+b_2'}, \quad g_{21} = \frac{2a_2' b_1'}{s+b_1'+b_2'} \\
 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1(0, s) \\ \theta_2(0, s) \\ \theta_3(0, s) \\ \theta_4(0, s) \end{bmatrix} \\
 + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1(1, s) \\ \theta_2(1, s) \\ \theta_3(1, s) \\ \theta_4(1, s) \end{bmatrix} &= \begin{bmatrix} \theta_{h1}(s) \\ \theta_{e1}(s) \\ \theta_{h1}(s) \\ \theta_{e1}(s) \end{bmatrix} \dots (27)
 \end{aligned}$$

The transfer function matrix will be composed by the sum of the first and the third columns, and the sum of the second and the fourth columns in Eq. (14) as this flow type has two inlet passages for the hot- and the cold-side fluids, respectively. The responses of the cold-side fluid at the outlet for the inlet temperature of the hot-side fluid may be equal to the average of the temperatures as there are two outlet passages.

It is also noted that for P.a in Fig. 2 (a) all the initial values of $\theta(0, s)$ can always be known, which does not lead to a two-point boundary-value problem and therefore $(P_1 + P_2 e^T)^{-1}$ does not have to be calculated.

4.4 Complex flow pattern

The basic equations of this flow pattern can always be derived by combining the equations in Series and Parallel flow patterns, and the additional assumptions are the same as those in Parallel flow pattern.

As an example of this flow pattern, C8c in Fig. 2(c) will be analysed. The equations corresponding to Eqs. (11) and (10) are as follows.

$$\frac{d}{d\xi} \begin{bmatrix} \theta_1(\xi, s) \\ \theta_2(\xi, s) \\ \theta_3(\xi, s) \\ \theta_4(\xi, s) \\ \theta_5(\xi, s) \\ \theta_6(\xi, s) \\ \theta_7(\xi, s) \\ \theta_8(\xi, s) \end{bmatrix} = \begin{bmatrix} -f_{11} & g_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ g_{21} & -f_{22} & g_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & g_{12} & -f_{33} & g_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & g_{21} & -f_{22} & g_{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & g_{12} & -f_{33} & g_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & -g_{21} & f_{22} & -g_{21} & 0 \\ 0 & 0 & 0 & 0 & 0 & g_{12} & -f_{33} & g_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & -g_{21} & f_{33} \end{bmatrix} \begin{bmatrix} \theta_1(\xi, s) \\ \theta_2(\xi, s) \\ \theta_3(\xi, s) \\ \theta_4(\xi, s) \\ \theta_5(\xi, s) \\ \theta_6(\xi, s) \\ \theta_7(\xi, s) \\ \theta_8(\xi, s) \end{bmatrix} \quad \theta(0, s) = \begin{bmatrix} \theta_{h1}(s) \\ \theta_{e1}(s) \\ \theta_{h1}(s) \\ \theta_{e1}(s) \\ \theta_{h1}(s) \\ \theta_{e1}(s) \\ \theta_{h1}(s) \\ \theta_{e1}(s) \end{bmatrix} \dots (28)$$

where

$$\begin{aligned}
 f_{11} &= 4 \left(s + \frac{a_1' s}{s+b_1'} + \frac{a_1' (s+b_2')}{s+b_1'+b_2'} \right), \quad f_{22} = 2 \left(rs + \frac{2a_2' (s+b_1')}{s+b_1'+b_2'} \right), \quad f_{33} = 4 \left(s + \frac{2a_1' (s+b_2')}{s+b_1'+b_2'} \right) \\
 f_{33} &= 2 \left(rs + \frac{a_2' (s+b_1')}{s+b_1'+b_2'} + \frac{a_2' s}{s+b_2'} \right), \quad g_{12} = \frac{4a_1' b_2'}{s+b_1'+b_2'}, \quad g_{21} = \frac{2a_2' b_1'}{s+b_1'+b_2'}
 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1(0, s) \\ \theta_2(0, s) \\ \theta_3(0, s) \\ \theta_4(0, s) \\ \theta_5(0, s) \\ \theta_6(0, s) \\ \theta_7(0, s) \\ \theta_8(0, s) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1(1, s) \\ \theta_2(1, s) \\ \theta_3(1, s) \\ \theta_4(1, s) \\ \theta_5(1, s) \\ \theta_6(1, s) \\ \theta_7(1, s) \\ \theta_8(1, s) \end{bmatrix} = \begin{bmatrix} \theta_{h1}(s) \\ \theta_{e1}(s) \\ \theta_{h1}(s) \\ \theta_{e1}(s) \\ \theta_{h1}(s) \\ 0 \\ \theta_{h1}(s) \\ 0 \end{bmatrix} \dots (29)$$

In this case, the transfer function matrix will be composed by the sum of four odd-numbered columns, and the sum of the second and the fourth columns in Eq.(14) because this flow type has four inlet passages for the hot-side fluid and two inlet passages for the cold-side fluid, respectively. The response of the cold-side fluid at the outlet for the inlet temperature of the hot-side fluid may be equal to the average of the temperatures since there are two outlet passages.

5. Comparison of characteristics among flow types based on numerical results

Since any flow type can be analysed using the approach in the previous chapter, the statics and dynamics of each flow type may be numerically investigated when the heat exchangers are used as heaters and each flow rate of the hot- and cold-side fluids remains constant, respectively. The nondimensional parameters are given in Table 1 and the numerical results with these parameters are shown in Table 2 or Figs. 4 and 5.

Table 2 shows temperature effectiveness with respect to the cold-side fluid for any flow type and for any number of flow passages (which is related to heat transfer area). It is clear that except S.a and P.a types temperature effectiveness of any flow type increases with the number of passages and that among all flow types S.d has the largest value of temperature effectiveness for the same number of passages. It is obvious that temperature effectiveness of P.a also increases with odd- or even-numbered passages and that since S4a has the largest value of temperature effectiveness among S.a, this type S.a should not be used with more than five passages.

Fig.4 shows steady state fluid temperature profiles of S8a, S8d, and C8c. Fig.4(a) shows that for S8a the temperature differ-

Table 1 Nondimensional parameters

		$a_1^{(r)}$	$a_2^{(r)}$	$b_1^{(r)}$	$b_2^{(r)}$
S	2~8	a_1	$\mu r^{1-\nu} a_1$	$\mu_p a_1$	$\mu_p r^{-\nu} a_1$
	3	$\frac{1}{2^v} a_1$	$\frac{1}{2^v} \mu r^{1-\nu} a_1$	$\frac{1}{2^v} \mu_p a_1$	$\frac{1}{2^v} \mu_p r^{-\nu} a_1$
P	4	$\frac{1}{2^v} a_1$	$\frac{1}{2^v} \mu r^{1-\nu} a_1$	$\frac{1}{2^v} \mu_p a_1$	$\frac{1}{2^v} \mu_p r^{-\nu} a_1$
	5	$\frac{1}{3^v} a_1$	$\frac{1}{3^v} \mu r^{1-\nu} a_1$	$\frac{1}{3^v} \mu_p a_1$	$\frac{1}{3^v} \mu_p r^{-\nu} a_1$
	6	$\frac{1}{3^v} a_1$	$\frac{1}{3^v} \mu r^{1-\nu} a_1$	$\frac{1}{3^v} \mu_p a_1$	$\frac{1}{3^v} \mu_p r^{-\nu} a_1$
	7	$\frac{1}{4^v} a_1$	$\frac{1}{4^v} \mu r^{1-\nu} a_1$	$\frac{1}{4^v} \mu_p a_1$	$\frac{1}{4^v} \mu_p r^{-\nu} a_1$
	8	$\frac{1}{4^v} a_1$	$\frac{1}{4^v} \mu r^{1-\nu} a_1$	$\frac{1}{4^v} \mu_p a_1$	$\frac{1}{4^v} \mu_p r^{-\nu} a_1$
C	3	$\frac{1}{2^v} a_1$	$\mu r^{1-\nu} a_1$	$\frac{1}{2^v} \mu_p a_1$	$\mu_p r^{-\nu} a_1$
	4	$\frac{1}{2^v} a_1$		$\frac{1}{2^v} \mu_p a_1$	
	5	$\frac{1}{3^v} a_1$		$\frac{1}{3^v} \mu_p a_1$	
	6	$\frac{1}{3^v} a_1$		$\frac{1}{3^v} \mu_p a_1$	
	7	$\frac{1}{4^v} a_1$		$\frac{1}{4^v} \mu_p a_1$	
	8ab	$\frac{1}{4^v} a_1$		$\frac{1}{4^v} \mu_p a_1$	
	8cd	$\frac{1}{2^v} a_1$	$\frac{1}{2^v} \mu r^{1-\nu} a_1$	$\frac{1}{2^v} \mu_p a_1$	$\frac{1}{2^v} \mu_p r^{-\nu} a_1$
	8cf	$\frac{1}{2^v} a_1$	$\frac{1}{2^v} \mu r^{1-\nu} a_1$	$\frac{1}{2^v} \mu_p a_1$	$\frac{1}{2^v} \mu_p r^{-\nu} a_1$

where $a_1=1.0$, $r=1.5$, $\mu=1.0$,
 $\mu=3.0$, $\nu=0.7$ used in this
 numerical analysis

Table 2 Numerical examples of statics (temperature effectiveness) for some flow types

	S		P		C	
	S-a	S-d	P-a	P-b	C-a	C-b
2	0.395	0.418	—	—	—	—
3	0.569	0.568	0.490	0.547	0.490	0.547
4	0.616	0.666	0.487	0.564	0.573	0.610
5	0.611	0.734	0.528	0.628	0.616	0.616
6	0.605	0.787	0.526	0.633	0.653	0.664
7	0.601	0.825	0.548	0.673	0.653	0.679
8	0.600	0.857	0.545	0.675	0.685	0.701

ence in only the second passage of the cold-side fluid is very large and fluids in the 4th, 6th, and 8th passages do not contribute to heat transfer. As seen in Fig.4(b) each passage of S8d has almost equal temperature difference. C8c in Fig. 4(c) is a popular flow type used practically and has a steady state temperature profile as expected from its flow pattern. C8d, C8e, and C8f also have such temperature profiles as expected from their flow patterns though their figures are not given in this paper.

Fig.5 shows frequency responses of the flow types in Table 2. Fig.5(a) explains that the phase angle and gain of S.a decrease monotonously with the number of passages just as a pure parallel-flow heat exchanger and that for S.a with even-numbered passages the last passage does not effectively serve as a heat transfer one. Fig.5 (b) show that though for S.d there is difference in the steady state gains and in the low frequency range, both the phase angle and gain curves of S.d with more than four passages become oscillatory in the same way independently of the number of passages in the high frequency range just as a pure counterflow heat exchanger. Fig. 5(c) explains that frequency responses of P.b are similar to a counterflow heat exchanger but are not so oscillatory as those of S.d in Fig. 5 (b) and C.b in Fig. 5(d). Fig. 5(d) shows that frequency responses of C.b are similar to those of S.d in Fig. 5(b) as expected from the flow patterns.

It is concluded from the above mentioned results that S.d is the best of flow patterns with respect to the statics and dynamics, followed by C.b, P.b, S.a, and P.a types in the order of performance. S.a

Fig.4 Steady state fluid temperature profiles for some flow types

with more than five passages should not be used and should be modified using plates with a larger heat transfer area to reduce the number of plates. Further, when a flow type is designed to have the condition where the inlet passage of hot-side fluid and the outlet passage of cold-side fluid are near by and both fluids flow there in a counter current, the steady state gain may become large and the phase lag may become small.

6. Conclusions

Though plate heat exchangers are widely used in many industries, there have been few researches on not only the statics but also the dynamics. This study may constitute a part of the researches on optimal control synthesis for the plate heat exchanger system. Basic governing equations have been derived first as a distributed system based on heat balance, and then according to the combination and the number of heat transfer plates various flow types have been classified into three flow patterns, and the dynamics has been systemat-

ically analysed for this exchanger system which has a number of heat transfer plates. Further, the results have been numerically compared under the condition that each flow rate of the hot- and cold-side fluids remains constant, respectively in any flow type.

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(a) S.a

(b) S.d

(c) P.b

(d) C.b

Fig.5 Frequency responses dependent on the number of heat transfer plates

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