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Flow of Low Reynolds Number in a Circular Straight Pipe with Various Protrusions*

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It is often experienced that a flow in practical pipe lines becomes a turbulent one under extraordinary low Reynolds number. In order to make clear the phenomena a flow in a circular pipe with a protrusion is examined by measuring fluid friction as a function of Reynolds number and by observing a streak line in it.

And it is obviously concluded that a laminar flow through a circular straight pipe is locally disturbed by a protrusion set in the pipe and that Reynolds number when the turbulence begins is constant if the diameter and dimension and shape of the protrusion are determined, that is, a quasi-critical Reynolds number is effectively defined. And the relation between dimension and shape of a protrusion and the quasi-critical Reynolds number is investigated.

1. Introduction

It is important to examine a flow in oil-hydraulic circuits to reveal their function in a wide range. In the treatment of these problems Hagen-Poiseuille law for laminar flow in pipes is useful, but the flow in the practical circuits does not always follow the law; Reynolds number much below the value of its lower critical number, that is, 2000 or 2320²⁾ is recommended for the practical circuits to keep a laminar flow in them³⁾. For example, in the Hydraulic Handbook⁴⁾ issued by Trade and Technical Press in 1974 it is presumed in a rough calculation that Reynolds number of about 1200 is recommended in a practical circuit.

The reason why such a low Reynolds number is recommended for practical purpose lies certainly in that the laminar flow becomes unstable at low Reynolds number due to the disturbance of joints or bends which are inevitable in practical pipe lines.

The existence of a turbulent flow in a pipe with protrusions in the range of

Reynolds numbers below the lower critical one has been mentioned in several reports. Osborne Reynolds⁵⁾ indicates that, when a coiled wire is inserted in a straight circular pipe, the laminar flow becomes unstable. The flow velocity is a half of the critical flow velocity which is expected in a pipe without any protrusion in it. Itaya and Iwanami⁶⁾ also indicate that the critical Reynolds number reduces to about 1000 when a Pitot tube is set in flow in a circular pipe. Fujimoto⁷⁾ indicates that a turbulent flow in a straight pipe at values below 2000 in Reynolds number, which is caused by a disturbance, is damped out by viscous friction. Prandtl⁸⁾ points out that contraction or expansion of the pipe itself affects the critical Reynolds number.

The reports mentioned above indicate the existence of turbulent flow when Reynolds number is less than its lower critical number, but it is important from an engineering standpoint to obtain the technical data of the turbulent flow which are available to the practical design of oil-hydraulic circuits, that is, the effects of dimensions and forms of the protrusions and the length of a pipe sufficient to damp up the turbulence must be examined.

For these purposes two kinds of experiments are carried out, that is, the head losses of flow in a circular pipe with various types of disturbances are measured to obtain friction factors of a pipe line, and visualization of turbulence is realized to discuss a more detailed conception of the extinction of turbulent flow in a pipe with protrusions of several

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types in it.

2. Experimental apparatus and method

2.1 Measurement of friction factor in a pipe line

2.1.1 Flow in an oil-hydraulic circuit

The state of flow in a pipe line with a protrusion is observed as a function of the Reynolds number or friction factor by changing the flow rate. The protrusion by which the flow is to be disturbed, is therefore set in a straight part of an oil-hydraulic circuit. And, the pressure loss in the down stream of the protrusion is measured in several divided parts along the pipe and thus the mean flow rates at each interval are calculated.

In the experiment a brass pipe of 10 mm in internal diameter with a bellmouth at its entrance is prepared. In front of the bellmouth a chamber is set to make the inlet stream uniform. One of the protrusions, shape and dimension of which are shown in Table 1, is set at a point M with an inlet length of 2m. In the down stream of the protrusion four static-pressure holes of 0.7mm in diameter are bored and reamed at A, B, C and D with distances of 0.1m for MA, 0.4m for AB and 0.5m for both BC and CD. The diameter of the hole is selected such as to minimize its influence on the flow in the pipe.

The differences in static pressure between the holes are measured with a strain-gauge type differential pressure gauge of $1705 \times 10^{-6}/5\text{kg/cm}^2$ in out-put strain sensitivity, and the flow rate is also measured by a flow meter of minimum reading of $0.001\text{m}^3/\text{min}$.

Oil is circulated through the oil-hydraulic circuit by a vane pump at the rate from 0.001 to $0.06\text{m}^3/\text{min}$. And, to

Fig. 1⁵⁾ Streak line of a flow with coiled wire

Table 1 Shape and dimensions of protrusions for oil flow

mm		
d	t	y
10.0	1.0	1.8, 3.0

prevent the occurrence of a pulsation in the flow through the circuit, an accumulator of 7kg/cm^2 in seal pressure is applied. During an experimental run, the pressure in the circuit is controlled to within 50kg/cm^2 by means of a pressure control valve.

The oil used is #90 turbine oil without any additives, and its properties tested are shown in Table 2. The temperature of oil during experiments is controlled at 40°C as the optimum temperature of oil for oil-hydraulic circuits⁹⁾. In the experiments, to prevent the influence of the external atmospheric temperature, the test pipe is immersed in a water bath controlled at 40°C .

2.1.2 Flow of water through a pipe

The change of kinematic viscosity of water due to temperature is far smaller than that of oils. Therefore, by performing the experiment with water the error in measurements caused by the temperature variation can be avoided. The head losses and the flow rates in the lower-stream of the protrusion on the inner surface of pipe are measured with an apparatus shown in Fig. 3.

In Fig. 3, an acryl pipe of 10mm in internal diameter with a bellmouth made of plastics at its entrance is used. One of the protrusions of annular shape with dimensions shown in Table 3 is set with an inlet length of 1.5m. The size of static pressure holes, their location and the number of holes shown in Fig. 2 are similar to those in the case of the oil-hydraulic circuit experiments.

Fig. 2 Pipe for experiment of oil-hydraulic circuit

Table 2 Physical properties of 90# turbine oil

Specific gravity	0.8683
Reaction test	Neutral
Kinematic viscosity 37.8°C	33.56 cSt
Kinematic viscosity 98.9°C	5.41 cSt
Viscosity index	105
Carbon residue Wt %	0.00
Total acid number KOH mg/g	0.01

The pressure difference between two neighboring static pressure holes is measured by a two-liquid U-tube manometer of chloroform and water, and the difference of heads of the U-tube is measured to an accuracy of 0.49mm Aq by a reading microscope, minimum one division of which is 0.01mm.

2.2 Apparatus and method for observation of streak line

An apparatus consisting of a tank shown in Fig. 3 and a pipe in Fig. 4 is made available to observe and photograph the behavior of a streak line. An acryl pipe of either 16mm or 20mm in internal diameter, with a bellmouth at its entrance is prepared.

A protrusion is set at a point of the pipe where the inlet flow settles. Two types of protrusions shown in Table 4 are used; one is an annular ring with different internal diameter, and the other is a splined ring with periodic triangular teeth on its inner surface.

A tracer of aqueous solution of methylene blue is injected into the flow at the center of bellmouth through a fine needle. The flow rate of water is measured with a messcylinder and a stop watch.

Fig. 3 Apparatus for measurement of friction factor of water flow

Table 3 Shape and dimensions of protrusions for water flow

mm		
d	t	y
10.3	1.0	1.5, 2.0, 2.5

Fig. 4 Apparatus for observation of streak line

Table 4 Shape and dimensions of protrusions for observation of streak line

mm			
d	y	annular ring type	triangular teeth type
16.0		1.3	2.3
		1.8	3.2
		2.7	5.3
		4.0	6.2
20.0		1.2	3.0
		2.1	4.0
		3.0	5.0
		3.9	6.0
			7.2
t=0.25mm

Fig. 5 Friction factor downstream of a protrusion

3. Results of the experiments

3.1 Friction factors

Varying the flow rate through the circular pipe in Fig. 2, the head losses between the neighboring static-pressure holes are measured.

Friction factors λ are obtained by substituting the result into the equation (1) derived from Darcy's formula,

$$\lambda = h \frac{2dg}{\ell v^2} \quad (1)$$

where h : head loss, ℓ : distance between static-pressure holes, d : internal diameter of the pipe, and v : mean velocity of flow through the pipe. The values are compared with Nikuradse's experimental results and the value in Moody diagram¹⁰.

In Fig. 5, the effects of the distances from the protrusion on friction factors in the oil-hydraulic circuit are summarized according to Reynolds number as a parameter. In the figure, each plot is the experimental value of head loss in each interval of two neighboring static pressure holes. The figure shows the difference between the holes. The figure shows the difference between the experimental value of friction factor and the friction factor of laminar flows, i.e. $\lambda = 64/Re$. In the interval A ~ B which is closest to the protrusion, the friction factor λ is equal to $64/Re$ when the Reynolds number is less than 500. When

Reynolds number exceeds 600, however, the factor becomes much larger than $64/Re$. And further downstream B ~ C the friction factor becomes larger than $64/Re$ when Reynolds number is over 1000. In C ~ D, the friction factor λ is equal to $64/Re$ at Reynolds number up to about 2000. The distance at which the increase of friction factor λ due to a protrusion decreases to the value of $64/Re$ in the circular pipe, can be estimated from the figure.

Fig. 6 shows the relation between friction factor λ and Reynolds number by the same experimental values as those in Fig. 5. When protrusion height is 3mm, the friction factor λ in the interval A ~ B shows a sudden and remarkable increase at Reynolds number of about 500. When Reynolds number exceeds 550, the factor becomes larger than $64/Re$. Downstream, in B ~ C the friction factor λ is larger than $64/Re$ at Reynolds number over 1000. With an increase of the Reynolds number, in B ~ C the value increases further away from the straight line of $\lambda = 64/Re$. In C ~ D, then, the friction factor λ agrees with Nikuradse's experimental results¹⁾, and the critical Reynolds number becomes 2300. In this way, further from the protrusion, the friction factors approach Nikuradse's experimental results.

When the protrusion height is 2mm, the friction factor in an interval A ~ B becomes larger than $64/Re$ at Reynolds number over 1100. The Reynolds number, at which the friction factor λ exceeds the

Fig. 6 $\lambda \sim Re$ relation of oil flow

Fig. 7 $\lambda \sim Re$ relation of water flow

values $64/R_e$ in laminar flow, differs with the height of the protrusion. The higher the protrusion, the lower the Reynolds number.

If a protrusion is set in a pipe, the friction factor in its downstream occasionally becomes larger than $64/R_e$ in laminar flow. In this case, the state of flow downstream of the protrusion may be other than a laminar flow. When the Reynolds number is less than the lower critical one, the flow converges to a laminar flow at a sufficient distance from the protrusion. Fig. 7 shows the effect of an annular protrusion on the flow through a circular pipe on the friction factor. A similar phenomenon of an increase in the friction factor is also observed, to the one which is shown in a flow downstream of a splined protrusion.

Fig. 8 shows the effect of the size of a static pressure hole (1mm and 3mm in internal diameter), on the state of the flow in an experiment using a brass straight pipe of 10mm in internal diameter. Varying the flow rate through the pipe, head loss between the two neighboring static pressure holes apart 1m from each other is measured, and the friction factors λ are obtained from eq. (1). In the figure, for comparison, the experimental values of friction factor in interval A ~ B, marked by Δ , for a case of triangularly splined protrusions with the height of 2mm and 3mm, are also shown.

Fig. 8 $\lambda \sim R_e$ relation

When the hole diameter is 1mm, the friction factor agrees with Nikuradse's experimental results. But, when the hole diameter is 3mm, the friction factor be-

Fig. 9 Photographic illustration of streak line

comes higher than $64/R_e$ even when the Reynolds number is lower than the critical one.

It is thus concluded from Fig. 8 that a larger value of the static-pressure holes causes a similar effect to that of a protrusion in the pipe. Therefore, when size of the static-pressure holes is not chosen carefully in such a measurement of a difference in the static-pressure in a pipe line, the measured values are liable to include the effect of turbulence caused by static pressure holes themselves.

3.2 Observation of the streak line

As is shown above the friction factor λ becomes larger than the value $64/R_e$ and thus it is presumed that the flow is turbulent, even when Reynolds number of the flow is less than its critical value. To clarify the state, the streak line of the flow is observed, using a pipe shown in Fig. 4, and the photographs thus obtained are shown in Fig. 9.

Fig. 9 (A) shows the streak lines of tracer in the vicinity of a triangularly toothed protrusion set in the acryl pipe of 20mm in internal diameter. When the flow rate is sufficiently small, that is, the Reynolds number is low, the flow of tracer makes a straight line. When the flow rate is increased and thus the Reynolds number reaches a certain limit, the flow of the tracer begins to wave at a short distance from the protrusion, its amplitude increases gradually and finally the tracer diffuses throughout the pipe, as the tracer flows down stream. And, with further increases of the flow rate,

Fig. 10 $Reg \sim y$ relation

the tracer is dispersed just after it passes through the protrusion. By increasing the flow rate still more, the tracer is disturbed and widely diffused in the pipe before the flow passes through the protrusion.

Fig. 9 (B) shows the streak lines in the case of an annular protrusion set up in an acryl pipe. Behavior of the tracer is similar to the case of a triangularly toothed protrusion.

It is shown in Fig. 9 that even when the Reynolds number of flow is less than the critical one, the streak line becomes a turbulent flow locally due to a protrusion in the pipe. And, the Reynolds number where the streak line begins to be disturbed differs depending on the dimension and shape of the protrusion.

The local turbulent flow below the critical Reynolds number may be considered a critical phenomenon different from that of a straight circular pipe without any protrusion. It is inappropriate, however, to term the Reynolds number the critical one as is formally defined, because it occurs locally, and so the Reynolds number when the turbulence of the flow takes place locally is preferably termed the "quasi-critical Reynolds number".

Fig. 10 shows the values of the quasi-critical Reynolds number obtained by the observation of the streak line in the flow in the acryl pipe with a protrusion shown in Table 4. In the case of a protrusion with a smaller internal diameter, the quasi-critical Reynolds number decreases in proportion to the increase of the protrusion height y .

Fig. 11 $Reg \sim y/d$ relation

For the same protrusion height y , the quasi-critical Reynolds number differs according to the shape of the protrusion. The value is lower for the annular protrusion than for the triangularly toothed one. That is, the quasi-critical Reynolds number is increased with a decrease of the protrusion area.

The pipe friction in a turbulent flow region is usually treated as a function of the ratio of inner surface roughness to internal diameter of the pipe to be applied. In Fig. 11 the results of observations of streak line are summarized with respect to y/d , i.e. ratio of protrusion height y and internal diameter d . With an increase of y/d , the quasi-critical Reynolds number decreases. And for the same value of y/d , the number differs according to the shape of the protrusion.

4. Discussion

4.1 Correspondence of the friction factor with the observed results of streak line

The results of experiments of flow of oil or water through a circular pipe are compared with respect to dimension of shape of protrusions in Fig. 6 and 7 and to the results of observation on the streak lines in Fig. 11. In the case of a triangularly toothed protrusion with $y/d = 0.2$, i.e. the protrusion of 2mm in height and internal diameter of 10mm, the minimum Reynolds number at which the friction factor exceeds the value of laminar flow is found to be 1100 from Fig. 6. On the other hand, the quasi-critical Reynolds number for $y/d = 0.2$ is 1200 from fig. 11. Therefore, the two Reynolds numbers at the quasi-critical state of the flow in two circular pipes, which have the same internal diameter and have the same protrusion in it, are approximately equal to each other. In Fig. 11, the plots marked by $\times$ are the minimum Reynolds number for the friction factor when they exceed the line of $\lambda = 64/R_e$ at each y/d value, which can be derived from Figs. 6 and 7. These two are also in coincidence. As the same phenomenon is observed by the two independent methods, it can be concluded that the quasi-critical Reynolds number corresponds with the minimum Reynolds number at which the friction factor becomes greater than the value in laminar flow in a circular pipe with a protrusion. That is to say, the quasi-critical Reynolds number coincides with the Reynolds number at which the flow of oil or water through a circular pipe brings about a local turbulence due to the protrusion.

4.2 Local turbulences of a flow through a pipe

If there exists a protrusion in a circular pipe of uniform internal diameter, a friction factor which is larger than that in laminar flow is observed in the down stream of the protrusion, even when the flow has a Reynolds number far below the critical one. Examination of the streak line in this case reveals the existence of a local turbulence of flow in the down stream of the protrusion. The state of the disturbed flow due to a protrusion agrees with that of the disturbed flow in which the turbulence disappears, as described by Fujimoto⁷⁾.

As already described, there is good correspondence between the minimum Reynolds number at which the friction factor exceeds the value in laminar flow below the critical Reynolds number and the Reynolds number at which a turbulence occurs due to a protrusion in the streak line of flow. It can thus be concluded that the rise in friction factor is caused by the local turbulence of the flow.

The local turbulence of a flow in a circular pipe with a protrusion appears at the quasi-critical Reynolds number. And this value is determined by the ratio of the height of a protrusion to the internal diameter of pipe, y/d . For the same value of y/d , the quasi-critical Reynolds number differs according to the shape of the protrusion.

Little consideration has been made of the effect of such a discontinuous variation in a pipe cross section. This is probably because roughness of inner surface of a pipe has little effect on the friction factor¹⁾ in a laminar flow, and further, the influence of disturbed flow on a friction factor has not been known well yet. And hence, if the quasi-critical Reynolds number of a pipe with protrusion, is obtained as a function of shape and dimension of protrusions, it must provide useful information for the design of oil-hydraulic circuits.

4.3 Protrusions and turbulences of a flow due to protrusions

A flow through a circular pipe is disturbed locally by a protrusion, when

Table 5 Shape of protrusion and coefficient

shape of protrusion	annular ring type	triangular teeth type
A	30	90
n	1.5	1.5

the Reynolds number exceeds the quasi-critical Reynolds number. In this case, there is a relation between the shape and size of a protrusion and the quasi-critical Reynolds number as shown in Fig. 8. From these results, the following empirical formula can be derived,

$$Re_q = A (y/d)^{-n} \quad (2)$$

and, the constants A and n are shown in Table 5, which are calculated from the data given above. The value A means a Reynolds number when y/d is equal to 1. For the same value of y/d , value A depends on the shape of protrusions, and value A is larger in a triangularly toothed protrusion than in an annular protrusion. That is, with an increase of the sectional area of a protrusion, the value of quasi-critical Reynolds number decreases. The index n means the logarithmic ratio of an increasing Re_q to an increasing y/d . The value n is independent of the shape of protrusion whether it is triangular or annular. If the shape and size of a protrusion and the internal diameter of the pipe are given, from the formula (2) it can be known whether the flow through it becomes locally turbulent or not.

As already indicated, local turbulence of a flow through a pipe is related to cross sectional area of a protrusion. The results in the observation of the streak line are summarized in Fig. 12, according to the solidity S_1/S_0 which is the ratio of the cross sectional area of the protrusion, S_1 , to the cross-sectional area of the pipe, S_0 . On a logarithmic scale, the relation between solidities and quasi-critical Reynolds number is linear.

In the case when a triangularly toothed protrusion is set in a pipe, the configuration approaches an annular protrusion.

Fig. 12 $Re_q \sim S_1/S_0$ relation

sion of the same height, as the number of triangularly toothed protrusion with the same height increases. The relation between quasi-critical Reynolds number and cross-sectional area of the annular protrusion thus formed, which is derived from observation of the streak line, is shown in Fig. 13. In the figure, S_{1r} is the cross-sectional area of the annular protrusion with outside diameter d and internal diameter $(d-2y)$.

For the same height of a protrusion, local turbulence of a flow through a pipe differs according to the contraction ratio in cross section of the pipe.

4.4 Friction factors

In Fig. 14 the values of friction factor in the present experiments is overlapped with the experimental values of Nikuradse¹⁾, the values in Moody diagram¹⁰⁾

Fig. 13 $Re_q \sim S_1/S_{1r}$ relation

Fig. 14 $\lambda \sim Re$ relation

and the values calculated from a chart in the Hydraulic Handbook.⁴⁾

The values marked by small dots $\cdot$ and large dots $\bullet$ are the experimental ones for a triangularly toothed protrusion of 2mm or 3mm in height.

Nikuradse's values marked $\circ$ are obtained from experiments on a circular pipe coated with sand grains on its inner surface. Surface roughness of the pipe is thus represented by the ratio r/k , where r is the pipe radius and k is the average irregularity of a larger sand grain, r/k is 507 at the minimum surface roughness and 15 at the maximum.

Moody presents a diagram showing the practical relation between Reynolds number and friction factor in a pipe, at which roughness ratio ϵ/d covers the range of $1 \times 10^{-6} \sim 5 \times 10^{-2}$ where ϵ is surface roughness and d is the internal diameter of a pipe. The solid line in Fig. 14 shows the values in a low Reynolds number region for both maximum and minimum values of ϵ/d .

The values marked with double dots-dash-line are derived from a diagram giving the relation between flow rate, pressure loss and pipe diameter in the Hydraulic Handbook.

Concerning the values of friction factor in the transition region of flow, Nikuradse shows the Reynolds number of transition point definitely. But, Moody does not give any definite values, but gives a domain by hatching. In the Hydraulic Handbook, on the other hand, the values are given as a transition point for Reynolds number about 1200; the values in the vicinity are then roughly indicated by hatched lines.

In a laminar flow through a circular pipe with a protrusion, the friction factor sufficiently downward from the protrusion agrees with Nikuradse's experimental value of the $64/R_e$ value in Moody diagram, such as the case of a laminar flow through the smooth-surface circular pipe. In the region close to the protrusion, however, the friction factor increases remarkably over $64/R_e$, due to the local turbulence of the flow at the part of a pipe.

The minimum Reynolds number for the friction factor to exceed the value in laminar flow is in agreement with the quasi-critical Reynolds number for a given y/d value, that is the ratio of protrusion height to pipe internal diameter. The quasi-critical Reynolds number for both annular and triangularly toothed protrusions is obtainable from eq. (2).

5. Conclusions

Varying the shape and size of a pro-

trusion set in a circular pipe, the behavior of a flow through the pipe is studied both by the experiments to obtain the friction factor and by observation of the streak line. The following conclusions are obtained.

(1) The flow in a circular pipe is locally turbulent due to the protrusion. The turbulence, however, is found to be damped to a laminar flow at a certain distance from the protrusion. And the distance sufficient to damp the local turbulence can approximately be estimated.

(2) When the flow is in disturbed state due to the effect of a protrusion, the friction factor becomes larger than $64/R_e$ at lower Reynolds number than the critical one.

(3) The value of quasi-critical Reynolds number differs according to the shape of the protrusion. For the same shape of a protrusion, the value is determined by the ratio y/d , i.e. the ratio of protrusion height to pipe internal diameter. The relation between quasi-critical Reynolds number and y/d is made clear.

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