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A Photoplasticity Utilizing the Stress-Optic Law of Polyester at the R.P.E. Transition Temperature*

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At the R.P.E. transition temperature, named by one of the authors, it was found that the stress-birefringence relation of polyester was linear even in inelastic range (in elastic range as well). By using this relation, factors of stress concentration and stress distributions at a free boundary were determined, and two-dimensional stress distributions at points apart from the free boundary were calculated by employment of the shear-difference method. Results obtained were in substantial agreement with the experimental results reported by M.M.Frocht. Moreover, the stress-strain relation of polyester at the R.P.E. transition temperature showed a good approximation to that of metal prototype, especially Al-alloy. Thus, it was proved that the photoplasticity utilizing the stress-optic law of polyester at the R.P.E. transition temperature is available to analyze the stress distributions in certain metals and polymeric materials.

1. Introduction

For many years, efforts have been directed towards the development of a technique of birefringent stress (or strain) analysis of inelastic deformations by an extension of the standard methods of photoelasticity. This technique is called "photoplasticity". Most of the efforts have been made to take the polymeric model materials having (1) a linear relationship between birefringence and stresses (or strains) in inelastic range, (2) a stress-strain curve similar to that of metal prototype, and (3) Poisson's ratio equal to that of metal prototype in inelastic range. Of the materials investigated previously, celluloid and polycarbonate appear to be most suitable for problems involving elastoplastic states of stress. Extensive studies using celluloid have been made by Hiltcher⁽³⁾, Fried⁽⁴⁾, Nishida^{(4),(5)}, Frocht and Thomson^{(6),(7)}, Frocht and Cheng^{(8),(9)}, Mönch^{(10),(11)} and Ohashi^{(12)~(14)}. Ito⁽¹⁵⁾ suggested the use of polycarbonate and detail studies of this material have been made by Brinson⁽¹⁶⁾ and Whitfield⁽¹⁷⁾. Hunter and Schwarts⁽¹⁸⁾ and Hunter⁽¹⁹⁾ made photoelastoplastic studies different from those of the authors mentioned above. They used the

creep and frozen-stress characteristics of polycarbonate and epoxy resin in order to obtain a nonlinear stress-strain curve whose shape could be adjusted by various thermal cycle treatments. In recent years, Morris and Riley⁽²⁰⁾ and Weis, Goldsmith and Chase⁽²¹⁾ conducted studies on the mechanical and optical properties of a polyester material to determine its applicability as a possible photoplasticity material.

For the practical utilization of these methods, however, more information seems to be necessary about the stress-optic law which expresses the relationship between birefringence (isochromatic fringes) and the stresses (or strains) of polymeric model materials.

One of the present authors has investigated the temperature dependence of the photoviscoelastic coefficients* of polymeric materials by means of creep tests^{(22)~(24)}. The main results obtained were: (1) A reversal in sign of the retarded-photoelastic coefficient took place in polyester-mixtures at each specific temperature. (2) The temperature at which the retarded-photoelastic effect vanishes, was found and named the "R.P.E. transition temperature"⁽²³⁾ (retarded-photoelastic effect transition

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* Photoviscoelastic coefficients can be considered as a linear combination of the photoelastic and retarded-photoelastic coefficients. The former means the stress-birefringence effect which appears during the loading, and the latter does the effect which appears under dead load and shows time dependence.

temperature) ".

Further study on the stress-optic law and stress-strain relation of polyester-mixtures at the R.P.E. transition temperature is carried out in this paper. It is revealed that (1) the optical creep (retarded-photoelastic fringes) does not appear whereas a remarkable mechanical creep does, (2) the fringe orders in birefringence are dependent on the stress values but not independent on creep time, and in addition a linear relationship exists between the fringe orders and stress values both in elastic and inelastic stress ranges, (3) the stress-strain relation of the mixtures shows a good agreement with those of some metal prototypes, (4) the values of Poisson's ratio ν of the mixtures in inelastic range are approximately the same as that of the metal prototype in the plastic flow. Thus, a new method of the photo-elasto-plastic analysis based on these four conditions could be proposed. In addition, factors of stress concentration and two-dimensional stress distributions of the specimen with a centrally located circular hole are examined by the new method.

2. Experimental procedure

2.1. Polymeric model materials and their R.P.E. transition temperatures

Two kinds of unsaturated polyester with approximately 30 wt% styrene monomer, Rigolac Nos. 1557 and 70F, provided by the Showa Gosei Jushi Ltd. were used. Polyester-mixtures (photoelasto-plastic model materials), were made by mixing of adequate amounts of these unsaturated polyesters. The mixtures were cast into glass plate molds and cured at room temperature by adding about 0.3 wt% Permex N (60 % solution of methyl ethyl keton peroxide in dimethyl phthalate) and about 0.3 wt% cobalt naphthenate. Dumbbell-shaped specimens were used for determining the R.P.E. transition temperature and the stress-strain curves. The specimens were 7 mm wide, 25 mm long in parallel part and 2.5 mm thick. Plate models with a centrally located circular hole were used for analysis of the factors of stress concentration and two-dimensional elasto-plastic stress distributions. The size of the plate model is shown in Fig. 1. Before testing, specimens were subjected to thermal treatment for about 5 hours at the temperature higher than the glass transition temperature by a few degrees in order to accom-

Fig. 1 The size of the specimen with a centrally located circular hole

plish the chemical reaction and removal of the initial strains. The compositions of the specimens examined here and their R.P.E. transition temperature are given in Table 1.

Table 1 Compositions of the specimens and each R.P.E. transition temperature

H : S	75 : 25	70 : 30	65 : 35	60 : 40
SIGN	75 - 25	70 - 30	65 - 35	60 - 40
R.P.E. Tran. Temp. (°C)	22 ~ 25	18 ~ 22	16 ~ 20	10 ~ 20

(H; Rigolac No.1557, S; Rigolac No.70F)

2.2. Apparatus

An automatic recording apparatus with the creep loading mechanism was used for creep tests and this apparatus combined with an Instron tensile testing machine was also used for uniaxial tensile tests. (Uniaxial tensile tests were made at the rate of 0.05 cm/min). This apparatus is so designed as to be able to measure continuously the stress, strain and birefringence during the creep and uniaxial tests. (For the details of the apparatus, refer to the previous paper⁽²⁵⁾). General view of this apparatus is given in Fig. 2. Two-dimensional elasto-plastic fringe patterns were measured by an Instron tensile testing machine attached to the ordinary photoelastic apparatus.

Fig. 2 General view of the apparatus

3. Results

Due to the limitations of space, the only results given in this paper are for 70-30 polyester-mixture of the mixtures listed up in Table 1. All the tests were carried out at the R.P.E. transition temperature.

3.1. Strain, birefringence (isochromatic fringes) and stress-birefringence (isochromatic fringes) during the creep tests

Birefringence and the strains were measured continuously for about 15 min after loading. Fig. 3 summarizes the results of creep tests for 70-30 polyester-

mixture; in this figure, (a) shows the strain and (b) shows birefringence (isochromatic fringes) depending on time under the constant stress. (The strain is expressed in percentage of elongation and the stress is the nominal stress in kg/mm^2). As can be seen in Fig. 3 (b), this mixture makes no optical creep (that is, the retarded-photoelastic fringe does not appear), although the mechanical creep occurs remarkably (Fig. 3 (a)). In other words, at the R.P.E. transition temperature, the isochromatic fringe orders are dependent on the stress but independent of time. The values of the stress and strain measured at

(a)

(b)

Fig. 3 (a) Strains depending on time under the constant stress for 70-30 polyester-mixture
(b) Birefringence depending on time under the constant stress for 70-30 polyester-mixture

0, 1, 3, 5, 10 and 15 min after loading are plotted in Fig. 4. Since all the curves show the non-linearity, this mixture was considered as having been loaded up to the inelastic range. Next, the stress-birefringence (isochromatic fringes) relation was examined. As seen in Fig. 5, the fringe order was always proportional to the stress value (even in inelastic region) at the R.P.E. transition temperature*. The photoelasto-plastic constant α calculated from this figure is about 0.35 mm/kg .

Fig. 4 Stress-strain curve for 70-30 polyester-mixture at 0, 1, 3, 5, 10 and 15 min after loading (in creep test)

Fig. 5 The relationship between the stress and birefringence at 0, 1, 3, 5, 10 and 15 min after loading for 70-30 polyester-mixture (in creep test)

* As the temperature deviated from the R.P.E. transition temperature, stress-birefringence (isochromatic fringes) relation depends on creep time and shows the non-linearity.

3.2. Stress-strain, stress-birefringence (isochromatic fringes) during the uniaxial tensile tests

The samples of 70-30 polyester-mixture were stretched up to the inelastic state at the rate of 0.05 cm/min. The stress-strain relation obtained by these tests is shown in Fig. 6. Generally, it is difficult to determine the yield point exactly for polymeric materials*. So we used the proportional limit for the yield point σ_0 in the present work ($\sigma_0 \approx 2.4$ kg/mm²). On the other hand, the stress-birefringence (isochromatic fringes) relation is plotted in Fig. 7. The results revealed that, even in inelastic state, the stress-birefringence (isochromatic fringes) showed a proportional relation except in the higher strain range over about 7 %

Fig. 6 The stress-strain curve for 70-30 polyester-mixture (in uniaxial tensile test)

Fig. 7 The relationship between the stress and birefringence for 70-30 polyester-mixture (in uniaxial tensile test)

strain. The value of the photoelasto-plastic constant α calculated from the gradient of the line was 0.31 mm/kg.

The shapes of the stress-strain curves of the materials of model and prototype can be compared by means of a non-dimensional representation, using σ / σ_1 for ordinate and $E\varepsilon / \sigma_1$ for abscissa; where E is Young's modulus, and σ_1 is the yield point decided from the secant modulus $E_s = 0.7E$. In this case, the stress-strain curve can be given by the so-called Ramberg-Osgood equation,⁽²⁶⁾

$$E\varepsilon / \sigma_1 = \sigma / \sigma_1 + 3/7 (\sigma / \sigma_1)^n$$
, where n is a parameter which is adjusted to give the best fit to the curve which it is desired to represent. Some types of Al-alloy and Ni-Cr steel have stress-strain curves which fall in the range $n=3$ to $n=11$. Figure 8 shows a comparison between the stress-strain curves for $n=5$ and $n=10$, and the curve for 70-30 polyester-mixture at the R.P.E. transition temperature. Experimental data lies in the range $n=5$ to $n=10$ except in the region of large plastic flow (over about 1.5 strain, which corresponds to 7 % strain shown in Fig. 4).

The results of a few tests made to determine the value of Poisson's ratio ν in the inelastic range give $\nu \approx 0.48$. In the case of metals, at plastic flow $\nu \approx 0.5$. Thus, the primary conditions for the transition from model to prototype have been satisfied.

Fig. 8 Comparison between the stress-strain curves given by Ramberg-Osgood equation for $n=5$ and $n=10$, and the experimental data for 70-30 polyester-mixture

* A number of investigators adopt the effective yield point for their studies; Hunter,⁽¹⁹⁾ and Frocht⁽⁹⁾ use the "proportional limit", Riley⁽²⁰⁾ use the "secant yield point" defined by Ramberg-Osgood, and Gurtman⁽²⁶⁾ takes the "limiting stress", which does not appear the mechanical creep, as the common yield stress.

3.3. Factors of stress concentration and the stress distributions around the rim of the circular hole

In both elastic and inelastic states, the stress distributions σ_θ , and iso-chromatic fringe orders N at the rim of the circular hole are plotted on a polar diagram (Fig. 9) for different load values. (For the symmetry, a quarter part of the circular hole is shown in this figure). Dotted line shown in Fig. 9 indicates the yield condition obtained from the Tresca criterion, $(\sigma_1 - \sigma_2) / 2 = \sigma_0 / 2$, where σ_1, σ_2 are principal stresses and σ_0 is yield strength determined by uniaxial tensile tests. The yield sites at the rim of the hole can be obtained as the intersecting points of the dotted line and the curves for different load values. This figure clearly shows that a maximum tension side (point A) already has come into an inelastic state even at a relatively light

Fig. 9 Stress distributions σ_θ and iso-chromatic fringe orders N at the rim of the circular hole

Fig. 10 Stress distributions at the rim of the circular hole in non-dimensional representation ($\sigma_\theta / \sigma_{av}$)

load 150 kg, while the maximum compression side (point C) still remains in an elastic state at the maximum load (350 kg). Stress distributions at the rim of the hole in non-dimensional representation ($\sigma_\theta / \sigma_{av}$) are shown in Fig. 10, in which three different values of the ratio $\sigma_{av} / \sigma_0 = 0.31, 0.61$ and 0.92 were used for parameters. (σ_θ : stress around the rim of the circular hole, σ_{av} : a average stress on net area of the section). In this figure, the curve obtained from an elastic theory is also given. The boundary between the elastic and inelastic zones is indicated by the arrow for each of the elasto-plastic stress distributions for $\sigma_{av} / \sigma_0 = 0.61$ and 0.92 . (in each case, the boundary was determined from the Tresca criterion). The curve in tension side obtained for $\sigma_{av} / \sigma_0 = 0.31$, which equivalent to 100 kg in load, and all curves in compression side, are in good agreement with the theoretical solution by Howland. The other curves show lower values than the theoretical one in inelastic stress range. Figure 11 shows the factors of stress concentration, $k = \sigma_\theta / \sigma_{av}$, at the point A in Figs. 9 and 10. (These factors were determined in the same way in the elastic state). In this figure, the experimental values of the factors of stress concentration in the elastic region ($k \approx 2.4$) agree well with the theoretical solution ($k \approx 2.3$), and gradually decrease beyond the yield point determined by the Tresca criterion.

Fig. 11 Factors of stress concentration at the point A.

3.4. Stress distributions along the line of symmetry normal to the applied tension

The stress distributions along the line of symmetry normal to the applied tension (i.e. along the X-axis shown in Fig. 12) were obtained. The relationship between the values of birefringence N , the principal stress difference ($\sigma = \sigma_y - \sigma_x$) and the non-dimensional distance X/a from the edge of the circular hole (point A

shown in Fig. 12) is plotted in Fig. 13 for six different load values. The elastic boundary is also shown in this figure by the dotted line with the application of the Tresca criterion. In order to separate the principal stresses, the shear-difference method* was employed in the present work. The isoclinic parameter θ ** (in degrees) measured along the line of a_1 - a_1 is shown in Fig. 14 as a function of non-dimensional distance X/a from the edge of a circular hole. The stress distributions σ_x and σ_y

Fig. 12 A sketch showing the measuring part of the specimen

Fig. 13 The relationship between the values of birefringence N , the principal stress difference ($\sigma = \sigma_y - \sigma_x$) and the non-dimensional distance X/a from the edge of the circular hole

* We confirmed that the isoclinic parameter gives the direction of the principal stresses.

** Measuring conditions of the isoclinic parameter were as follows; (1) temperature; R.P.E. transition temperature, (2) loading range; beyond the yield strength, (3) samples; 70-30 polyester-mixture prepared from the same stock sheet as used in the tests of the isochromatic fringes.

were determined by means of the same shear-difference method as in the elastic state. The principal stress distributions σ_x and σ_y determined on the section of symmetry normal to the applied load are shown in Fig. 15 for six different stress values. The non-dimensional principal stress distributions σ_x / σ_{av} and σ_y / σ_{av} determined on the section of symmetry normal to the applied loads are shown in Fig. 16 for $\sigma_{av} / \sigma_0 = 0.31, 0.61$ and 0.92 ; where σ_x and σ_y are the stresses in the direction of X - and Y -axis, respectively. Also, the approximate boundaries between the elastic and inelastic zones are shown by arrows for each of the elasto-plastic stress distributions for $\sigma_{av} / \sigma_0 = 0.61$ and 0.92 . (In each case the boundaries were located with the application of Tresca criterion and

Fig. 14 Isoclinic parameter θ measured along the line of a_1 - a_1

Fig. 15 The principal stress distributions σ_x and σ_y determined on the section of symmetry normal to the applied loads

Fig. 16 Non-dimensional principal stress distributions σ_x / σ_{av} and σ_y / σ_{av} determined on the section of symmetry normal to the applied loads

under the assumption that the elastic limit is equal to the proportional limit). To compare with the experimental stress distributions for σ_x and σ_y , the theoretical solution⁽²⁷⁾ in the elastic stress range is also shown in this figure; the experimental results are in good agreement with the theoretical value except in the vicinity of the parallel edges. Non-coincidence appearing in the vicinity of the parallel edges seems to be due to trivial experimental error such as an effect of bending stresses resulting from only a little asymmetrical stretching.

4. Discussion

The present method seems to be applicable to determine the two-dimensional elasto-plastic stress distributions according to the results obtained here; that is, (1) at the R.P.E. transition temperature, the relationship between the stress and birefringence (isochromatic fringes) of the polyester-mixtures showed a simple proportionality even in inelastic range, (2) the stress-strain relation of the polyester-mixtures at the R.P.E. transition temperature showed a good approximation to that of metal prototype, especially Al-alloy, (3) moreover, two-dimensional stress distributions determined here were in substantial agreement with the experimental results reported by Frocht.

The linearity of the stress-birefringence relation deviated in the range beyond 7 % strain and the non-dimensional stress-strain curve began to separate from the Ramberg-Osgood equation in the range beyond 1.5 non-dimensional strain. So, utilizing the present method for large plastic deformation, the stress-birefringent proportionality and the stress-strain curve of polyester-mixtures should

be improved so as to correspond more closely in higher strain range to those of some metal. One way of doing these is to decrease the contents of styrene monomer in polyester-mixtures. Another way of varying the slope of the curve in the plastic region is to change the values of the strain rate. Details of these will be published and reported in near future.

5. Conclusions

Conclusions derived from the present experiments are:

- (1) At the R.P.E. transition temperature, the stress-birefringence of polyester-mixtures showed a simple proportional relation even in inelastic state (in elastic state as well), except in the higher strain range over about 7 % strain.
- (2) The stress-strain relation of polyester-mixtures showed a good agreement with that of metal prototype, especially Al-alloy and Ni-Cr steel.
- (3) Factors of stress concentration and stress distributions around the rim of the circular hole in a thin plate were in good agreement with the theoretical solution by Howland and the results reported by Frocht. Also, the stress distributions along the line of symmetry normal to the applied tension agree well with the results reported by Frocht.
- (4) It was proved that the photoplasticity utilizing the stress-optic law of polyester-mixtures at the R.P.E. transition temperature was available to determine two-dimensional elasto-plastic stress distributions of some types of Al-alloy and Ni-Cr steel as well as polymeric materials.
- (5) By improving the stress-birefringent proportionality and the stress-strain curve of polyester-mixtures so as to correspond more closely in higher strain range to those of some metal, the present method can be used for large plastic deformation.

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