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Study on Optimum Design of Machine Structures
with respect to Dynamic Characteristics*
(Approach to Optimum Design of Machine Tool
Structures with respect to Regenerative Chatter)

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In order to attain dynamically optimum design of machine tools which would have minimum chance of machining chatter, an approach based on energy balances of a mathematical system at the resonance is developed and analysed theoretically. This method aims that the maximum compliance of the tool-work relative displacement in the direction normal to cut surface caused by the cutting force at the cutting point must be minimum across all frequency ranges. Using the computer simulations of machine tool structures, modal flexibilities are computed, by the magnitude of which the chance of regenerative chatter is judged. Focusing on the particular mode where the modal flexibility is predominantly high, and by consulting with the energy distribution through the complete structure at the mode, a design change is proposed such that the static flexibility is small and evenly shared by modal flexibilities in many modes. The outline of a computer program is described and demonstrated by computed and experimental results on a practical machine structure.

1. Introduction

In order to analyse structural dynamics, dynamic characteristics should be not only measured by experiment but calculated at the design stage by computer program. For this purpose, many computer program systems have been produced; but one such system¹⁾ was developed by the author, based on receptance synthesis, in order to analyse both static and dynamic characteristics. These characteristics were computed for actual machine structures and models by using the program, whose practical validity was demonstrated.^{2),3)}

The dynamic characteristics of machine structures can be analysed at the design stage so that the computed results have fairly good correspondence with the measured results. The next step is for an optimum-design method to be developed in order to improve these characteristics. In general, machine tools are very complex structures and it is difficult to apply the usual mathematical optimization methods as they stand. Therefore, machine structures should be designed so that they will cause the minimum of trouble due to vibration, using heuristic optimization methods.

This paper describes an optimum-design method using the energy-balance principle as a heuristic approach. The method can be applied not only to machine tools but

also to ordinary machine structures. In this paper, the receptance synthesis program system is used for the computation of dynamic characteristics, but the method can be applied easily using many other computer programs written according to other computation principles also.

2. Problems in achieving optimum design of machine tool structures free from chatter vibration

The following objectives are considered for optimum design in respect to the dynamic characteristics of machine structures, according to the functions and requirements of the machines.

- (1) Static stiffness
- (2) First resonance frequency
- (3) Elimination of resonance modes over a certain frequency range
- (4) Minimization of the maximum value of compliances over the whole frequency range (in regenerative chatter of machine tools, the maximum real negative part of the vector loci).

Of the above, the studies carried out up to the present time have dealt with optimum design in relation to (1) or (2).

When the machine in question is a machine tool, the following must be considered for optimum design analysis;

- (i) Machine tool structures are generally very complex and three-dimensional vibration analyses are required.
- (ii) The problem is the relative displacement or receptance between the tool and the points at which cutting or grinding, etc., is to be done.

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- (iii) The self-excited chatter vibration⁴⁾ is the most troublesome and difficult to prevent of all types of chatter. Therefore optimum design with respect to factor (4) above is needed.
- (iv) As the stability limits are determined by the magnitude of the compliance neighborhood of the resonance mode, not only the resonance frequencies and mode shapes but also the damping characteristics should be analysed fundamentally.
- (v) The stability limit of self-excited vibration lies in the range of small-vibration displacement. Consequently, the dynamic characteristics can in many cases be analysed as a linear vibration system.

3. Theory of optimization of structural dynamics based on the energy-balance principle

A machine structure comprises many members joined to each other to form the whole. In analysing the dynamics of a complex three-dimensional structure, it is essential to observe the whole system and analyse it macroscopically. For this purpose, the vibration system can be considered based on the balance between input, kinetic, potential and dissipated energies. The following theory as far as eq.(21) is based on that previously published.³⁾

A machine structure consists of sub-systems which are connected to one another by combinations of bolted joints, welded joints, bearings, slideways and so on. By way of explanation, let us assume that a system A consists of N sub-systems $B_1, B_2, \dots, B_N$, and that each junction (bolted joint, bearing, or other) between sub-systems also forms a sub-system. In general, a system has an infinite degree of freedom, and so an infinite number of natural frequencies. For a structure in resonance at any resonance frequency, the following two relations must be true;

(I) The maximum kinetic energy T_{Am} of the whole system is equal to the maximum potential energy V_{Am} of the whole system.

(II) At steady resonance, the input energy per cycle I_{Am} in a resonant condition is equal to the summation of the energies per cycle D_{Am} dissipated through whole system.

At the mth resonance frequency, the N sub-systems have the energies (T_{1m}, V_{1m}, D_{1m}), (T_{2m}, V_{2m}, D_{2m}), ..., (T_{Nm}, V_{Nm}, D_{Nm}) where the first subscripts 1, 2, ..., N indicate sub-system numbers. Joints do not generally have equivalent kinetic energy. The potential and kinetic energies of the total system are equal to the sum of those of all sub-systems, and are expressed by:

$$T_{Am} = T_{1m} + T_{2m} + \dots + T_{Nm} \dots (1)$$

$$V_{Am} = V_{1m} + V_{2m} + \dots + V_{Nm} \dots (2)$$

Using $T_{Am} = \omega_m^2 T_{Am}'$, the resonance frequency is

$$\omega_m = \sqrt{\frac{V_{Am}}{T_{Am}'}} \dots (3)$$

Assuming that the damping energy of each sub-system is proportional to its potential energy, the following equations are obtained:

$$\frac{D_{1m}}{2\pi V_{1m}} = \eta_1, \frac{D_{2m}}{2\pi V_{2m}} = \eta_2, \dots, \frac{D_{Nm}}{2\pi V_{Nm}} = \eta_N \dots (4)$$

Where $\eta_1, \eta_2, \dots, \eta_N$ are the so-called loss-coefficients. Their values are inherent to the material of the sub-system and the condition of the junction.

So far as two assumptions, (a) that the potential energy is proportional to the square of the amplitude of the displacement, and (b) that D_{Am}/V_{Am} is far less than 0.5, hold, the logarithmic decrement δ_{Am} at the mth mode of the total system A is expressed by

$$\delta_{Am} = -\frac{1}{2} \ln \left(1 - \frac{D_{Am}}{V_{Am}} \right) \approx \frac{D_{Am}}{2V_{Am}} \dots (5)$$

Denoting the ratio of potential energy in each sub-system to the potential energy of the total system by

$$\frac{V_{1m}}{V_{Am}} = \zeta_{1m}, \frac{V_{2m}}{V_{Am}} = \zeta_{2m}, \dots, \frac{V_{Nm}}{V_{Am}} = \zeta_{Nm} \dots (6)$$

and substituting eq.(4) and eq.(6) into eq.(5), we obtain

$$\delta_{Am} = \pi \sum_{i=1}^N \eta_i \zeta_{im} \dots (7)$$

The ratio of kinetic energy in each sub-system to the kinetic energy of the total system is expressed by

$$\frac{T_{1m}}{T_{Am}} = r_{1m}, \frac{T_{2m}}{T_{Am}} = r_{2m}, \dots, \frac{T_{Nm}}{T_{Am}} = r_{Nm} \dots (8)$$

Denoting the mass of each sub-system by W_i , the total mass W_A through the complete structure is expressed by

$$W_A = \sum_{i=1}^N W_i \dots (9)$$

The potential and kinetic energies per unit mass of the whole structure are expressed as

$$\frac{V_{Am}}{W_A} = \zeta_{Am}', \frac{T_{Am}}{W_A} = r_{Am}' \dots (10)$$

The potential and kinetic energies per unit mass of each sub-system are

$$\frac{V_{im}}{W_i} = \zeta_{im}'', \frac{V_{im}}{W_i} = r_{im}'' \dots (11)$$

From eq.(10) and eq.(11), the energy distribution ratios per unit mass are derived as

$$\frac{\zeta_{im}''}{\zeta_{Am}'} = \zeta_{im}''', \frac{r_{im}''}{r_{Am}'} = r_{im}''' \dots (12)$$

When ζ_{im}'' and r_{im}'' are nearly 1, it can be said that the potential and kinetic energies are uniformly distributed.

Let us observe the relative displacement between tool and work at the cutting point C in the framework of a machine tool, as in Fig. 1. Denoting the angle between

the cutting force direction and the principal direction in the m th order mode by θ_m , the angle between the cutting direction and the principal direction by φ_m , the cutting force amplitude by F_e , the tool-work relative displacement in the cutting direction by X_c , and that in the principal direction by X_m , relation (II) yields an expression of cutting point compliance in the m th order mode as follows:

$$F_e X_m \pi \sin \beta \cos \theta_m = D_{Am} \quad \dots (13)$$

In this equation, β stands for the phase difference between the force F_e and the displacement X_c .

Using $X_c = X_m \cos \varphi_m$ (14) and transforming eq.(13), we obtain

$$\left(\frac{X_c}{F_e}\right)_m = \sin \beta \cos \theta_m \cos \varphi_m \frac{\pi X_m}{D_{Am}} \dots (15)$$

Substituting eq.(5) into eq.(15), we obtain

$$\left(\frac{X_c}{F_e}\right)_m = \sin \beta \cos \theta_m \cos \varphi_m \frac{\pi f_m}{f_{Am}} \dots (16)$$

Where

$$k_m = \frac{2E_m}{X_m^2} = \frac{1}{f_m} \quad \dots (17)$$

and E_m stands for the maximum instantaneous value of either the potential or the kinetic energy in the system. Further, k_m and f_m as defined by eq.(17) are the modal stiffness and the modal flexibility respectively, of the relative motion at the cutting point in the principal direction in the m th order mode, due to the cutting force. If we denote the relative displacement in the cutting force direction by X_e , and that in the cutting direction by X_c , eq.(15) and eq.(16) can be written as follows:

$$\left(\frac{X_c}{F_e}\right)_m = \sin \beta \frac{\pi}{D_{Am}/X_e X_c} \quad \dots (18)$$

$$\left(\frac{X_c}{F_e}\right)_m = \sin \beta \frac{\pi f_m'}{f_{Am}} \quad \dots (19)$$

$$\text{Where } f_m' = \cos \theta_m \cos \varphi_m f_m = \frac{X_e X_c}{2E_m} = \frac{1}{k_m'} \quad \dots (20)$$

Now, k_m' and f_m' as defined by eq.(20) are the modal stiffness and the modal flexibility respectively, of the relative motion at the cutting point in the cutting direction in the m th order mode, due to the cutting force.

The total sum of the modal flexibility f_m' over all resonance modes is equal to the static compliance f_s of the relative motion in the direction normal to the cut surface, due to the cutting force, as shown in eq.(21).

$$f_s = \sum_{m=1}^{\infty} f_m' \quad \dots (21)$$

In the above way, the resonance frequency ω_m , the logarithmic decrement δ_{Am} and the modal flexibility f_m' were expressed by energy distributions having definite physical properties.

In relation to the minimization of the possibility of regenerative chatter, it is known that the maximum real negative part of the compliance of the relative motion in the direction normal to the cut surface, due to the cutting force, is small. This is equivalent to the statement that the compliance as formulated in eq.'s (15), (16), (18), or (19) is small in modes of any order.

When the structure has light damping, the value computed from eq.(19) is subject to very little change due to variations in damping. Hence we can say that f_m is an invariant which depends only on the allocation of masses and springs in the system. Even if the designer does not have sufficient knowledge about the damping, f_m can be calculated exactly. The values of the modal flexibility are directly dependent on the design modification, but cannot be controlled effectively after the design is finalized and the machine is actually manufactured.

On the other hand, the value of the logarithmic decrement is inherent to each resonance mode, and generally exhibits little interaction with the values in other modes. The logarithmic decrement also has nonlinear and indeterminate characteristics. Furthermore, its value can be adjusted by altering the tightness of bolted joints, the preloading of bearings or the addition of the proper damping, even after the machine has been manufactured.

Therefore, in order to minimize the possibility of regenerative chatter, first, the design should be changed so that the modal flexibility f_m over the whole frequency range becomes small. Next, optimization with respect to the logarithmic decrement should be carried out. Using the above principle of optimization, the method whereby a series of design changes may be made according to changes of potential and kinetic energy distribution through the

Fig.1 Illustration of the coordinate system used in theoretical analysis (Horizontal milling machine)

whole system, is described below.

The procedure for producing an optimum design with respect to regenerative chatter vibrations in a machine tool whose original design is given are as follows:

(1) Using eq.(20), the modal flexibilities in several resonance modes are calculated in respect to the relative exciting force in the cutting force direction and the relative displacement normal to the cut surface. As shown in eq.(21), the total sum of the modal flexibility over the whole frequency range is equal to the static compliance. Therefore, if the sum of the modal flexibilities in a limited number of resonance modes is almost equal to the static compliance, the flexibilities in modes not used for determining the stability limit of chatter can be neglected, and so it is unnecessary to compute them. Using calculations of modal flexibility, the resonance modes having relatively high f_m/f_s values are selected for further analysis for the purpose of design modification.

(2) After consideration of the energy distribution in the mode, design changes such that energy is more evenly distributed in the system are made.

(3) In a resonance mode having large modal flexibility, attempts are made to allocate greater latent damping at those parts of the machine which have greater potential energy, and the logarithmic decrement δ_{Am} shown in eq.(7) is increased.

4. Examples of modal flexibility and logarithmic decrement

The factors which influence the magnitude of the modal flexibility and the logarithmic decrement are exemplified below.

4.1 Modal flexibility

Fig. 2(a) shows the computed results of the modal flexibility in each resonance mode of a bending vibration model of a cantilever beam having uniform cross-section and a length of 1000 mm. The computed model was excited in a direction orthogonal to the center line of the beam at point A, X mm from the clamped end and the displacement of the same point is picked up. The solid line in Fig. 2 is the modal flexibility in the first resonance mode; the broken line, in the second; and the dotted line, in the third.

Fig. 2(b) shows the ratio of modal flexibility f_m for each resonance mode at point A to the static compliance f_s at the same point, plotted against X. The ratios f_m/f_s vary greatly according to X. By applying this model to eq.(16) or (19), it was seen that the variation of the maximum value of compliance near the resonance frequency depends on the modal flexibility, because the logarithmic decrement value of a model with no joints is the same in each resonance mode. It was concluded that the

modal flexibility is the most important factor for optimum design with respect to maximum compliance.

4.2 Logarithmic decrement

In order to exemplify the fact that the value of the logarithmic decrement depends on the energy distribution, an experiment was carried out using the model shown in Fig. 3. The 30-mm diameter steel bar, which had a gas weld in it, was clamped at both ends. The welding junction point X was varied, but the distance between one end and the other was kept constant at 1000mm, and the frequency responses at each junction point were obtained. The bar was clamped on clamping stands, each with 6 bolts, which were bolted to the base. To change the location of the joint, the bolts holding the steel bar were released, and it was moved along its own axis and then clamped again.

Modal stiffness k_i and equivalent damping ratio ζ_i were obtained using the curve fitting methods, from the frequency responses. The logarithmic decrement was calculated from the relation $\delta_i = \pi \zeta_i / \sqrt{1 - \zeta_i^2}$. Fig. 4 shows the theoretical values of the maximum potential energy distributions in 1st - 3rd resonance modes of the bending vibration of the uniform bar. Fig. 5 shows the experimental values of the maximum compliance, modal stiffness and logarithmic decrement plotted against the varying locations, X, of the joint in the 1st resonance mode.

Fig.2 Computed results of modal flexibility in bending resonance modes and ratio of modal flexibility to bending static compliance

In spite of the changes of location of the joint and the presence of the weld, the first resonance frequency was nearly constant; and the modal flexibility was nearly constant, as shown in Fig. 5. From these results, the modal shapes and energy distributions seem to be constant for all cases. In fig. 5, the value of the logarithmic decrement is minimum where the location of joint $X = 250$ mm, and the curve is conical. These results verify that the loss-coefficients at a welded joint are far larger than the material damping of a steel bar, and that the values of the logarithmic decrement depend on the potential energy distribution, as indicated in eq.(7).

Fig.3 Experimental model of double-clamped bar with welded-joint

Fig.5 Changes of dynamic characteristics in 1st resonance mode in relation to the location of welded joint

5. Optimization procedure by computer program system

Fig. 6 shows the flowchart of design optimization by means of computer program, such that the modal flexibility f_m in respect to the relative exciting force in the cutting force direction and the relative displacement normal to the cut surface are decreased, and that the static compliance f_s is evenly shared by modal flexibilities in many modes. The procedure for optimizing the design of either an already manufactured machine or a new machine is as follows.

Fig.4 Computed results of potential energy distributions of a uniform beam in three resonance modes (doubled-clamped beam)

Fig.6 Flow chart of process for design optimization of structures by energy balance method

Table 1 Factors contributing to stiffness and inertia of beams

	Factors contributing to stiffness	Factors contributing to inertia
Flexural vibration	Modulus of elasticity: E	Density: ρ
	Geometrical moment of inertia: I	Cross-sectional area: A
	Beam length: l	Beam length: l
Torsional vibration	Shear modulus: G	Density: ρ
	Torsional stiffness: J	Polar moment of inertia: I_p
	Beam length: l	Beam length: l
Longitudinal vibration	Modulus of elasticity: E	Density: ρ
	Cross-sectional area: A	Cross-sectional area: A
	Beam length: l	Beam length: l

First, a mathematical model is made from the blueprint or the initial design, and is simulated. The compliance of the relative force between tool and work, and of the relative displacement in the direction normal to the cut surface, is calculated for the simulation model. Next, the resonance frequency, the modal flexibility f_m and the ratio f_m/f_s of the modal flexibility f_m to the static compliance f_s are calculated. Thus, the resonance modes with greater modal flexibility than the others are found. In these resonance modes, the potential and the kinetic energy distribution through the whole system are calculated. Potential and kinetic energies per unit mass are calculated and energy distributions in respect to flexural, torsional and longitudinal vibrations are obtained separately. From these results, the subsystems which have the greatest values for energy distribution or for energy per unit mass are detected, since they are the ones requiring redesign.

The concentration of the energy distribution in each subsystem is considered in the following three ways:

- (1) Concentration of potential energy only
- (2) Concentration of kinetic energy only
- (3) Concentration of both potential and kinetic energies

In case (1), the stiffness referent to the coordinate direction of the subsystem should be increased. In case (2), the inertia referent to the coordinate direction of the subsystem should be decreased. In case (3), design changes should be carried out, such as increasing the stiffness and decreasing the inertia, though if it turns out in practice to be impossible to do both of these, the more important one should be selected.

Table 1 shows the factors for the stiffness and the inertia related to the flexural, torsional and longitudinal vibrations of the beam element. In many machine tool structures, the frameworks and lengths of the members are already set, and the same material is almost always used throughout. Therefore, in the receptance synthesis program system, the optimum design is made using the cross-sectional shape as the variable design parameter. Cross-sectional shapes include circular, rectangular, thin-walled cylinder, hollow circular and box-type cross-sections, as well as I- and J-sections. For each cross-section, the relations between the cross-sectional area A, the geometrical moment of inertia I, the torsional stiffness J

and the polar moment of inertia I_p are derived, and data for A, I, J and I_p in the redesign are produced.

The modified coefficients for the design are specified as percentages of the input data. When the new data are determined, the same calculations are repeated through the loop shown by the broken line in Fig. 6. If the design parameters of the revised design do not satisfy the restraint function, the data of the next subsystem under consideration are changed. When there are no further resonance modes to be considered on repetition of the above procedure, the computation ends as the optimized design has been obtained. The above processes are automatically carried out in the receptance synthesis program. When the computing time is restricted, the process of calculation may be divided into parts and carried out at different times until the optimum design is obtained.

6. Applied example of optimum design theory

An example of design optimization for the structure in Fig. 7, using a computer program based on the receptance program system, follows.

The structure is a dressing unit to be installed on an internal grinding machine. It has a height of 427 mm, a weight of 54 kg, and is made of cast iron. The machine was manufactured on a trial basis. The outer surface of the grinding wheel is dressed by a diamond tool mounted at the tip of arm A. The arm, hinged by a pair of roller bearings, assumes a horizontal position for dressing supported by the arm rest which is a cantilever beam extended from the axle bracket.

Fig.7 Illustration of the dressing unit

Fig. 8 shows the mathematical model for the analysis of its dynamic characteristics. The structure is divided into 16 beam element subsystems, and synthesized into a complete system. The rotating part of the arm is simulated by a flexible element with springs whose coefficients are calculated from the bearing stiffness of the roller bearings.

The computed and experimental values of the frequency responses at the dressing point A of the model were derived. The solid line in Fig. 9 gives the experimental results of the frequency responses of receptances on measurement of the displacement in Z direction when the dressing point A of the original design was excited in Z direction. The solid line in Fig. 10 shows the results computed with the mathematical model. The computed results corresponded well with the experimental ones, including those related to the modal shapes. Table 2 summarises the computed results of modal flexibility and energy distributions for the original design, up to the 3rd resonance mode.

It is immediately seen that in the 1st order mode, the modal flexibility in Z direction is large, accounting for more than 98 percent of the total in that direction. Thus, it is seen that the structure can be greatly improved by improving the rigidity with respect to this mode.

Considering the results listed for the 1st mode, it is seen that 60 percent of the potential energy is stored in the arm rest. Energy distributions per unit mass corresponding to eq. (12) show wide scatter, and the armrest has the largest value for concentration of strain. Therefore, increase of the stiffness of the arm rest is indicated.

The first modification has been carried out: the design of the axle bracket has been changed so that the flexural rigidity of its connecting armrest is about ten times as great as in the original design. A mathematical model which differs in the neighborhood of the armrest has been made for the revised design, also shown in Fig. 8.

The arm element has the largest kinetic energy distribution in Z direction in the results computed for revised design 1. Consequently, the thickness of the arm was decreased by 30 percent for revised design 2.

As revised design 2 has a concentration of kinetic energy at the micro-adjuster element, the weight of that element was decreased by 55 percent for revised design 3.

Fig. 8 Mathematical model of the dressing unit

The measured and computed results of frequency responses shown in Figs. 9 and 10 show a considerable degree of agreement. Table 3 shows the static compliance f_s in Z direction, the resonance frequency ω_m , which warrants further consideration, the modal flexibility f_m and f_m/f_s . In revised design 1, the static compliance f_s , the modal flexibility f_m , and f_m/f_s are greatly improved. In revised designs 2 and 3, the resonance frequency ω_m increases and f_m/f_s decreases, representing an improvement of the dynamic characteristics.

7. Conclusions

In order to attain a dynamically optimum design with minimum danger of machining chatter, a design optimization method based on the analysis of modal flexibilities and energy distributions was developed and theoretical analysis was carried

Fig. 9 Measured results of frequency responses

Fig. 10 Computed results of frequency responses

Table 2 Computed results of modal flexibility and energy distribution of the dressing unit

		Original design				
		Static compliance	Mode 1 108.7Hz (Experimental result 129.6Hz)	Mode 2 122.1Hz (113.0Hz)	Mode 3 267.0Hz (279.3Hz)	Model 108.7Hz Energy distribution per unit mass
Static & modal flexibility in μ/kg	X direction	13.9		13.1		
	Y direction	2.74	2.73		0.000131	
	Z direction	18.8	18.4		0.372	
Ratios of kinetic energy	Dressing tool & micro adjuster		0.353	0.326	0.0302	30.8
	Arm (Subsystem 3)		0.472	0.414	0.0465	7.28
	(Subsystem 4,5,6,8)		0.130	0.188	0.0630	3.45
	.		.	.	.	.
	.		.	.	.	.
	Total		1.0	1.0	1.0	
Ratios of potential energy	.		.	.	.	.
	.		.	.	.	.
	Roller bearing		0.00587	0.0200	0.00415	
	Axle bracket	Subsystem 9 (bending	0.0456	0.0796	0.00283	1.65
		torsion		0.169		
	Subsystem 10, 11	(bending	0.103	0.0392	0.0471	3.93
		torsion		0.173		
	Arm rest	(bending	0.600	0.00019	0.0937	48.0
		Subsystem 7 (torsion				
	Slide rail	(bending	0.178	0.0679	0.721	3.40
		torsion		0.212		
Column and base	(bending	0.0572	0.111	0.127	0.0212	
	torsion		0.0629			
Total		1.0	1.0	1.0		

Table 3 Computed results of dynamic characteristics of the dressing unit at the dresser point A in the revised designs as compared with the original design

	Original design	Revised design 1	Revised design 2	Revised design 3
Design change		Increase rigidity of armrest	Reduce arm weight	Reduce micro adjuster weight
Direction of excitation and pickup	Z direction	Z direction	Z direction	Z direction
Static compliance $f_s \mu/\text{kg}$	18.8	3.63	4.31	4.37
Resonance frequency m, Hz	108.7	165.9	170.2	201.1
Modal flexibility $f_m, \mu/\text{kg}$	18.4	2.44	2.88	2.14
f_m/f_s	0.979	0.705	0.693	0.490

out. The procedure and method of optimization by means of a computer program system were described and their validity was demonstrated by computed and experimental results with an actual structure.

The optimization method proposed in this paper, which is based on the energy balance principle, is very useful, because actual machine structures are complex and have many parameters for design change, and application of the mathematical method is difficult.

The method described in this paper can be applied to many other computer program systems for calculating dynamic characteristics and carried out not only for machine tools but for other machine structures as well.

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