Radiative Heat Transfer in Cylinder-Row Systems

(Part 2, Variation of Heat-Transfer

Characteristics with Number of Rows)*

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In succession to the previous study, radiative heat transfer in three-row and four-row gray cylinder systems with square-center arrangements is exactly analyzed to discuss the influence of the number of rows on the characteristics of heat transfer of these systems. Radiative heat transfer in semi-infinite rows of gray cylinders as a limiting case of a large number of rows is also analyzed approximately by using a simplified model, and the accuracy of the solutions is evaluated. On the basis of equivalent emissivity, the characteristics of cylinder-row systems are examined in relation to the number of rows, surface emissivity and center-to-center distance of cylinders.

1. Introduction

A bundle of cylinders aligned and arranged in parallel rows at an equal interval has been used as a favorable configuration of heat-transfer surfaces. Such arrangement of cylinder rows certainly can provide favorable surfaces for convective heat transfer in many cases, but under specific conditions, radiative heat exchanges between the cylinder-surfaces and the environment apparently share a significant proportion in the total heat-transfer rate of this system of cylinders. The pursuit in this regard, however, so far has remained unsatisfactory. Previous reports concerned with this problem include one on two rows of black tubes within the furnace⁽¹⁾⁽²⁾, and another on recent approximations of radiative transfer rates in pin-fin-type radiators⁽³⁾. In other words, research still remains short of pursuing the influences of the emissivity of cylinder surfaces and the number of rows on general characteristics in the radiative transfer of such a system.

In the preceding report⁽⁴⁾, the radiative transfer was analyzed in the system of two-row gray cylinders arranged on square-centers to discuss the variations in heat flux on the cylinder circumference as well as the influences of emissivity and center-to-center distance of cylinders on the heat flux of the system. Also in the system of semi-infinite-row black cylinders, the charac-

teristics of radiative transfer were discussed. The present study is planned to find a clue to more comprehensive understanding of the radiative transfer characteristics of gray-cylinder-row systems. Efforts are made to identify the influence of the number of rows on the radiative transfer with relation to the surface emissivity and center-to-center distance of cylinders. Basic equations are derived for the m -row system, similar in manner to the preceding two-row system. In order to make computation simple, numerical solutions are obtained limitedly for three- and four-row cylinder systems, while for the semi-infinite-row system (a typical limiting arrangement for a considerably large number of rows), approximate solutions are obtained with a simplified model. The equivalent emissivity evaluated from the present and the preceding results, expressing the radiative heat-transfer characteristics of cylinder-row systems, is discussed in relation to the number of rows.

2. Nomenclature

- $B, \bar{B}$: radiosities
 C : center-to-center distance
 F : angle factor
 H_{i-j} : irradiation (i.e., incident energy flux) from cylinders in j -th row to cylinder i
 $l \equiv |i - j|$
 $q, \bar{q}$: net heat fluxes
 R : cylinder radius
 T_w : temperature of cylinder surface
 $\beta, \bar{\beta}$: dimensionless radiosities, $B/\epsilon\sigma T_w^4, \bar{B}/\epsilon\sigma T_w^4$
 γ : dimensionless distance, C/R
 ϵ : emissivity of cylinder surface
 ϵ_e : equivalent emissivity, Eq. (18)
 ϵ_p : emissivity of plane P (Fig.3)

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- η_{i-j} : dimensionless irradiation,
 $H_{i-j} / \epsilon \sigma T_w^4$
 θ : angular position coordinate
 on basic cylinder (Fig. 2)
 σ : Stefan-Boltzmann constant
 ϕ : incident angle (Fig. 2)
 ϕ', ϕ'' : incident angles from both
 edges of visible arc of one
 cylinder, $\phi' > \phi''$ (Fig. 2)
 ψ : angular position coordinate
 on cylinder radiating to
 basic cylinder

Subscripts

- $1, 2, \dots, m$: first, second, $\dots$, m -th
 row
 $i0, i1, \dots$: cylinder $i0, i1, \dots$
 $1^*, 1^{**}, \dots$: divided surface $1^*, 1^{**}, \dots$

3. Analysis

3.1 Basic equations and numerical solution

The analytical procedures in the preceding study⁽⁴⁾ are applied to obtain basic equations for radiative transfer in the system of cylinders arranged in arbitrary finite rows.

Figure 1 shows a schematic diagram of

the system of cylinders, radius R , finite m rows in a square-center arrangement of center-to-center distance C . The radiative transfer between these cylinder rows and the environment (imagined as surfaces indicated by the dotted lines) is analyzed under the conditions that each cylinder is infinite in length and each row has cylinders lined infinitely. In performing the analysis, the following assumptions are provided: (i) Each cylinder surface is gray, and diffuse in emission and reflection, having the same emissivity ϵ . (ii) Temperature T_w is uniform on each surface and the same in all cylinders. (iii) The environment is black with zero temperature, and the intervening medium is transparent for radiation.

With these assumptions, the radiative heat fluxes on the cylinder circumference can be identified by solving the simultaneous equations for the radiosities of the cylinders in the rows concerned. For the analysis, cylinders are longitudinally denoted by $i0, i1, \dots$, and $m0$ to represent basic cylinders for the first, second, $\dots$, and m -th row, respectively (Fig. 1). In an arbitrary i -th row, an angular

Fig. 1 Configuration of cylinder-row system

Fig. 2 Angular coordinate system

position on basic cylinder $i0$ is represented by angle θ_i ($i = 1, 2, \dots, m$), clockwise positive (Fig. 2). Because of the symmetric arrangement of cylinders, analysis of the system is made on the cylinder semicircumference $\theta_i = 0 \sim \pi$. The dimensionless radiosity $\beta_i(\theta_i)$ at an arbitrary point A_i on cylinder $i0$ can be expressed with the dimensionless irradiation η_{i-j} from the cylinders in the j -th row ($j = 1, 2, \dots, m$) using the following

$$i = j; \quad \eta_{i-j}(\theta_i) = \frac{1}{2} \int_{\phi_{i,i1}''}^{\phi_{i,i1}'} \beta_i(\psi_{i1}) \cos \phi_i d\phi_i \quad (2)$$

$$i \neq j; \quad \eta_{i-j}(\theta_i) = \frac{1}{2} \sum_k \int_{\phi_{i,jk}''}^{\phi_{i,jk}'} \beta_j(\psi_{jk}) \cos \phi_i d\phi_i + \frac{1}{2} \sum_{k'} \int_{\phi_{i,j'k'}''}^{\phi_{i,j'k'}'} \beta_j(\psi_{j'k'}) \cos \phi_i d\phi_i \quad (3)$$

In Eqs. (2) and (3), the angle ϕ shown in Fig. 2, is clockwise positive from the normal at A_i ; the angle ψ indicative of the angular position on the cylinder circumference radiating energy toward A_i is counterclockwise positive for the cylinders lining rightward from the basic cylinder, while clockwise positive for others lining leftward. In Eq. (3), the right member which shows the sum of irradiation from cylinders in the j -th row is expressed in two terms for the convenience of analysis; the first term indicates the part from the cylinders lining rightward from cylinder $j0$ and the second term the part from others lining leftward. In the same equation, k and k'

integral equation within $0 \leq \theta_i \leq \pi$:

$$\beta_i(\theta_i) \equiv \frac{B_i(\theta_i)}{\epsilon \sigma T_w^4} = 1 + (1 - \epsilon) \sum_{j=1}^m \eta_{i-j}(\theta_i) \quad (1)$$

($i = 1, 2, \dots, m$)

where the second term in the right-hand side accounts for the sum of reflected energies. The irradiation η_{i-j} can be expressed as

are the values related inherently to the radiating cylinder-row j , representing k_j and k'_j , respectively (suffix j omitted to simplify the symbol). In Eq. (3), k , k' and $\phi'_{i,j}$, $\phi''_{i,j}$ in each row, varying according to the position of A_i , can be determined geometrically.

In Eqs. (2) and (3), ψ and $\phi'_{i,j}$, $\phi''_{i,j}$ are given as follows:

(i) $i = j$:

ψ_{i1} , and $\phi'_{i,i1}$ and $\phi''_{i,i1}$ in Eq. (2) respectively agree with those derived in the preceding report [Eqs. (2.a) and (2.b) and Eqs. (6) and (7) in Reference (4)].

(ii) $i < j$:

ψ_{jk} , and $\phi'_{i,jk}$ and $\phi''_{i,jk}$ in Eq. (3) become

$$\psi_{jk}(\theta_i, \phi_i) = \frac{3}{2} \pi - \theta_i - \phi_i - \cos^{-1} \{ [1 + \gamma^2(\ell^2 + k^2) - 2\gamma(k \sin \theta_i + \ell \cos \theta_i)]^{1/2} \sin \alpha_j \} \quad (4.a)$$

$$\alpha_j = \theta_i + \phi_i - \tan^{-1} \frac{k\gamma - \sin \theta_i}{\ell\gamma - \cos \theta_i} \quad (4.b)$$

$$0 \leq \theta_i \leq \frac{\pi}{2} - \tan^{-1} \frac{\ell}{k} + \tan^{-1} \{ \frac{\gamma^2(\ell^2 + k^2)}{4} - 1 \}^{1/2};$$

$$\phi'_{i,jk} = \phi_{i,jk}^+, \quad \phi''_{i,jk} = \phi_{i,jk}^- \quad (5.a)$$

$$\frac{\pi}{2} - \tan^{-1} \frac{\ell}{k} + \tan^{-1} \{ \frac{\gamma^2(\ell^2 + k^2)}{4} - 1 \}^{1/2} \leq \theta_i \leq \pi - \tan^{-1} \frac{\ell}{k};$$

$$\phi'_{i,jk} = \phi_{i,jk}^+, \quad \phi''_{i,jk} = -\frac{\pi}{2} \quad (5.b)$$

where

$$\begin{aligned} \phi_{i,jk}^+ &= -\theta_i + \tan^{-1} \frac{k\gamma - \sin \theta_i}{\ell\gamma - \cos \theta_i} + \tan^{-1} \frac{1}{\{ \gamma^2(\ell^2 + k^2) - 2\gamma(k \sin \theta_i + \ell \cos \theta_i) \}^{1/2}} \\ \phi_{i,jk}^- &= -\theta_i + \tan^{-1} \frac{k\gamma - \sin \theta_i}{\ell\gamma - \cos \theta_i} - \tan^{-1} \frac{1}{\{ \gamma^2(\ell^2 + k^2) - 2\gamma(k \sin \theta_i + \ell \cos \theta_i) \}^{1/2}} \end{aligned} \quad (6)$$

And $\psi_{j,k}$, and $\phi'_{i,j,k}$, and $\phi''_{i,j,k}$, are

$$\psi_{j,k}(\theta_i, \phi_i) = \frac{\pi}{2} + \theta_i + \phi_i + \cos^{-1} \{ [1 + \gamma^2(\ell^2 + k'^2) + 2\gamma(k' \sin \theta_i - \ell \cos \theta_i)]^{1/2} \sin \alpha'_j \} \quad (7.a)$$

$$\alpha'_j = \theta_i + \phi_i + \tan^{-1} \frac{k'\gamma + \sin \theta_i}{\ell\gamma - \cos \theta_i} \quad (7.b)$$

$$0 \leq \theta_i \leq -\frac{\pi}{2} + \tan^{-1} \frac{\ell}{k'} + \tan^{-1} \{ \frac{\gamma^2(\ell^2 + k'^2)}{4} - 1 \}^{1/2} ;$$

$$\phi'_{i,j,k} = \phi^+_{i,j,k}, \quad \phi''_{i,j,k} = \phi^-_{i,j,k} \quad (8.a)$$

$$-\frac{\pi}{2} + \tan^{-1} \frac{\ell}{k'} + \tan^{-1} \{ \frac{\gamma^2(\ell^2 + k'^2)}{4} - 1 \}^{1/2} \leq \theta_i \leq \tan^{-1} \frac{\ell}{k'} ;$$

$$\phi'_{i,j,k} = \phi^+_{i,j,k}, \quad \phi''_{i,j,k} = -\frac{\pi}{2} \quad (8.b)$$

where

$$\begin{aligned} \phi^+_{i,j,k} &= -\theta_i - \tan^{-1} \frac{k'\gamma + \sin \theta_i}{\ell\gamma - \cos \theta_i} \\ &\quad + \tan^{-1} \frac{1}{\{ \gamma^2(\ell^2 + k'^2) + 2\gamma(k' \sin \theta_i - \ell \cos \theta_i) \}^{1/2}} \\ \phi^-_{i,j,k} &= -\theta_i - \tan^{-1} \frac{k'\gamma + \sin \theta_i}{\ell\gamma - \cos \theta_i} \\ &\quad - \tan^{-1} \frac{1}{\{ \gamma^2(\ell^2 + k'^2) + 2\gamma(k' \sin \theta_i - \ell \cos \theta_i) \}^{1/2}} \end{aligned} \quad (9)$$

(iii) $i > j$:

From the geometry, these angles can be adequately expressed with equations corresponding to Eqs. (4) through (9) by replacing, in the equations in (ii), θ_i with $(\pi - \theta_i)$; ϕ_i with $-\phi_i$; ϕ' with $-\phi''$; ϕ'' with $-\phi'$ and so on.

Equations (5) and (8) for ϕ' and ϕ'' are valid provided that irradiation from the object cylinder is not obstructed at all on the way to A_i by any other cylinder. If the irradiation is partly intercepted by another cylinder, an angle between the normal at A_i and a tangent from A_i to the intercepting cylinder should be employed in place of ϕ' or ϕ'' in these equations, as described in the preceding report.

In the basic integral equations, Eqs. (1) through (3), containing m unknown functions ($\beta_1, \beta_2, \dots, \beta_m$), the dimensionless radiosities are equal in symmetrically arranged cylinders. Therefore, according as m is an even or an uneven number, these basic equations can be solved simultaneously for the $(m/2)$ set with $i = 1, 2, \dots, m/2$, or for the $(m+1)/2$ set with $i = 1, 2, \dots, (m+1)/2$, in Eq. (1). Once the solutions are obtained, the net local heat flux $q_i(\theta_i)$ can be obtained for $\varepsilon < 1$ from

$$\frac{q_i(\theta_i)}{\varepsilon \sigma T_w^4} = \frac{1}{1 - \varepsilon} \{ 1 - \varepsilon \beta_i(\theta_i) \} \quad (10)$$

and for $\varepsilon = 1$ (without solving the basic equations) from

$$\begin{aligned} \frac{q_i(\theta_i)}{\sigma T_w^4} &= 1 - \left(\sum_j \sum_k F_{d\theta_i-jk} \right. \\ &\quad \left. + \sum_{j'} \sum_{k'} F_{d\theta_i-j'k'} \right) \end{aligned} \quad (11)$$

where $F_{d\theta_i-jk}$, for instance, is the angle factor between an infinitesimal element at A_i and the whole area of cylinder jk .

Radiative heat fluxes on a system of finite rows may be obtained theoretically by solving Eqs. (1) through (3), but practically, even for not so many rows, computational processes are complex, making these equations considerably difficult to solve. In the present study, we performed computations only for three- and four-row systems ($m = 3, 4$) to estimate the general characteristics of the influence of an increase in row on radiative heat fluxes in such a system. To solve the integral equations, the iterative method was employed with $\Delta\theta_i =$

0.01π as the step size of θ_i . In the process of numerical solution of Eq. (3) to evaluate the irradiation from the cylinders in the adjacent row, k and k' become infinite at $\theta_i = 0$ or π , or considerably large in its neighborhood, requiring approximations for the corresponding integral terms of extremely small values, as described in the preceding report⁽⁴⁾.

3.2 Approximation for system of semi-infinite-row gray cylinders

Gray cylinders in multiple rows can be typified by a system of such cylinders in semi-infinite rows, that is, in rows arrayed infinitely on one side from the first. The radiative transfer in such a system of cylinder rows is worth attention to ascertain the upper limit of heat transfer in a system of multiple rows or to estimate the rate of heat transfer that is difficult to obtain numerically in arbitrary finite rows as described in the foregoing section.

Application of the basic equations in section 3.1 is unavailable to obtain exact values of local heat fluxes in this cylinder-row system, so that a simplified model is employed to obtain approximate values of the radiative heat fluxes in this system. The geometrical and physical conditions of this model are assumed identical with the foregoing finite-row systems except for the number of rows. Additionally postulated in the simplification procedures are: (i) All the rows arrayed further than an arbitrary one are replaced with a plane P which is parallel to the rows and has an emissivity ϵ_p and the same temperature T_w as on the cylinders. (ii) Each cylinder surface is

Fig. 3 Simplified model of semi-infinite-row system

† Because of the symmetric distribution of radiosities, this twofold division is essentially equal to a fourfold division further partitioned with another center line vertical to the plane P.

roughly divided into several sections; radiosity is uniform on each section and on the plane P.

To make calculation easy, the plane P is placed at the fifth row, and the surface of each cylinder in the first through the fourth row is halved by its circle-center line parallel to the plane as shown by the model in Fig. 3†. The plane P is an imaginary plane approximating the whole of the fifth and further rows arrayed infinitely. Its emissivity ϵ_p may be regarded to be $\epsilon < \epsilon_p < 1$, but is retained as unknown until a definite choice is available. For eight of divided cylinder surfaces (denoted by 1^* , 1^{**} , ..., 4^* , 4^{**}) and for the plane P the following nine equations can respectively be given to express their $\bar{\beta}$ — the dimensionless radiosity in this approximate analysis:

$$\begin{aligned}\bar{\beta}_1^* = \{ & 1 + (1 - \epsilon)(F_{1^*-1^{**}R}\bar{\beta}_1^{**} \\ & + F_{1^*-2^*R}\bar{\beta}_2^* + F_{1^*-2^{**}R}\bar{\beta}_2^{**} \\ & + F_{1^*-3^*R}\bar{\beta}_3^* + F_{1^*-3^{**}R}\bar{\beta}_3^{**} \\ & + F_{1^*-4^*R}\bar{\beta}_4^* + F_{1^*-4^{**}R}\bar{\beta}_4^{**} \\ & + F_{1^*-P}\bar{\beta}_P)\} / \{1 - (1 - \epsilon)F_{1^*-1^*R}\} \\ & \dots\dots\dots\end{aligned}$$

$$\begin{aligned}\bar{\beta}_4^{**} = \{ & 1 + (1 - \epsilon)(F_{4^{**}-1^*R}\bar{\beta}_1^* \\ & + F_{4^{**}-1^{**}R}\bar{\beta}_1^{**} + F_{4^{**}-2^*R}\bar{\beta}_2^* \\ & + F_{4^{**}-2^{**}R}\bar{\beta}_2^{**} + F_{4^{**}-3^*R}\bar{\beta}_3^* \\ & + F_{4^{**}-3^{**}R}\bar{\beta}_3^{**} + F_{4^{**}-4^*R}\bar{\beta}_4^* \\ & + F_{4^{**}-P}\bar{\beta}_P)\} / \{1 - (1 - \epsilon)F_{4^{**}-4^{**}R}\} \end{aligned} \quad (12)$$

$$\begin{aligned}\bar{\beta}_P = \frac{\epsilon_p}{\epsilon} + (1 - \epsilon_p) \{ & F_{P-1^*R}\bar{\beta}_1^* \\ & + F_{P-1^{**}R}\bar{\beta}_1^{**} + F_{P-2^*R}\bar{\beta}_2^* \\ & + F_{P-2^{**}R}\bar{\beta}_2^{**} + F_{P-3^*R}\bar{\beta}_3^* \\ & + F_{P-3^{**}R}\bar{\beta}_3^{**} + F_{P-4^*R}\bar{\beta}_4^* \\ & + F_{P-4^{**}R}\bar{\beta}_4^{**} \} \end{aligned} \quad (13)$$

In Eq. (12), $F_{1^*-1^{**}R}$, for example, represents the angle factor from a half-cylinder 1^* to all of half-cylinders 1^{**} in sight. When several of such angle factors have been obtained in some equation, all other angle factors involved can be obtained on the following reciprocal relations:

$$\begin{aligned}F_{1^*-1^*R} = F_{1^{**}-1^{**}R} = F_{2^*-2^*R} = \dots \\ F_{1^*-1^{**}R} = F_{1^{**}-1^*R} = F_{2^*-2^{**}R} = \dots \\ \dots\dots\dots\end{aligned} \quad (14)$$

The angle factors in Eq. (13) can be obtained by using the angle factors involved in Eq. (12) and with F_{P-4}^{**R} in the following equation in the order of F_{P-4}^{*R} , F_{P-3}^{**R} , ..., F_{P-1}^{*R} :

$$F_{P-4}^{**R} = \frac{\pi}{\gamma} F_{4}^{**P} = 1 + \frac{1}{2} \frac{\pi}{\gamma} - \frac{1}{\gamma} \{ (\gamma^2 - 2\gamma)^{1/2} + \tan^{-1} \frac{1}{(\gamma^2 - 2\gamma)^{1/2}} \} \quad (15)$$

The irreducible minimum numbers of angle factors for solving Eqs. (12) and (13) were obtained by the crossed-string method(5) as in the preceding study.

To solve simultaneously Eqs. (12) and (13) that have unknowns including ϵ_P beside nine of $\bar{\beta}$, it is necessary to determine ϵ_P anyway. Accordingly, ϵ_P is approximated by using an equivalent emissivity derived through the following procedures:

The relation between the heat flux $\bar{q}$, in the approximate analysis, on the divided cylinder surface and the radiosity $\bar{\beta}$ can be expressed for an arbitrary i -th row as

$$\bar{q}_i^* + \bar{q}_i^{**} = \frac{\epsilon \sigma T_w^4 \{ 2 - \epsilon (\bar{\beta}_i^* + \bar{\beta}_i^{**}) \}}{1 - \epsilon} \quad (i = 1, 2, 3, 4) \quad (16)$$

and on the plane P as

$$\bar{q}_P = \frac{\epsilon_P \sigma T_w^4 (1 - \epsilon \bar{\beta}_P)}{1 - \epsilon_P} \quad (17)$$

For the radiation heat loss from this cylinder-row system to the environment, the equivalent emissivity ϵ_e is defined as

$$\epsilon_e = \frac{q_e}{e_b} \quad (18)$$

where q_e and e_b respectively represent a mean heat-transfer rate per unit area to the environment from a plane replacing in parallel the cylinder rows, and the emissive power of a black surface at T_w . The numerator in the above equation corresponds to the mean heat flux through a plane placed outside the system in parallel to rows. By using the heat flux $\bar{q}$, this equivalent emissivity can be expressed as

$$\epsilon_e = \frac{\pi \sum_{i=1}^4 (\bar{q}_i^* + \bar{q}_i^{**})}{\gamma \sigma T_w^4} + \frac{\bar{q}_P}{\sigma T_w^4} \quad (19)$$

Substituting Eqs. (16) and (17) into Eq. (19), we obtain

$$\epsilon_e = \frac{\pi \epsilon}{\gamma (1 - \epsilon)} \left\{ 8 - \epsilon \sum_{i=1}^4 (\bar{\beta}_i^* + \bar{\beta}_i^{**}) \right\} + \frac{\epsilon_P (1 - \epsilon \bar{\beta}_P)}{1 - \epsilon_P} \quad (20)$$

Now, ϵ_P , the emissivity on the plane P, in the above equation is approximated by the equivalent emissivity ϵ_e as

$$\epsilon_P = \epsilon_e \quad (21)$$

With this approximation, Eq. (20) can be rewritten as

$$\epsilon_e^2 - \epsilon \epsilon_e \bar{\beta}_P + \frac{\pi \epsilon (1 - \epsilon_e)}{\gamma (1 - \epsilon)} \times \left\{ 8 - \epsilon \sum_{i=1}^4 (\bar{\beta}_i^* + \bar{\beta}_i^{**}) \right\} = 0 \quad (22)$$

On the divided cylinder surfaces and the plane P, dimensionless radiosities can respectively be obtained together with the equivalent emissivity by replacing ϵ_P with ϵ_e in Eq. (13) and solving simultaneous equations composed of Eqs. (12), (13) and (22). In practical solution, the Newton-Raphson method was used because of Eq. (22) being non-linear.

4. Results and discussion

4.1 Three-row and four-row cylinder systems

In three-row and four-row cylinder systems, as examples of the system of cylinders in finite rows, numerical solutions were obtained in a range of $\gamma = 2 \sim 10$. Figures 4 and 5 indicate the

Fig. 4 Local heat flux distributions on cylinders of three-row system, $\gamma = 5$

Fig. 5 Local heat flux distributions on cylinders of four-row system, $\gamma = 5$

Fig. 6 Mean heat fluxes of each row cylinder and plane P, $\epsilon = 0.7$

solutions, respectively showing the heat flux distributions on the circumferences of cylinders in three- and four-row systems. Both distribution curves over the first-row cylinders show a trend nearly similar to that of the two-row system studied previously⁽⁴⁾. However, as evident from comparison between Figs. 4 and 5, the heat fluxes in the range of $\theta_1 = 0 \sim \pi/2$ tend to decrease with an increase in row, obviously due to the interference of cylinders in the second and any further row obstructing the direct radiation to the environment. The heat flux distribution over the second-row cylinders generally takes a gently wavy curve in both groups (those in the three-row system are symmetric about $\theta_2 = \pi/2$), each showing a minimum around $\theta_2 = \pi/2$. Similar trends of curves can be presumed for the third and further rows in a multiple-row system. In both Figs. 4 and 5 the heat fluxes show curves ascending higher for surfaces with lower ϵ , just as observed in the preceding two-row system⁽⁴⁾.

4.2 Semi-infinite-row gray cylinder system

The curves in Fig. 6 show examples of mean heat fluxes over the cylinders in each row obtained on the present model for a semi-infinite-row system. The dotted curves indicate the mean heat flux on the plane P that substitutes for the fifth and further infinite rows. Besides the variations of mean heat fluxes with γ , this figure indicates the contribution of cylinders in each row to radiation heat loss to the environment. Shown in Fig. 7 is the relation between the equivalent emissivity ϵ_e and the emissivity ϵ . As evident in the trend of each curve, ϵ_e increases monotonously with ϵ , and also with γ , particularly prominent at lower ϵ and smaller γ .

As these results have been obtained through several simplification and approximation procedures, their accuracy must be evaluated in respective instances. What should first be discussed is whether positioning the plane P at the fifth row in the system is appropriate or not. An adequate position of the plane P in the simplified model is one where solutions of

Fig. 7 Equivalent emissivity of semi-infinite-row system

ϵ_e do not vary even if the plane P is shifted somewhat forward or backward. If the plane P is placed far backward, a satisfactory convergence of solutions may be expected. Such a positioning of the plane P, however, needs the calculations of too many angle factors, eventually making ϵ_e extremely difficult to solve. Therefore, in the present study, we chose the fifth row as an appropriate position of the plane P, because the maximum difference was only 0.3 % between the solutions (the equivalent emissivities) obtained from two models with the plane P at the fourth and the fifth row, indicating an acceptable convergence in the fifth row.

In the second place, evaluation must be made on the adequacy of the approximation of the emissivity of the plane P, in Eq. (21). To estimate the extent of error for ϵ_e in the present calculations, we set up, as previously stated, the following expressions indicative of the upper- and the lower-limit values of ϵ_P , respectively:

$$\epsilon_P = 1 \quad (23)$$

$$\epsilon_P = \epsilon \quad (24)$$

Solving simultaneously Eqs. (12) and (13) under the conditions of Eq. (23) and Eq. (24), and obtaining values of equivalent emissivity with Eq. (20), we find the highest and the lowest values of ϵ_e , respectively. The differences between these values and those obtained in the present calculations may be regarded as indicating the limits of error extent in this approximation. These differences are graphically shown in Fig. 8. $\epsilon_{e,b}$ and $\epsilon_{e,\epsilon}$ in the ordinate show the values of equivalent emissivity obtained with Eqs. (23) and (24), respectively. As clear in the figure, the extents of errors expand with an increase in γ or a decrease in ϵ , finally showing maximum error of about 5 % within the present range of γ and ϵ . The following findings may well suggest the extent of error virtually far narrower: Figure 8 additionally shows the extents of errors in the approximate solutions with the plane P replaced by the fourth row. Comparison of these and the results with

Fig. 8 Evaluation of approximation for ε_P

the fifth row indicates that a shift of the plane P backward by one row nearly halves the error extent. This means that the error extent becomes progressively and infinitely less with a more backward shift of the plane P reducing its contribution to the radiation heat loss from the system. Therefore, if the present results were satisfactorily converged solutions with regard to the position of the plane P, the error extents could be replaced by an infinitesimal obtainable with the plane P placed infinitely backward, leaving no problem of error on the approximation. But the actually obtained results are suspected not so well converged as to allow such replacement. However, when the error extents in the present results are supposed to be replaceable by those with the plane P placed backward by two or three rows, and the variations are estimated in the light of Fig. 8, it can well be presumed that the replaced error extents would turn out considerably insignificant, implying the actual extents still more trifling. Another simplification procedure of halving a cylinder is evaluated in the following section.

4.3 The number of rows and equivalent emissivity

In section 3.2, the radiative transfer in the system of semi-infinite rows was approximately analyzed by using the equivalent emissivity defined with Eq. (18). This equivalent emissivity, as clear in the equation, expresses one of the heat-transfer characteristics of cylinder-row systems. It is surely necessary to clarify its changes with the number of rows in the system, and γ or ε .

Figures 9, 10 and 11 show the variations in equivalent emissivity with γ that have been obtained from the results in the present study and those in the preceding one- and two-row systems. In these figures, only the curves for the semi-

infinite-row system represent the results obtained by the approximate technique. Referring to the one-row through the four-row system, it is evident that at $\gamma = 2$, representing the cylinders closely contiguous to each other, all the curves have the same value of ε_e and converge to one point. For $\gamma > 2$, ε_e decreases with an increase in γ in the one-row system, the curve declining monotonously. In the systems of two and more rows, the curves first rise transiently to form peaks (maximum values of ε_e) and then decline with an increase in γ . Keeping such specific trend, these curves shift upward

Fig. 9 Variation of equivalent emissivity with center-to-center distance, $\varepsilon = 0.5$

Fig. 10 Variation of equivalent emissivity with center-to-center distance, $\varepsilon = 0.7$

Fig. 11 Variation of equivalent emissivity with center-to-center distance, $\varepsilon = 0.9$

Fig. 12 Variation of equivalent emissivity with number of rows, $\epsilon = 0.5$

with an increase in the number of rows. The peaks of these curves shift rightward for larger γ with increased rows as indicated by a chain line in Fig. 9. The curves for the semi-infinite-row system, once falling slightly near $\gamma = 2$, generally follow a rising trend but show no peak. These findings suggest that for the systems with a relatively larger number of rows, the curves within a certain range of $\gamma > 2$ form no peaks and approach the curve for the semi-infinite-row system with an increase in the number of rows. This characteristic of equivalent emissivity in some systems of cylinder rows showing its maximum value within a certain range of γ seems an important factor to be taken into consideration in a design of heat-transfer surfaces of cylinder-row-type.

The curves of semi-infinite-row system in Figs. 9 through 11 are inconsistent in trend with other curves at $\gamma = 2$. This apparently indicates the less accuracy of approximate solutions due to the simplification procedure roughly halving the cylinder surfaces and assuming the radiosity uniform on each divided surface. The solutions, if exact, would be consistent with those of the finite-row systems at $\gamma = 2$. The errors attributable to halving the cylinder surfaces, usually larger than those already discussed for other two approximations, tend to lead to solutions considerably large in value. As can be seen in the figures, the errors become larger for surfaces with lower ϵ and for cylinder rows with smaller γ (maximum appears at $\gamma = 2$). In the present calculations, the maximum errors at $\gamma = 2$ are 9.2, 5.6 and 1.9 % for $\epsilon = 0.5$, 0.7 and 0.9, respectively. These errors, however, as demonstrated in other work⁽⁶⁾, are understood to lessen sharply with the expansion of distance between surfaces to a certain extent. At $\gamma = 2.5$, the above

Fig. 13 Variation of equivalent emissivity with number of rows, $\epsilon = 0.7$

Fig. 14 Variation of equivalent emissivity with number of rows, $\epsilon = 0.9$

values are presumed to reduce to nearly half (see Appendix).

The relation between the equivalent emissivity and the number of rows may more clearly be understood referring to Figs. 12, 13 and 14. In the figures, symbols $\circ$ and $\bullet$ respectively indicate the solutions for finite- and semi-infinite-row systems expressed with γ as a parameter; dotted lines connect the results for the same γ , plainly showing their upward trends with an increase in the number of rows. As evident in the figures, the equivalent emissivity is greater for cylinder rows with smaller γ in the one-row system, but this trend is disturbed with an increase in the number of rows, finally showing a complete reversal in the semi-infinite-row system. This, as can also be understood well from Figs. 9, 10 and 11, is attributed to the fact that ϵ_e has its maximum in two or more rows for γ and the rate of increase in ϵ_e with an increase in the number of rows is greater for cylinder rows with larger γ . The differences in the values of ϵ_e at infinite rows (the semi-infinite-row system) are greater for surfaces with lower ϵ as shown in Figs.

12 through 14, but at $\varepsilon = 1$, as easily understood, the values converge to one point, $\varepsilon_e = 1$.

5. Conclusions

Radiative transfer was analyzed in finite-row and semi-infinite-row gray cylinder systems; numerical solutions were obtained for three- and four-row systems, and approximate solutions for semi-infinite-row system. The heat-transfer characteristics clarified here are summarized as follows:

(1) In finite-row systems, heat fluxes on the first row cylinders decrease within $\theta_1 = 0 \sim \pi/2$ with an increase in the rows of the system.

(2) In three- and four-row systems, the curves of heat flux distributions on the second row are distinctly gentle compared with those on the first row, and each has a minimum value around $\theta_2 = \pi/2$.

(3) The approximate solutions obtained for the semi-infinite-row system are estimated high in accuracy except in the range of relatively small γ .

(4) In each of cylinder-row systems with two or more rows there appears a maximum value of equivalent emissivity in relation to γ , but, when the number of rows increases excessively, such a maximum does not appear within a certain range of $\gamma > 2$.

(5) The equivalent emissivity on the cylinder-row system rises with an increase in the rows of the system when γ is constant. The tendency of equivalent emissivity to be larger for a cylinder row with smaller γ in the one-row system is disturbed with an increase in row in the system until it is completely reversed in the semi-infinite-row system.

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Appendix: Evaluation of approximation by halving cylinder surfaces

The approximation by halving cylinder surfaces in cylinder-row systems results in considerably large errors in cases with small γ . In view of the difficulty in exact evaluation of the errors in the semi-infinite-row system over the range of $\gamma > 2$, the errors are evaluated in the three-row and the four-row system to presume therewith those in the semi-infinite-row system.

Figure 15 shows their error variations with γ . In the ordinate, $\varepsilon_{e, \text{approx}}$ denotes the equivalent emissivity evaluated from the solutions obtained with this approximation, and $\varepsilon_{e, \text{exact}}$ denotes that evaluated from numerical solutions by the exact formulation. With an increase in γ , the error extents decrease and sharply drop in the range of smaller γ ; the values at $\gamma = 2$ dwindle to nearly half at $\gamma = 2.5$. This trend is more conspicuous for surfaces with lower ε . With the same ε , the errors in the three-row and the four-row system are almost consistent in the range from $\gamma = 2$ to $\gamma = 2.5 \sim 3$. But at larger γ , errors become larger for the four-row than for the three-row system, suggesting the error growing larger with the larger number of cylinder rows. The consistency of the two curves for the four-row and the three-row system in the range of small γ can be ascribed to the fact that in that range the radiation heat loss is almost exclusive only in the two outermost cylinder-rows in each system, with little contributions from inner cylinder-rows, as can well be understood in the complete agreement of both systems at $\gamma = 2$ irrespective of the number of rows. Then it can be estimated that the errors in the range of $\gamma = 2 \sim 2.5$ for the semi-infinite-row system are almost equivalent to those for the four-row system, and in the range of $\gamma > 2.5$ their decrease turns sluggish with an increase in γ .

Fig. 15 Evaluation of approximate solutions by halving cylinders