Vibration Characteristics of Loaded Vibratory Feeder*

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This paper describes the vibration characteristics of a resonance feeder which is susceptible to the reaction force of the load. When the conveyed bodies move on the vibrating plate, the resonance frequency of the system decreases and the damping effect increases. It is found that the resonance frequency of the feeder varies not only with the inertia force of the load but also with the frictional force. The effect of the load is substituted by the equivalent weight and the equivalent coefficient of viscous damping so that the most suitable exciting force and the exciting frequency are found easily. To keep the amplitude constant in spite of variations of the exciting frequency and the load, the exciting frequency has to be set 2-3 % lower than the resonance frequency.

1. Introduction

The former report⁽¹⁾ described the feeding mechanism of machine parts on the vibrating plate whose amplitude was constant. However, it seems that the amplitude of the practical plate varies with an increasing weight of parts. Therefore, it is necessary to clarify the vibration characteristics including the effects of the conveyed load for the sake of effective use of the feeder. Some studies⁽²⁾⁻⁽⁵⁾ on that characteristics have been reported already, but they seem to lack analytical considerations.

This report makes clear the influence of the load quantitatively and shows charts of equivalent weight and equivalent coefficient of viscous damping in order to apply the analysis to the design of feeder.

2. Analysis on vibration of vibratory machines

Figure 1 shows a simplified vibration model of the vibratory machines which is a one degree of freedom. On the vibrating plate the loads slide, jump or repose relatively.

Notations:

A_i, A_o : amplitude of the vibrating plate in the cases with and without load

W_o, W : weights of plate and load

κ : load ratio $= W/W_o$

k : spring constant

C_o : coefficient of viscous damping without load

F, ω : exciting force and angular frequency

quency

P, Q, L : reaction forces of load

μ : coefficient of friction

g : acceleration of gravity

Z, X, Y : displacements of vibrating plate and load shown in Fig. 1.

v, V : velocity of conveyance and its coefficient

θ_i : phases when the load changes its basic form of motion ($=\omega t_i, i=1, \dots$)

ω_{no} : natural angular frequency without load ($=\sqrt{k g/W_o}$)

$K = a\omega^2/g, K_R = (a\omega^2 \cos(\beta - \alpha))/(g \cos \alpha),$

$K_v = (a\omega^2 \sin(\beta - \alpha))/(g \cos \alpha)$

The vibrating plates are excited by the reaction forces P and Q of the load in addition to the exciting force $F \sin \omega t$. The Z -components L of the resultant force of P and Q are obtained as follows:

$$\left. \begin{aligned} L &= P, & \text{in the case of Fig. 1(a)} \\ L &= -Q, & \text{in the case of Fig. 1(b)} \\ L &= -Q \sin(\beta - \alpha) + P \cos(\beta - \alpha), & \text{in the case of Fig. 1(c)} \end{aligned} \right\} \dots (1)$$

As described in the former report⁽¹⁾, the amplitude determines at any load W both the basic form of motion and the phases θ_i when the basic form changes. Therefore, P, Q and L are proportional to W . L can be separated into W and the reaction force $\bar{L}$ per unit weight. So, we obtain:

$$L = W \cdot \bar{L} = (W_o \cdot \kappa) \cdot \bar{L} \dots (2)$$

It is useful for the later application of the reaction force to expand Eq.(2) in the following Fourier series:

(a) Shaking feeder ($\beta = \alpha$) (b) Vibratory grinder ($\beta = 90^\circ, \alpha = 0^\circ$) (c) Vibratory feeder ($0^\circ < \beta < 90^\circ$)

Fig.1 Models of vibratory machines

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$$\begin{aligned} \bar{L} &= \frac{1}{2} \bar{L}_{c0} + \sum_{n=1}^{\infty} (\bar{L}_{cn} \cos n\theta + \bar{L}_{sn} \sin n\theta) \\ \bar{L}_{cn} &= \frac{\omega}{\pi} \int_0^{2\pi/\omega} \bar{L} \cos n\omega t \cdot dt \\ \bar{L}_{sn} &= \frac{\omega}{\pi} \int_0^{2\pi/\omega} \bar{L} \sin n\omega t \cdot dt \end{aligned} \quad \left. \begin{array}{l} (n=0, 1, \dots) \\ \dots \end{array} \right\} \quad (3)$$

The equation of motion of the vibrating plate in Fig.1 is obtained as follows:

$$\frac{W_0}{g} \frac{d^2 Z}{dt^2} + C_0 \frac{dZ}{dt} + kZ = L + F \sin(\omega t + \varphi) \quad \dots (4)$$

This equation is nonlinear because $\bar{L}$, $\bar{L}_{cn}$ and $\bar{L}_{sn}$ are functions of K_v , K_h and $\theta_1 \sim \theta_6$ as shown in Eqs.(8), (11) and (13). So, the solution Z of Eq.(4) is expanded in the following Fourier series:

$$Z(\theta) = a_{10} + a_{11} \sin \theta + a_{12} \sin(2\theta - \varphi_2) + \dots (5)$$

The phase of the first order term is assumed to be zero in Eq.(5), but progress of phase of exciting force is assumed to be φ in Eq.(4) in the place.

Substituting Eqs.(2), (3) and (5) into Eq.(4), we obtain the first order equilibrium equations as:

$$\left. \begin{aligned} (1 - (\frac{\omega_{no}}{\omega})^2) \frac{a_{11} \omega^2}{g} + \kappa \bar{L}_{s1} &= -\bar{F} \cos \varphi \\ -\frac{C_0 g}{W_0 \omega} \cdot \frac{a_{11} \omega^2}{g} + \kappa \bar{L}_{c1} &= -\bar{F} \sin \varphi \end{aligned} \right\} \dots (6)$$

where $\bar{F} = F/W$. Solving the second order equilibrium equations, and calculating the second order amplitude a_{12} in the case of a practical example, we find that a_{12} is less than 1/20 of a_{11} near the resonance frequency. Therefore, the high order terms in Eq.(5) may be neglected. Equation (6) are able to be transformed as follows:

$$\left. \begin{aligned} (1 - (\frac{\omega_{no}}{\omega})^2) \cdot K + \kappa \cdot \bar{L}_{s1} &= -\bar{F} \cos \varphi \\ -\frac{C_0 g}{W_0 \omega} \cdot K + \kappa \cdot \bar{L}_{c1} &= -\bar{F} \sin \varphi \end{aligned} \right\} \dots (7)$$

These equations are the fundamental equations for obtaining vibration characteristics of the vibratory machine. Though a_{11} is able to be calculated under given $\bar{L}$, the calculation is very complicated in the iteration method, because θ_i and $\bar{L}$ also vary with variation of the amplitude. For simplicity, therefore, an adequate amplitude a_{11} and K are assumed, and θ_i and $\bar{L}$ in the case of the amplitude a_{11} are calculated. Then, ω/ω_{no} and φ are derived from Eq.(7). Thus we can obtain the frequency response of the vibratory machine. $\bar{L}$ has to be examined with every vibratory machine.

2.1 Vibration characteristics of the shaking feeder

Figure1(a) shows a vibration model of the shaking feeder. The load repeats forward sliding motion (Form A), backward sliding motion (Form B) or relatively reposed motion (Form C) on the shaking plate. These three basic motions are derived from simple equation of motion as follows:

$$\text{Form } \left\{ \begin{array}{l} A \\ B \end{array} \right. \bar{x}' = (\pm \frac{\mu}{K_h} - \frac{\tan \alpha}{K_h}) \cdot (\theta - \theta_0) + \bar{x}'_0$$

$$\text{Form } C \quad \bar{x}' = \cos \theta$$

where, $\bar{x}' = \dot{x}/(a\omega)$, $\bar{x}'_0$ is the initial velocity at $\theta = \theta_0$, and $K_h = K$. The load repeats the

Fig.2 Modes of motion of the load on the shaking feeder

basic motions according to the orders shown in Fig.2. Mode 1 is a case of very small amplitude and Mode 6 one of very large amplitude. In Mode 6 the sliding motion takes longer time in a period than the relatively reposed motion.

The mode of motion under given K_h is obtained from the condition that the load finishes its motion in every cycle of the vibrating plate. For example, for $K_h/\mu = 3$ and $(\tan \alpha)/\mu = -0.6$, we obtain Mode 4, $\theta_1 = 0.14$, $\theta_2 = \theta_3 = 5.07$ and $\theta_4 = 6.3$ rad.

The reaction forces of the load moved thus are as follows:

$$\begin{aligned} \text{Form } \left\{ \begin{array}{l} A \\ B \end{array} \right. & P = \mp W \mu \cos \alpha - W \sin \alpha \\ \text{Form } C & P = \frac{W}{g} a \omega^2 \sin \theta - W \sin \alpha \end{aligned}$$

Substituting the above equations into the relation $\bar{L} = L/W = P/W$, we obtain:

$$\left. \begin{aligned} \bar{L}_{s1} &= -\frac{\mu}{K_h} (\cos \theta_2 - \cos \theta_1 - \cos \theta_4 + \cos \theta_3) \\ &\quad + \frac{K_h}{2\pi} ((\theta_1 - \theta_4 + 2\pi - \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_4)) \\ &\quad - \frac{1}{2}(\sin 2\theta_3 - \sin 2\theta_2) + \theta_3 - \theta_2) \\ \bar{L}_{c1} &= \frac{\mu}{K_h} (\sin \theta_2 - \sin \theta_1 - \sin \theta_4 + \sin \theta_3) \\ &\quad - \frac{K_h}{4\pi} (\cos 2\theta_1 - \cos 2\theta_4 + \cos 2\theta_3 - \cos 2\theta_2) \end{aligned} \right\} \quad (8)$$

Calculating $\bar{L}_{s1}$ and $\bar{L}_{c1}$ from the values of $\theta_1 \sim \theta_4$ obtained beforehand and substituting $\bar{L}_{s1}$ and $\bar{L}_{c1}$ into Eq.(7), we can obtain ω/ω_{no} and φ under given K_h , κ and F . The full lines of Fig.3 show the frequency response in the case of the load ratio $\kappa = 0, 0.1, 0.2$. Resonance frequency and maximum value of K_h decrease with an increase of κ . The curves are found to rise gently on the left hand side of the top, and to fall steeply on the right side.

Fig.3 Frequency response of the loaded shaking feeder

2.2 Vibration characteristics of the vibratory grinder

The load jumps or reposes relatively on the vertically vibrating plate as shown in Fig.1(b). This plate is assumed to vibrate harmonically with amplitude a_{11} . Here, $K_v = K$ in this system. The phase θ_2 of jumping and the phase θ_3 of landing are represented in the former report as follows:

$$\begin{aligned}\theta_2 &= \sin^{-1}(1/K_v) \\ K_v \sin \theta_3 &= 1 + (\theta_3 - \theta_2) \sqrt{K_v^2 - 1} - 0.5(\theta_3 - \theta_2)^2 \\ (1 \leq K_v \leq 3.3)\end{aligned}$$

In this study we assume no repulsion of the load. The reaction forces consist of two forces, that is, inertia force and impulsive force. At first, inertia force of the load that flies from θ_2 to θ_3 is represented as follows:

$$Q = P = 0, \text{ therefore } \bar{L} = 0 \quad \dots\dots(9)$$

Inertia forces of reposing motion between θ_3 and $\theta_2 + 2\pi$ are:

$$\begin{aligned}Q &= W(1 - K_v \sin \theta), \quad P = 0, \text{ therefore } \\ \bar{L} &= -1 + K_v \sin \theta\end{aligned} \quad \dots\dots(10)$$

From Eqs.(9), (10) and (3) the first terms of $\bar{L}_s$ and $\bar{L}_{c1}$ in Eq.(11) are derived.

Then, based on the fact that the impact increases the energy dissipation of the system, the impulsive force is replaced by equivalent viscous damping force $C_e \cdot dZ/dt$. The dissipated energy due to impact in every cycle of the plate is described in the former report⁽¹⁾ as:

$$0.5Wg \frac{1}{\omega^2} (\theta_3 - \theta_2 + K_v (\cos \theta_3 - \cos \theta_2))^2, \quad (1 \leq K \leq 3.3)$$

while, the energy due to the viscous damping is:

$$\pi C_e \omega a_{s1}^2 (= C_e \frac{\pi g^2}{\omega^3} K_v^2)$$

Since these two terms have to be equal, the coefficient C_e can be calculated. The second term of $\bar{L}_{c1}$ in Eq.(11) represents the effect of impact obtained in this method.

Thus, all the effects of the load of a vertically vibrating system are expressed as follows:

$$\begin{aligned}\bar{L}_{s1} &= \frac{1}{\pi} (\cos \theta_2 - \cos \theta_3 + \frac{1}{2} K_v (\theta_2 + 2\pi - \theta_3) \\ &\quad - \frac{1}{4} K_v (\sin 2\theta_2 - \sin 2\theta_3)) \\ \bar{L}_{c1} &= \frac{1}{\pi} (\sin \theta_3 - \sin \theta_2 + \frac{1}{4} K_v (\cos 2\theta_3 - \cos 2\theta_2))\end{aligned}$$

Fig.4 Frequency response of the loaded vibratory grinder

$$-\frac{1}{2\pi} \frac{1}{K_v} (\theta_3 - \theta_2 + K_v (\cos \theta_3 - \cos \theta_2))^2 \quad \dots\dots(11)$$

(1 ≤ K_v ≤ 3.3)

We can obtain the frequency response of the vertical system from Eqs.(7) and (11). The full lines in Fig.4 show the calculated results. The resonance frequency decreases with an increase of the load as in the case of the shaking feeder.

2.3 Vibration characteristics of the vibratory feeder

The motion of the load shown in Fig.1(c) is very complicated as described in the former report⁽¹⁾. The results are summarized below. The basic forms of motion are:

Form A : sliding motion in the case of $\bar{x}' \leq \bar{x}'$

$$\begin{aligned}\bar{x}' &= \frac{\mu \tan(\beta - \alpha)}{K_v} (\theta - \theta_0 + K_v (\cos \theta - \cos \theta_0)) \\ &\quad - \frac{\tan \alpha}{\mu} \frac{\mu \tan(\beta - \alpha)}{K_v} (\theta - \theta_0) + \bar{x}'_0\end{aligned}$$

Form C : relatively reposed motion

$$\bar{x}' = \cos \theta$$

Form D : jumping motion

$$\bar{x}' = -\frac{\tan \alpha}{\mu} \frac{\mu \tan(\beta - \alpha)}{K_v} (\theta - \theta_0) + \bar{x}'_0$$

where, $\bar{x}' = \dot{x}/(a\omega \cos(\beta - \alpha))$ and $\bar{x}'_0$ is an initial velocity. The load moves periodically in every cycle of the plate according to the mode of motion composed of four basic forms. Sixteen modes occur stably in the range of $0 < K_v < 3.3$. Mode 9 shown in Fig.5 is a typical example of modes. The load in the mode moves in the order of B → D → B → C → A → C. The adequate mode of motion and the phases $\theta_1 \sim \theta_6$ are able to be calculated if the amplitude a_{11} is given.

The reaction forces P and Q are represented in every basic form as follows:

$$\begin{aligned}\text{Form A : } Q &= W(1 - K_v \sin \theta) \cdot \cos \alpha \\ P &= \mp \mu Q \\ \text{Form C : } Q &= W(1 - K_v \sin \theta) \cdot \cos \alpha \\ P &= -W(\tan \alpha - K_v \sin \theta) \cdot \cos \alpha \\ \text{Form D : } Q &= P = 0\end{aligned} \quad \dots\dots(12)$$

Fig.5 Motion of Mode 9 of the vibratory feeder

Substituting Eq.(12) into Eqs.(1) and (3), we obtain the integrations of $\bar{L}$ as:

$$\begin{aligned} \bar{L}_{S1} = & -(-\cos\theta_2 + \cos\theta_3 - \frac{K_V}{2}(\theta_2 + 2\pi - \theta_3) + \frac{K_V}{4}(\sin 2\theta_2 - \sin 2\theta_3)) \frac{\cos\alpha \sin(\beta - \alpha)}{\pi} \\ & -((-\cos\theta_5 + \cos\theta_4 - \cos\theta_1 + \cos\theta_6) \tan\alpha - \frac{K_h}{2}(\theta_5 - \theta_4 + \theta_1 + 2\pi - \theta_6)) \\ & + \frac{K_h}{4}(\sin 2\theta_5 - \sin 2\theta_4 + \sin 2\theta_1 - \sin 2\theta_6)) \frac{\cos\alpha \cos(\beta - \alpha)}{\pi} + \mu(-\cos\theta_2 + \cos\theta_1 \\ & - \cos\theta_4 + \cos\theta_3 + \cos\theta_6 - \cos\theta_5 - \frac{K_V}{2}(\theta_2 - \theta_1 + \theta_4 - \theta_3 - \theta_6 + \theta_5)) \\ & + \frac{K_V}{4}(\sin 2\theta_2 - \sin 2\theta_1 + \sin 2\theta_4 - \sin 2\theta_3 - \sin 2\theta_6 + \sin 2\theta_5)) \frac{\cos\alpha \cos(\beta - \alpha)}{\pi} \\ \bar{L}_{C1} = & -(\sin\theta_2 - \sin\theta_3 + \frac{K_V}{4}(\cos 2\theta_2 - \cos 2\theta_3)) \frac{\cos\alpha \sin(\beta - \alpha)}{\pi} - (\frac{K_V}{4}(\cos 2\theta_5 - \cos 2\theta_4 \\ & + \cos 2\theta_1 - \cos 2\theta_6) + (\sin\theta_5 - \sin\theta_4 + \sin\theta_1 - \sin\theta_6) \tan\alpha) \frac{\cos\alpha \cos(\beta - \alpha)}{\pi} \\ & - (\theta_3 - \theta_2 + K_V(\cos\theta_3 - \cos\theta_2))^2 \frac{\sin(\beta - \alpha)}{2\pi K_V} + \mu(\frac{K_V}{4}(\cos 2\theta_2 - \cos 2\theta_1 + \cos 2\theta_4 \\ & - \cos 2\theta_3 - \cos 2\theta_6 + \cos 2\theta_5) + \sin\theta_2 - \sin\theta_1 + \sin\theta_4 - \sin\theta_3 - \sin\theta_6 + \sin\theta_5) \frac{\cos\alpha \cos(\beta - \alpha)}{\pi} \end{aligned} \quad \dots(13)$$

The 4th term of $\bar{L}_{C1}$ in Eq.(13) represents the effect of impact, which is replaced by the equivalent viscous damping in a similar way to Eq.(11).

The acceleration K_V and the phases θ_i decided by K_V determine ω/ω_{n0} and φ through Eqs. (13) and (7). Because peaks of K_V in Fig.6 incline to the right, with the frequency variation the vibration is unstable at the right of resonance frequency, but is fairly stable at the left. The relation between K_V and κ is shown in Fig.7. When $\omega/\omega_{n0} = 0.98$, the value of K_V is large at $\kappa = 0$, but it decreases rapidly with an increase of κ . Meanwhile, if $\omega/\omega_{n0} = 0.94$, K_V is small at $\kappa = 0$, but it increases stably with an increase of κ . This increasing area is not infinite and K_V reverses over the limit of κ . Therefore, it is possible to obtain for a given range of κ the optimum condition that the curve incline to the left.

Fig.6 Frequency response of the vibratory feeder

Fig.7 Relation between acceleration and load ratio of the vibratory feeder

It seems that the vibration of the feeder varies transiently under the step-type inflow of the granular materials, and the amplitude exceeds the stable one, and it may diverge infinitely. This transient characteristics of the feeder have been also confirmed analytically and experimentally.

Thus, it is seen that the amplitude varies transiently, then converges to constant amplitude.

3. Experiments

The feeder used in the experiments consisted of a single degree of freedom vibrating system. It was supported by parallel leaf springs. And it was excited by rotation of unbalanced rotors. The direction of vibration of the vibrating plate (1200x150 mm) was variable by the change of angle of the springs. We obtained $\omega_{n0} = 17.2 \text{ rad/s}$ and $C_0/C_c < 0.05$ in this apparatus.

3.1 Frequency response of the shaking feeder

The leaf springs were set at $\beta = \alpha = -1^\circ 45'$ so as to vibrate as Fig.1(a). The conveyed body was a wooden plate. The shaking acceleration K_V varied as shown by marks ($\bullet, \circ, \ominus$) in Fig.3 in the range of frequency ratios $0.8 < \omega/\omega_{n0} < 1.1$ under the constant unbalance of the rotor.

Because the experimental results coincide well with the analytical ones, it is confirmed that the resonance characteristics of the shaking feeder are able to be analyzed by the approximate equations such as Eqs. (7) and (8).

3.2 Frequency response of the vibratory grinder

For lack of any practical grinder the experiments were performed using a vertically vibrating vessel (diameter 100mm) containing a great number of small screws. The vessel was excited by the electro-magnetic vibrator in the same method explained in the other report⁽⁶⁾. The exciting frequency was 60 Hz only because of the commercial current. So, the natural frequency ω_n and the frequency ratio ω/ω_n were varied not by change of exciting frequency, but by placing dead weights on the vibrating plate.

The marks (•, o, e, o) in Fig. 4 denote the experimental results. The curves are the analytical results calculated adjusting to the experimental conditions. Though the curves do not coincide so well with the experimental results as in Fig. 3, the phenomena that the resonance frequency decreases with the load are confirmed qualitatively.

3.3 Vibration characteristics of the vibratory feeder

The direction of springs of the shaking feeder was changed $\beta=45^\circ$, $\alpha=0$ in order to use the apparatus as a vibratory feeder. The experiments were performed to measure not only the acceleration but also the phase angle. Figure 8 represents the comparison between the experimental results and the theoretical ones. Both values agree well with each other.

As described above, the reaction force of the load and its direction vary suddenly, and so the force can not be considered to be harmonic. However, it is possible to analyze the vibration characteristics of the vibratory machines by adopting only the fundamental terms of the Fourier series of the reaction forces, and by approximating the machines as a linear vibrating system. The approximate solution analyzed in this report is useful enough for practical application.

Transient vibration of the feeder was tested by sand poured from a hopper. The experiments were performed under various conditions of flow rate, frequency and exciting force. As shown in Fig. 9, for example, the acceleration K_v of the plate increased transiently up to 2 times as large as the final

Fig. 8 Comparison between the experimental results and the theoretical ones

K_v , but it converged to constant acceleration.

4. Application of the equivalent weight and the equivalent coefficient of viscous damping to the design of the feeder

Substituting the resonance condition $\varphi=90^\circ$ into the fundamental equations (7), we obtain the resonance frequency ω_{nL} of the system with the load as:

$$\left(\frac{\omega_{nL}}{\omega_{n0}}\right)^2 = \frac{1}{1 + \kappa \cdot \bar{L}_{S1} / K} \quad \dots (14)$$

Since $\omega_{n0}^2 = k g / W_0$, we have:

$$\omega_{nL}^2 = \frac{k g}{W_0 + \bar{L}_{S1} \cdot W / K} \quad \dots (15)$$

It is explained in the denominator of Eq. (15) that a part of the load W vibrates as if it were fixed on the vibrating plate. Considering $\bar{L}_{S1}$ of Eq. (13) which increases ω_{nL} , we find that $\bar{L}_{S1}$ consists of terms proportional to the coefficient μ of friction and those without any relation to μ . The former terms are derived from frictional forces which occur in the sliding motion between $\theta_1 \rightarrow \theta_2$, $\theta_3 \rightarrow \theta_4$ and $\theta_5 \rightarrow \theta_6$ in Fig. 5. The latter terms are derived from inertia forces which occur in the relatively reposed motions between $\theta_4 \rightarrow \theta_5$ and $\theta_6 \rightarrow \theta_1 + 2\pi$. It is natural that the inertia forces should vary the resonance frequency of the system. In addition, the frictional forces also affect the natural frequency in this feeder system. Because the frictional force lags in phase compared with the vibration velocity of the plate, the components of frictional forces appear in terms of $\bar{L}_{S1}$.

The inertia forces similarly appear in $\bar{L}_c$, which represents the damping of vibration. Thus, the frictional force and the inertia one intricately affect the vibration characteristics of the vibratory feeder.

Then, we consider the equivalent weight and equivalent damping useful in the design of the feeder. Equation (15) means that $\bar{L}_{S1} / K$ times the weight W of load has to be added to the weight W_0 of the plate. Therefore, we obtained the equivalent weight W_{eq} and the ratio γ_m of W_{eq} as:

$$\left. \begin{aligned} W_{eq} &= W \cdot \gamma_m \\ \gamma_m &= \bar{L}_{S1} / K \end{aligned} \right\} \quad \dots (16)$$

Then, if both sides of the second equation of Eq. (7) are multiplied by $0.5 a \omega W$, the right side of the equation expresses

Fig. 9 Example of transient vibration of the vibratory feeder

the power supplied by the exciter. So, replacing the left side of the equation by the viscous damping, we obtain the equivalent coefficient C_{eq} of viscous damping and the ratio γ_c of C_{eq} as follows:

$$\left. \begin{aligned} C_{eq} &= \gamma_c \cdot W / a\omega \\ \gamma_c &= -\bar{L}_{c1} \end{aligned} \right\} \dots (17)$$

The calculated results of γ_m and γ_c are indicated by the full lines and the dotted lines in Fig.10.

Fig.10 Distribution of the equivalent weights and the equivalent coefficients of viscous damping ($\beta=30^\circ$, $\alpha=0^\circ$. Fine lines show the coefficient of conveying velocity, and broken line separate modes of motion)

In this figure γ_m and γ_c vary with $\mu \tan(\beta-\alpha)$ or modes of motion described already. In the region of Mode 7 which does not contain the relatively reposed motion both γ_m and γ_c are smaller compared with ones in Mode 2 which contains much longer relatively reposed motion. We have to select a mode of motion adequate to the conveying purposes, considering the conveying velocity and velocity variation by modes. γ_m and γ_c don't vary so much with $\mu \tan(\beta-\alpha)$ in the range of $K_v > 1$ in which the load jumps upon the plate. In this range γ_m is to be decided for $\gamma_m = 0.1 \sim 0.4$. C_e has to be added between the plate and the stable base.

In order to stabilize the amplitude against the variation of the load and the exciting frequency ω we have to set the curve (Fig.7) in gentle inclination and to

keep the exciting frequency 2~3 % lower than ω_{n1} . On the contrary for the sake of the efficiency of power we have to set the curve (Fig.7) in steep inclination, and to keep x under the peak point of K_v in Fig.7 even when x is maximum.

6. Conclusions

The results of analysis and experiments are as follows:

- (1) The vibration characteristics of the loaded vibratory machine are clarified.
- (2) The resonance frequency ω_{n1} decreases and damping increases with the load. The frequency response curves are not symmetric on both sides of ω_{n1} , but they incline to the right. Therefore, the amplitude is stable in lower frequency than ω_{n1} against variation of ω .
- (3) The resonance frequency of the feeder decreases not only by the inertia force of the load but also by the frictional force.
- (4) The influence of the load is substituted by the equivalent weight W_{eq} and the equivalent coefficient C_{eq} of damping. W_{eq} and C_{eq} are considered useful to the design of the vibratory feeder.

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