621-752

Studies on Vibration Control of Beams Supporting Machine

(Case in Which Machine Is Worked Like

Dynamic Absorber and Case in Which

Dynamic Absorber Is Attached to Machine) *

By Ken-ichiro OHMATA**

To suppress the resonance of a simply supported beam and a clamped-clamped beam supporting a machine, the case in which the machine-suspension system is worked like a dynamic absorber and in which a dynamic absorber is attached to the machine are discussed theoretically. The adjusting conditions of the springs and the dashpots to the first or second resonances of the beams are determined and the effects of vibration isolation are discussed. Roughly speaking, the former method is particularly effective with respect to the suppression of vibration of the beams and the latter method is particularly effective with respect to that of the machine.

1. Introduction

Examples of a machine attached to a beam occur in many applied situations. When the machine is excited by a sinusoidally varying force, the machine connected to a beam by a spring and dashpot which are tuned to the forcing frequency like a dynamic absorber and a dynamic absorber attached to the machine (or the beam) might be useful to reduce the vibration of the beam and the machine, the stress of the beam and the force being transmitted to the foundation.

In this report, the adjusting conditions of such a machine-spring-dashpot system and a dynamic absorber to the first or second resonance of beams are determined and the effects of vibration isolation are discussed theoretically. The analyses are made of a simply supported beam and a clamped-clamped beam.

2. Case in which machine-spring-dashpot system attached to simply supported beam is worked like dynamic absorber

2.1 Solution for forced vibration

Consider a simply supported beam of uniform cross-section which carries a machine of mass m through a suspension spring of stiffness k and a dashpot of damping coefficient c at a point $x = l_1$, as in Fig. 1. The machine is excited by a sinusoidally varying force $F_0 e^{j\omega t}$. Neglecting the effects of shear and rotary

inertia of the beam, the steady-state displacements y_1 and y_2 of the regions $0_1 0_2$ and $0_2 0_3$ can be written in the form⁽¹⁾

$$y_i = X_i(x) e^{j\omega t} \quad (i=1, 2) \quad \dots\dots\dots (1)$$

$$X_1(x) = A_1 \sin \lambda x + A_2 \cos \lambda x + A_3 \sinh \lambda x + A_4 \cosh \lambda x \quad (0 \leq x \leq l_1) \quad \dots\dots\dots (2)$$

$$X_2(x) = A_5 \sin \lambda(l-x) + A_6 \cos \lambda(l-x) + A_7 \sinh \lambda(l-x) + A_8 \cosh \lambda(l-x) \quad (l_1 \leq x \leq l) \quad \dots\dots\dots (3)$$

where

$$\lambda = (\rho S \omega^2 / EI)^{1/4} \quad \dots\dots\dots (4)$$

In this equation ρ, E, S and I represent the density, Young's modulus, the area of cross-section, and the second moment of area of cross-section of the beam, respectively. The boundary conditions for this case are

$$X_1 = 0, \quad d^2 X_1 / dx^2 = 0 \quad \text{at } x = 0 \quad \dots\dots\dots (5)$$

$$X_2 = 0, \quad d^2 X_2 / dx^2 = 0 \quad \text{at } x = l \quad \dots\dots\dots (6)$$

$$\left. \begin{aligned} X_1 &= X_2, \quad dX_1/dx = dX_2/dx \\ d^2 X_1/dx^2 &= d^2 X_2/dx^2 \end{aligned} \right\} \text{at } x = l_1 \quad \dots\dots\dots (7)$$

$$EI \left(\frac{\partial^3 y_2}{\partial x^3} - \frac{\partial^3 y_1}{\partial x^3} \right) = k(z - y_1) + c(\dot{z} - \dot{y}_1) \quad \text{at } x = l_1 \quad \dots\dots\dots (8)$$

Fig. 1

* Received 2nd October, 1977.

** Assistant Professor, Faculty of Engineering, Meiji University, Kawasaki.

The equation of motion for m is

$$m\ddot{z} + k(z - y_{z=l_1}) + c(\dot{z} - \dot{y}_{z=l_1}) = F_0 e^{j\omega t} \dots (9)$$

We assume a particular solution of Eq. (9) of the form

$$z = A_9 e^{j\omega t} \quad (j = \sqrt{-1}) \dots (10)$$

Substituting Eqs. (1)~(3) and (10) into Eqs. (5)~(9), we obtain nine equations containing $A_1 \sim A_9$. Solving these equati-

$$\left. \begin{aligned} \Delta &= r_1 \gamma \lambda l (\sin \lambda l \sinh \lambda l_1 \sinh \lambda l_2 - \sin \lambda l_1 \sin \lambda l_2 \sinh \lambda l) + 2r_2 \sin \lambda l \sinh \lambda l \\ \Delta_1 &= r_1 \sin \lambda l_2 \sinh \lambda l, \quad \Delta_2 = -r_1 \sin \lambda l \sinh \lambda l_2 \\ \Delta_3 &= 2 \sin \lambda l \sinh \lambda l - r_1 (\sin \lambda l \sinh \lambda l_1 \sinh \lambda l_2 - \sin \lambda l_1 \sin \lambda l_2 \sinh \lambda l) \end{aligned} \right\} \dots (13)$$

and

$$r_1 = \frac{\alpha}{\lambda^2 l^2} + j \frac{\beta}{\lambda l}, \quad r_2 = \frac{\alpha}{\lambda^2 l^2} - \gamma \lambda l + j \frac{\beta}{\lambda l}, \quad \alpha = \frac{k}{k_B}, \quad \beta = \frac{c}{\sqrt{m_B k_B}}, \quad \gamma = \frac{m}{m_B}, \quad k_B = \frac{EI}{l^3}, \quad m_B = \rho S l \dots (14)$$

Δ_5 and Δ_7 are obtained by replacing l_2 in Δ_1 and Δ_3 , respectively, with l_1 .

Furthermore, if the machine is attached to the beam directly, the displacement at any point in the beam is also given in the form of Eqs. (1)~(3), (11) and (12). In this case, however, Δ_1 , Δ_7 and Δ_9 are given by

$$\left. \begin{aligned} \Delta &= \gamma \lambda l (\sin \lambda l \sinh \lambda l_1 \sinh \lambda l_2 - \sin \lambda l_1 \sin \lambda l_2 \sinh \lambda l) + 2 \sin \lambda l \sinh \lambda l \\ \Delta_1 &= \sin \lambda l_2 \sinh \lambda l, \quad \Delta_3 = -\sin \lambda l \sinh \lambda l_2 \end{aligned} \right\} \dots (15)$$

2.2 Equivalent mass and stiffness of beam

In general, a dynamic absorber attached to a multiple degree-of-freedom system or a continuous system will be effective only for the particular natural mode of system vibration to which the absorber is tuned. Therefore, the optimum values of absorber tuning and damping depend on the resonance concerned. It is difficult to obtain these optimum values analytically and they have been found by trial and error in previous analyses. For example, Snowden⁽²⁾ determined the optimum values of the dynamic absorber tuned to the first or second resonance of a cantilever, Fig. 2(b), by trial and error; and Neubert⁽³⁾ investigated those of the absorber attached to an axially excited bar.

If we can replace a beam excited near the n th resonance by a one degree-of-freedom system consisting of equivalent mass m_{BE} and equivalent stiffness k_{BE} [see Fig. 2(b)], the expressions⁽⁴⁾ for tuning the absorber on a single-mass system would apply to the absorber employed to suppress the n th resonance of the beam. Applying such an equivalent expression to the

ons simultaneously, we obtain

$$A_2 = 0, \quad A_4 = 0, \quad A_6 = 0, \quad A_8 = 0 \dots (11)$$

and A_1, A_3, A_5, A_7 and A_9 may be written in the form

$$A_r = \frac{F_0/k_B}{\lambda^2 l^2} \frac{\Delta_r}{\Delta} \quad (r=1, 3, 5, 7, 9) \dots (12)$$

where

system shown in Fig. 1, its equivalent system may be represented as Fig. 3. As the system shown in Fig. 3 differs from one in Fig. 2(b) in the location of application of driving force, the optimum values of k and c to suppress the resonance of the primary system ($m_{BE}-k_{BE}$) differ from the case in Fig. 2(b) too. Let us consider the values of m_{BE} and k_{BE} prior to determining these optimum values.

The equivalent mass of a beam m_{BE} which is concentrated at the point of attachment of a machine is found by equating the complete kinetic energy of the vibrating beam to that of the equivalent mass. Thus

$$\frac{1}{2} m_{BE} \dot{y}_{z=l_1}^2 = \frac{\rho S}{2} \int_0^l \dot{y}^2 dx \dots (16)$$

where y is the displacement of the beam at x when a machine is not attached. If ω approaches the n th natural angular frequency, y will be expressed approximately by the n th normal function $\phi_n(x)$ only:

$$y = \phi_n(x) e^{j\omega t} \dots (17)$$

Substituting Eq. (17) into Eq. (16), we obtain

Fig. 2 Cantilever beam with a dynamic absorber

Fig. 3 Equivalent system to the system shown in Fig. 1

$$m_{BE} = \frac{\rho S \int_0^l \phi_n^2(x) dx}{\phi_n^2(l_1)} = \frac{m_B \int_0^l \phi_n^2(x) dx}{l \phi_n^2(l_1)} \quad (m_B: \text{mass of beam}) \quad (18)$$

The equivalent stiffness k_{BE} of a beam is chosen so that the natural frequency of the equivalent system is equal to the n th natural frequency of the beam. Therefore

$$k_{BE} = \omega_n^2 m_{BE} \quad (19)$$

where ω_n is the n th natural angular frequency of the beam when a machine is not attached and it is related to the n th characteristic value $\lambda_n l$ by

$$\omega_n = (\lambda_n l)^2 \sqrt{k_B / m_B} \quad (20)$$

In the case of a simply supported beam, it is apparent that

$$\left. \begin{aligned} \lambda_n l &= n\pi \quad (n=1, 2, 3, \dots) \\ \phi_n(x) &= C_n \sin(n\pi x / l) \quad (C_n: \text{constant}) \end{aligned} \right\} \quad (21)$$

Hence, we have

$$m_{BE} = \frac{m_B}{2 \sin^2(n\pi l_1 / l)}, \quad k_{BE} = \frac{(n\pi)^4 k_B}{2 \sin^2(n\pi l_1 / l)} \quad (22)$$

2.3 Optimum conditions of suspension spring and dashpot to control n th beam resonance

Next, let us consider the optimum conditions of a suspension spring and dashpot to control the n th beam resonance by means of an equivalent system shown in Fig. 3. The equations of motion of the system shown in Fig. 3 are

$$\left. \begin{aligned} m_{BE} \ddot{y}_E + k_{BE} y_E + k(y_E - z_E) + c(\dot{y}_E - \dot{z}_E) &= 0 \\ m \ddot{z}_E + k(z_E - y_E) + c(\dot{z}_E - \dot{y}_E) &= F_0 e^{j\omega t} \end{aligned} \right\} \quad (23)$$

Introducing the notations

$$\left. \begin{aligned} \omega_n &= \sqrt{k_{BE} / m_{BE}}, \quad \nu = \sqrt{k / m}, \quad \mu = m / m_{BE} \\ \delta &= F_0 / k_{BE}, \quad Y_E = b / \delta, \quad f = \nu / \omega_n \\ h &= c / 2m\omega_n, \quad g = \omega / \omega_n \end{aligned} \right\} \quad (24)$$

the magnification factor of the steady-state forced vibration may be written as

$$Y_E^2 = \frac{f^4 + 4h^2 g^2}{[(1-g^2)(f^2 - g^2) - \mu f^2 g^2]^2 + 4h^2 g^2 (1 - (1+\mu)g^2)^2} \quad (25)$$

Substituting $h=0$ and $h=\infty$ into Eq.(25), we obtain the expressions for response curves when damping is zero and infinitely large.

$$Y_E = \frac{f^2}{(1-g^2)(f^2 - g^2) - \mu f^2 g^2} \quad \text{for } h=0 \quad (26)$$

$$Y_E = \frac{1}{1 - (1+\mu)g^2} \quad \text{for } h=\infty \quad (27)$$

Using Eqs.(25)~(27), the relationships

between g and Y_E for $h=0, 0.22$ and ∞ when $\mu=0.5$ and $f=0.47$ are plotted in Fig. 4. It can be seen from Fig. 4 that all of the response curves of the primary system (the equivalent system of the beam) pass through the points P, Q and R, and the ordinate of R is always lower than that of Q. Therefore, it is necessary to choose such a value of the frequency ratio f that the ordinate at P is equal to that at Q and such a value of the damping parameter h that the response curve is maximum in the close vicinity of these points, thereby decreasing the amplitude of the primary system as much as possible.

Firstly, let us consider the condition for which the ordinate at P equals to that at Q. By taking $(Y_E)_{h=\infty} = -(Y_E)_{h=0}$, we obtain the abscissa at P to be

$$g_P^2 = \frac{1}{2} [1 + 2(1+\mu)f^2 - \sqrt{1 + 2(1+\mu)f^2 - 8f^2}] \quad (28)$$

The abscissa at Q is also given by taking $(Y_E)_{h=\infty} = (Y_E)_{h=0}$.

$$g_Q = 1 \quad (29)$$

Substituting g_P and g_Q into Eq.(27) and taking $(Y_E)_P = -(Y_E)_Q$ we obtain

$$(1+\mu)(g_P^2 + g_Q^2) - 2 = 0 \quad (30)$$

Substituting Eqs.(28) and (29) into Eq. (30), we find the value of f for which the ordinate at P equals to that at Q.

$$f = \frac{\sqrt{1-\mu}}{1+\mu} \quad (31)$$

When f takes above value, the abscissa at P and the ordinates of the points P and Q become

$$g_P = \sqrt{(1-\mu)/(1+\mu)} \quad (32)$$

$$|Y_E|_{P,Q} = 1/\mu \quad (33)$$

It can be seen from Eq.(31) that when $\mu > 1$, it is impossible to give the same ordinate at points P and Q. In this case the ordinate at P becomes the highest of all fixed points and approaches 1 as the value of f decreases. In such a case it will be effective for controlling the vibration of the primary system to choose the value of f as small as possible and the value of h such that the response curve is maximum

Fig. 4 Amplitude of the equivalent mass

at the point P. However, as the value of μ hardly becomes larger than 1 in practical cases, we will consider the case when $\mu < 1$ in the present paper.

Next, we consider the condition that the response curve is maximum at the points P or Q. From Eq. (25), calculating $dY_E^2/dg^2 = 0$, and solving it for h^2 , we obtain

$$h^2 = \frac{1}{2} \frac{[Y_E^2((1-g^2)(f^2-g^2)-\mu f^2 g^2)] \times (2g^2-1-(1+\mu)f^2)}{1-Y_E^2(1-(1+\mu)g^2)(1-3(1+\mu)g^2)} \quad (34)$$

Substituting Eqs. (31), (33) and (32) or (29) into Eq. (34), we find the values of h such that the ordinate at P is equal to that at Q and the response curve is maximum at P or Q.

$$h_P^2 = \frac{\mu}{2(1+\mu)^3}, \quad h_Q^2 = \frac{\mu(1-\mu)}{2(1+\mu)^4} \quad (35)$$

The optimum value of h is defined by

$$h^2 = \frac{h_P^2 + h_Q^2}{2} = \frac{\mu}{2(1+\mu)^4} \quad (36)$$

The solid curve in Fig. 4 represents the response when f and h take the values given by Eqs. (31) and (36) respectively and shows that the machine-suspension system is effective in suppressing the resonance of the primary system like a dynamic absorber. This effect of vibration isolation increases as the value of μ increases, i.e. m_{BE} decreases. Therefore, it will be most effective to attach the machine-suspension system to the point in such manner that $\phi_n(l_1)$ is maximum.

Calculating the optimum values of f and h for the equivalent system shown in Fig. 3, we obtain the adjusting conditions of α and β to the n th beam resonance as

$$\alpha = \frac{k}{k_B} = \frac{m\omega_n^2 f^2}{k_B} = \frac{\gamma(\lambda_n l)^4(1-\mu)}{(1+\mu)^2} \quad (37)$$

$$\beta = \frac{c}{\sqrt{m_B k_B}} = \frac{2m\omega_n h}{\sqrt{m_B k_B}} = \frac{2\gamma(\lambda_n l)^2}{(1+\mu)^2} \sqrt{\frac{\mu}{2}} \quad (38)$$

In the case of a simply supported beam, it is apparent that

$$\left. \begin{aligned} \alpha &= \frac{\gamma(n\pi)^4(1-\mu)}{(1+\mu)^2}, & \beta &= \frac{2\gamma(n\pi)^2}{(1+\mu)^2} \sqrt{\frac{\mu}{2}} \\ \mu &= 2\gamma \sin^2(n\pi l_1/l) \end{aligned} \right\} \quad (39)$$

The values of α and β given by Eqs. (37)~(39) deviate from the optimum values as the mode number n increases and may be regarded approximately as the optimum values to the first two resonances of the beam.

2.4 Numerical example

Let us consider a case in which a machine-spring-dashpot system attached to the midpoint of a simply supported beam is tuned to the first beam resonance like a dynamic absorber. The values of α and β calculated from Eq. (39) and the optimum values of α and β obtained by trial and

error are given in Table 1. The relationship between the amplitude $a_1(l/2)$ at the midpoint of the beam and the frequency when $\gamma = 0.2$ and α and β take the values given by Eq. (39) is plotted in Fig. 5. It will be seen from Table 1 that, if the suspension system is tuned to the first beam resonance, the values of α and β given by Eq. (39) approach the optimum values as the value of μ decreases and error is less than 2% when $\mu < 0.6$. It is apparent from Fig. 5 that the machine-suspension system is effective in suppressing the first beam resonance. In this case, however, the amplitude of the machine at the first resonance is larger than that of the beam by 5~10 times when $\mu < 0.7$ and increases more and more as the value of μ increases.

The amplitude of a simply supported beam at $x = l_1$ when a machine-suspension system is attached at $l_1/l = 1/4$ and is tuned to the first or second beam resonance as given by Eq. (39), is plotted in Fig. 6 for the value of $\gamma = 0.2$. It can be seen from Fig. 6 that the values of α and β given by Eq. (39) slightly deviate from the optimum values, and these differences do not affect the effect of vibration isolation.

Next, we develop a way to get the values of α and β shown in Table 1. Denoting by Y_1 the ratio of the amplitude $a_1(l/2)$ at the midpoint of the beam when $l_1/l = 0.5$ to the static deflection x_{st} at the midpoint due to F_0 , the values of Y_1 when $\beta = 0$ and $\beta = \infty$ are

Table 1 Adjusting conditions of α and β to the first resonance of a simply supported beam ($l_1/l = 0.5$)

γ	μ	α		β	
		Optimum value	Eq. (39)	Optimum value	Eq. (39)
0.1	0.2	5.393	5.412	0.447	0.434
0.2	0.4	5.905	5.964	0.926	0.901
0.3	0.6	4.454	4.566	1.247	1.267
0.4	0.8	2.253	2.405	1.360	1.541

Fig. 5 Amplitude of the beam at the point of attachment of a machine ($l_1/l = 0.5$)

$$\left. \begin{aligned} (Y_1)_{\beta=0} &= \frac{48}{\lambda^3 l^3} \frac{q}{4(1-\gamma\lambda^4 l^4/\alpha) - \gamma\lambda l q} \\ (Y_1)_{\beta=\infty} &= \frac{48}{\lambda^3 l^3} \frac{q}{4 - \gamma\lambda l q} \\ q &= \tan(\lambda l/2) - \tanh(\lambda l/2) \end{aligned} \right\} \dots(40)$$

The abscissa at the point P in Fig. 5 is found by putting $(Y_1)_{\beta=\infty} = -(Y_1)_{\beta=0}$. Namely, it is equal to the positive minimum value of λl satisfying the following equation:

$$4 - 2\gamma(\lambda l)^4/\alpha - \gamma\lambda l\{\tan(\lambda l/2) - \tanh(\lambda l/2)\} = 0 \dots(41)$$

Denoting by $\lambda_P l$ this value of λl , the ordinate at the point P is

$$|Y_1|_P = 48\{2\alpha - \gamma(\lambda_P l)^4\}/\gamma^2(\lambda_P l)^8 \dots(42)$$

Likewise by putting $(Y_1)_{\beta=\infty} = (Y_1)_{\beta=0}$, we can get the abscissa at the point Q'.

$$\lambda_Q l = \pi \dots(43)$$

The ordinate at the point Q' is

$$|Y_1|_{Q'} = 48/\gamma\pi^4 \dots(44)$$

The value of α which gives the same ordinate at the points P' and Q' is given by putting $|Y_1|_{P'} = -|Y_1|_{Q'}$ as

$$\alpha = \frac{\gamma}{2}(\lambda_P l)^4 \left\{ 1 + \frac{(\lambda_P l)^4}{\pi^4} \right\} \dots(45)$$

When the optimum value of α has been determined, the optimum value of β can be found by trial and error starting off with the value given by Eq. (39).

3. Control of vibration of simply supported beam supporting machine by dynamic absorber

3.1 Solution for forced vibration

Next, let us consider a case in which a dynamic absorber of mass m_1 , stiffness k_1 and damping coefficient c_1 is attached to a machine in order to suppress the n th resonance of a simply supported beam supporting the machine of mass m , as shown in Fig. 7. The machine is excited by a sinusoidally varying force $F_0 e^{j\omega t}$. Neglecting the effects of shear and rotary inertia of the beam, the displacements y_1

Fig. 6 Amplitude of the beam at the point of attachment of a machine ($l_1/l = 1/4$)

and y_2 of the regions $O_1 O_2$ and $O_2 O_3$ can be expressed in the same forms as Eqs. (1)~(3). The boundary conditions for this case are given by Eqs. (5)~(7) and

$$\begin{aligned} EI \left(\frac{\partial^3 y_2}{\partial x^3} - \frac{\partial^3 y_1}{\partial x^3} \right) &= -m\ddot{y}_1 + k_1(u - y_1) \\ &+ c_1(\dot{u} - \dot{y}_1) + F_0 e^{j\omega t} \quad \text{at } x = l_1 \end{aligned} \dots(46)$$

The equation of motion for m_1 is

$$m_1 \ddot{u} + k_1(u - y_{x=l_1}) + c_1(\dot{u} - \dot{y}_{x=l_1}) = 0 \dots(47)$$

For the displacement u of the dynamic absorber, we assume the form of z given by Eq. (10). Substituting this equation for u and Eqs. (1)~(3) into Eqs. (5)~(7), (46) and (47), $A_1 \sim A_9$ may be expressed in the same forms as Eqs. (11) and (12). In this case, however, Δ , Δ_1 and Δ_2 are given by

$$\left. \begin{aligned} \Delta &= 2r_3 \sin \lambda l \sinh \lambda l + r_4 (\sin \lambda l \sinh \lambda l_1 \sinh \lambda l_2 \\ &- \sin \lambda l_1 \sin \lambda l_2 \sinh \lambda l) \\ \Delta_1 &= r_3 \sin \lambda l_2 \sinh \lambda l, \quad \Delta_2 = -r_3 \sin \lambda l \sinh \lambda l_2 \end{aligned} \right\} \dots(48)$$

where

$$\left. \begin{aligned} r_3 &= \frac{\eta}{\lambda^3 l^3} - \varepsilon \lambda l + j \frac{\xi}{\lambda l} \\ r_4 &= (\varepsilon + \gamma) \left(\frac{\eta}{\lambda^2 l^2} + j \xi \right) - \varepsilon \gamma \lambda^2 l^2 \\ \varepsilon &= m_1/m_B, \quad \eta = k_1/k_B, \quad \xi = c_1/\sqrt{m_B k_B} \end{aligned} \right\} \dots(49)$$

Δ_5 and Δ_7 are obtained by replacing l_2 in Δ_1 and Δ_3 , respectively, with l_4 .

3.2 Optimum absorber tuning and damping

In this case, an equivalent vibration system to the n th beam resonance is obtained by replacing a mass m_{BE} in Fig. 2(b) with

$$m_E = m + m_{BE} \dots(50)$$

where m_{BE} is the equivalent mass of the beam only and is given by

$$m_{BE} = \frac{m_B \left\{ \int_0^{l_1} \phi_{n1}^2(x) dx + \int_{l_1}^l \phi_{n2}^2(x) dx \right\}}{l \phi_{n1}^2(l_1)} \dots(51)$$

Fig. 7

In this equation $\phi_{n1}(x)$ and $\phi_{n2}(x)$ represent the n th normal functions of the regions O_1O_2 and O_2O_3 , respectively:

$$\left. \begin{aligned} \phi_{n1}(x) &= C_n \left[\sin \lambda_n x - \frac{\sin \lambda_n l \sinh \lambda_n l_2}{\sin \lambda_n l_2 \sinh \lambda_n l} \sinh \lambda_n x \right] \quad (0 \leq x \leq l_1) \\ \phi_{n2}(x) &= C_n \left[\sin \lambda_n (l-x) - \frac{\sin \lambda_n l \sinh \lambda_n l_1}{\sin \lambda_n l_1 \sinh \lambda_n l} \sinh \lambda_n (l-x) \right] \quad (l_1 \leq x \leq l) \end{aligned} \right\} \dots\dots\dots (52)$$

where C_n is a constant, and the characteristic value $\lambda_n l$ may be found by setting Δ of Eq. (15) equal to zero. Namely

$$2 \sin \lambda l \sinh \lambda l + \gamma \lambda l (\sin \lambda l \sinh \lambda l_1 \sinh \lambda l_2 - \sin \lambda l_1 \sin \lambda l_2 \sinh \lambda l) = 0 \dots\dots\dots (53)$$

The equivalent stiffness for this case is

$$k_E = \omega_n^2 m_E \dots\dots\dots (54)$$

The optimum conditions of a dynamic absorber attached to a one degree-of-freedom system consisting of m_E and k_E are given by

$$\frac{\sqrt{k_1/m_1}}{\sqrt{k_E/m_E}} = \frac{1}{1+\mu'}, \quad \frac{c_1}{2m_1\omega_n} = \sqrt{\frac{3\mu'}{8(1+\mu')^3}} \quad \left(\mu' = \frac{m_1}{m_E} \right) \dots\dots\dots (55)$$

The response curve of the equivalent mass when the dynamic absorber is tuned as given by Eq. (55) shows a curve like the solid line in Fig. 4 and passes through the two fixed points S and T corresponding to the points P and Q. The ordinates at the points S and T are given by

$$|Y_D|_{S,T} = \left| \frac{d}{F_0/k_E} \right|_{S,T} = \sqrt{1 + \frac{2}{\mu'}} \dots\dots\dots (56)$$

where d represents the amplitude of the equivalent mass. From Eq. (55) we obtain the adjusting conditions of the dynamic absorber to the n th beam resonance as

$$\eta = \frac{k_1}{k_E} = \frac{\varepsilon(\lambda_n l)^4}{(1+\mu')^2} \dots\dots\dots (57)$$

$$\xi = \frac{c_1}{\sqrt{m_E k_E}} = 2\varepsilon(\lambda_n l)^2 \sqrt{\frac{3\mu'}{8(1+\mu')^3}} \dots\dots\dots (58)$$

When $l_1/l = 0.5$, the value of μ' becomes

$$\mu' = \varepsilon \left[\gamma + \frac{6 \cos^2 \frac{\lambda_n l}{2} \tanh \frac{\lambda_n l}{2} - 3 \sin \lambda_n l + \lambda_n l \left(1 - \cos^2 \frac{\lambda_n l}{2} / \cosh^2 \frac{\lambda_n l}{2} \right)}{2 \lambda_n l \left(\sin \frac{\lambda_n l}{2} - \cos \frac{\lambda_n l}{2} \tanh \frac{\lambda_n l}{2} \right)^2} \right] \dots\dots\dots (59)$$

The values of η and ξ given by Eqs. (57) and (58) may be regarded approximately as the optimum values to the first two resonances of the beam and deviate from the optimum values as the mode number n increases more and more.

The amplitude of a simply supported beam at the midpoint when $l_1/l = 0.5$,

Fig. 8 Amplitude of the beam at the point of attachment of a machine

$\gamma = 0.2$, $\varepsilon = 0.1$ and the dynamic absorber is tuned to the first beam resonance as given by Eqs. (57) and (58) is plotted in Fig. 8. It will be seen from Fig. 8 that the conditions given by Eqs. (57) and (58) are approximately equal to the optimum conditions.

3.3 Comparison between effects of vibration isolation in two cases

The effects of vibration isolation in the above-mentioned two cases can be compared by the maximum amplitudes of beams at the resonance concerned. It is difficult to obtain these amplitudes exactly, but let us compare the ordinates at the two fixed points (P, Q and S, T) of the response curves of the equivalent system. From Eq. (33) we obtain

$$\frac{|b|_{P,Q}}{x_n} = \frac{3l^4}{l_1^2 l_2^2} \frac{1}{\gamma(\lambda_n l)^4} \dots\dots\dots (60)$$

where x_n is the static deflection at the point of attachment of the machine due to F_0 and is given by $x_n = F_0 l^3 / 3EI$. From Eq. (56) we obtain

$$\frac{|d|_{s,r}}{x_{st}} = \frac{3l^4}{l_1^2 l_2^2} \frac{\mu'}{s(\lambda_n l)^4} \sqrt{1 + \frac{2}{\mu'}} \dots (61)$$

Substituting Eq.(21) into Eq.(60) and substituting Eqs.(53) and (59) into Eq.(61), we obtain the ordinates at the fixed points P', Q' in Fig. 5 and S', T' in Fig. 8 respectively.

For the case when $l_1/l = 0.5$ and the springs and dashpots are tuned to the first beam resonances, the ordinates of the two fixed points given by Eqs.(60) and (61) are plotted as functions of γ and μ in Fig. 9. It will be seen from Fig. 9 that excepting for the case when $\mu < 0.1$, the case in which a machine is worked like a dynamic absorber is more effective than the case in which a dynamic absorber is attached to a machine for suppressing the amplitude of a beam at a concerned resonance. In the former case, however, the amplitude of the machine becomes larger than in the latter case, especially when $\mu > 0.7$.

4. Case of clamped-clamped beam

4.1 Case in which machine-suspension system is worked like a dynamic absorber

Let us next consider the case in which the beam in Fig. 1 is clamped at both ends. The displacement at any point in the beam is also given in the form of Eqs.(1)~(3) and (12). In this case, however, $\Delta_1, \Delta_2, \dots, \Delta_4$ are given by the following expressions:

$$\left. \begin{aligned} \Delta_1 &= 2r_1 \gamma \lambda l (\sin \lambda l \cosh \lambda l_1 \cosh \lambda l_2 - \cos \lambda l_1 \cos \lambda l_2 \sinh \lambda l - \sin \lambda l_1 \cosh \lambda l_1 + \cos \lambda l_1 \sinh \lambda l_1 \\ &\quad - \sin \lambda l_2 \cosh \lambda l_2 + \cos \lambda l_2 \sinh \lambda l_2) + 4r_2 (1 - \cos \lambda l \cosh \lambda l) \\ \Delta_1 &= -\Delta_3 = r_1 (\sin \lambda l_2 \sinh \lambda l - \sin \lambda l \sinh \lambda l_2 - \cos \lambda l_2 \cosh \lambda l - \cos \lambda l \cosh \lambda l_2 + \cos \lambda l_1 + \cosh \lambda l_1) \\ \Delta_2 &= -\Delta_4 = r_1 (\sin \lambda l \cosh \lambda l_2 - \sin \lambda l_2 \cosh \lambda l - \cos \lambda l \sinh \lambda l_2 + \cos \lambda l_2 \sinh \lambda l - \sin \lambda l_1 - \sinh \lambda l_1) \end{aligned} \right\} \dots (62)$$

where r_1 and r_2 are given by Eq.(14). Δ_5 and Δ_6 are obtained by replacing l_1 and l_2 in Δ_1 and Δ_2 , respectively, and $\Delta_7 = -\Delta_5$ and $\Delta_8 = -\Delta_6$. The nth characteristic value when the machine is not attached is obtained from

$$1 - \cos \lambda l \cosh \lambda l = 0 \dots (63)$$

and the nth normal function for the nth characteristic value $\lambda_n l$ is given by the following equation⁽⁴⁾:

$$\phi_n(x) = C_n \left[\cos \lambda_n x - \cosh \lambda_n x + \frac{\sin \lambda_n l + \sinh \lambda_n l}{\cos \lambda_n l - \cosh \lambda_n l} (\sin \lambda_n x - \sinh \lambda_n x) \right] \dots (64)$$

4.2 Case in which dynamic absorber is attached to machine

In the case in which a dynamic absorber is attached to a machine connected to a clamped-clamped beam, Δ is given by

$$\Delta = 2r_3 (\sin \lambda l \cosh \lambda l_1 \cosh \lambda l_2 - \cos \lambda l_1 \cos \lambda l_2 \sinh \lambda l - \sin \lambda l_1 \cosh \lambda l_1 + \cos \lambda l_1 \sinh \lambda l_1 \\ - \sin \lambda l_2 \cosh \lambda l_2 + \cos \lambda l_2 \sinh \lambda l_2) + 4r_3 (1 - \cos \lambda l \cosh \lambda l) \dots (65)$$

and $\Delta_1 \sim \Delta_4$ are given by replacing r_1 in $\Delta_1 \sim \Delta_4$ of Eq.(62) with r_3 . The nth characteristic value for this case can be obtained from

$$2(\cos \lambda l \cosh \lambda l - 1) + \gamma \lambda l (\cos \lambda l_1 \cos \lambda l_2 \sinh \lambda l - \sin \lambda l \cosh \lambda l_1 \cosh \lambda l_2 - \cos \lambda l_1 \sinh \lambda l_1 \\ - \cos \lambda l_2 \sinh \lambda l_2 + \sin \lambda l_1 \cosh \lambda l_1 + \sin \lambda l_2 \cosh \lambda l_2) = 0 \dots (66)$$

and the nth normal function of the region $0_1 0_2$ is given by

Fig. 9

Fig. 10 Amplitude of the beam at the point of attachment of a machine

$$\phi_{n1}(x) = C_n [B_1(\sin \lambda_n x - \sinh \lambda_n x) + B_2(\cos \lambda_n x - \cosh \lambda_n x)]$$

$$\left. \begin{aligned} B_1 &= \cos \lambda_n l_2 \cosh \lambda_n l - \sin \lambda_n l_2 \sinh \lambda_n l + \sin \lambda_n l \sinh \lambda_n l_2 + \cos \lambda_n l \cosh \lambda_n l_2 - \cos \lambda_n l_1 - \cosh \lambda_n l_1 \\ B_2 &= \sin \lambda_n l_2 \cosh \lambda_n l - \cos \lambda_n l_2 \sinh \lambda_n l - \sin \lambda_n l \cosh \lambda_n l_2 + \cos \lambda_n l \sinh \lambda_n l_2 + \sin \lambda_n l_1 + \sinh \lambda_n l_1 \end{aligned} \right\} \dots (67)$$

For the case when $\gamma = 0.2$ and $l_1/l = 0.5$, the amplitudes at the midpoints of clamped-clamped beams in which a machine-suspension system is worked like a dynamic absorber and a dynamic absorber of $\varepsilon = 0.1$ is attached to a machine in order to suppress the first beam resonance are shown in Fig. 10. The machine-suspension system and the dynamic absorber are tuned to the conditions given by Eqs. (37), (38) and Eqs. (57), (58), respectively. It will be seen from Fig. 10 that these conditions are approximately equal to the optimum conditions.

5. Conclusions

In this paper, the case in which a machine-suspension system is worked like a dynamic absorber and in which a dynamic absorber is attached to a machine in order to suppress the first or second resonance of a beam supporting a machine, are discussed theoretically. As the effects of vibration isolation in these two methods depend on the mass ratio of the machine to the beam, the location of attachment of the machine and the dimensions of the dynamic absorber, roughly speaking, the

former method is particularly effective for the suppression of vibration of the beam and the latter method for that of the machine.

Acknowledgement

The author wishes to thank Professor H. Fukuda of Meiji University for his guidance and Professor S. Hoshino of Muroran Institute of Technology for his helpful advice. The author also wishes to express his thanks to the Computer Centre and the Institute of Science and Technology in Meiji University for their kind cooperations.

References

- (1) e.g., Saito, H., Trans. Japan Soc. Mech. Engrs. (in Japanese), Vol. 30, No. 218(1964-10), p. 1246.
- (2) Snowdon, J. C., J. Acoust. Soc. Amer., Pt. 1, Vol. 39, No. 1 (1966), p. 878.
- (3) Neubert, V. H., J. Acoust. Soc. Amer., Vol. 36, No. 4 (1964), p. 673.
- (4) e.g., Timoshenko, S., et al., Vibration Problems in Engineering, John Wiley & Sons (1974)