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Studies of Thermal-Hydrodynamic Flow Instability*

(2nd Report: Effects of Heating Condition of Thermal-Hydrodynamic Flow Instability)

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For reliable prediction of flow stability in sodium-heated steam generators, a dynamic model was proposed for boiling flow oscillation in parallel channel systems, and an analysis code was developed. The model contains a description of a sodium flow exchanging heat with a water flow in counter-current fashion. The code was applied to three representative flow systems whose heating conditions differed from each other, whereby their flow stabilities were compared with a focus on the effects of heating condition.

Eigenvalues and flow impedances of the oscillation determined for each system reveal that:

- (1) Two fundamental systems for the steam generator, parallel tube system in an evaporator and steam generator modules arranged in parallel, have different stabilities under low frequency oscillation.
- (2) Existing analysis model conditioned on constant heat flux gives different results on stability from those of either steam generator model under low frequency oscillation.

1. Introduction

For ensuring sound performance of a sodium heated steam generator (hereinafter referred to as SG) for a long period, evaluation of its flow stability is necessitated in the stage planning. In the previous report¹⁾, the author developed a general description for flow stability analysis of parallel channel system, thereby proposing a new method of analysis based on the root of the characteristic equation (i.e. eigenvalues) obtained by placing matrix transfer functions expressing propagation of the multidimensional disturbances under boundary conditions. It was also noted that, in setting up an analysis model, consideration of heating conditions is as important as that of boundary conditions.

The instability phenomenon which poses a problem in the SG is considered to be classified as a density wave oscillation and in this type of oscillation the propagation behavior of enthalpy disturbance is highly concerned with the stability of a system.¹⁾

Consequently, the effects of heating condition on stability are considered to be fairly large, but any quantitative evaluation has not been attempted so far.

In this report, with a focus on the effect of heating condition on flow stability an analytical study is made on the three representative models shown in Fig. 1. Two different types of models are conceivable as SG models; one, model B, is of the temperature dominating type and represents a parallel heat transfer tube and the other, model C, is a heat-exchanger type and represents a parallel loop or parallel module of

SG's. What difference can exist between these two models in respect to the stability threshold and oscillation mode is the main theme in this report. The existing analysis codes²⁾⁻⁴⁾ were originally developed for use with nuclear reactors and boilers and for this reason they adopted the model A of the heat flux dominating type. To determine how far can they be applied in the evaluation of flow stability of an SG is another important subject of this study.

Fig. 1 System model for heating condition.

2. Principal symbols

- A: heat equivalent of work
- A, B: matrix coefficients
- a: tube inner radius
- b: tube outer radius
- C, D: matrix transfer functions
- d: tube inner diameter
- de: equivalent diameter of sodium side flow channel
- F: specified friction force per unit volume
- f: coefficient related to heating condition
- G: weight velocity
- g: gravitational acceleration
- H: specific enthalpy
- J: vector composed of flux terms of X
- P: pressure
- Q: specified heat input rate per unit volume
- q: heat flux

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- s: Laplace transform variable or eigenvalue
 T: temperature
 t: time
 V: specific volume
X, Y: vectors composed of system state variables
 Z: flow impedance
 z: distance
 α : heat transfer coefficient
 γ : logarithmic amplification factor (real part of s)
 κ : thermal diffusivity
 θ : angle of tube to the horizontal
 λ : friction coefficient or thermal conductivity
 Φ : vector composed of production terms of **X**
 ω : frequency (imaginary part of s)
 τ : characteristic time
Suffixes
 I: channel inlet or imaginary part
 E: channel exit
 R: real part
 N: sodium side
 a: inner surface of tube
 b: outer surface of tube
 *: total enthalpy or total pressure
 -: Laplace transformed value

3. Theoretical analysis

3.1 Matrix transfer functions

In the previous report models A and B, whose heat input rate can be determined by the water side alone, were taken up and their matrix transfer functions were derived to describe propagation of disturbances. In this report, using transport equations prepared for model B as a foundation, the author derives transfer functions for model C by arranging them with an energy equation for sodium side into simultaneous equations.

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ H + \frac{3AG^2V^2}{2g} & G + \frac{AG^3V}{g} \left(\frac{\partial V}{\partial P} \right) & \frac{AG^3V}{g} \left(\frac{\partial V}{\partial P} \right) & 0 \\ \frac{2GV}{g} & \frac{G^2}{g} \left(\frac{\partial V}{\partial H} \right) & 1 + \frac{G^2}{g} \left(\frac{\partial V}{\partial P} \right) & 0 \\ 0 & 0 & 0 & -G_N \end{bmatrix} \quad (3')$$

$$A_x = \begin{bmatrix} 0 & -\frac{1}{V^2} \left(\frac{\partial V}{\partial H} \right) & -\frac{1}{V^2} \left(\frac{\partial V}{\partial P} \right) & 0 \\ \frac{AGV}{g} & \frac{1}{V} - \frac{1}{V^2} \left(H - \frac{AG^2V^2}{2g} \right) \left(\frac{\partial V}{\partial H} \right) & -A - \frac{1}{V^2} \left(H - \frac{AG^2V^2}{2g} \right) \left(\frac{\partial V}{\partial P} \right) & 0 \\ \frac{1}{g} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/V_N \end{bmatrix} \quad (4')$$

$$A_\phi = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \left(\frac{\partial Q}{\partial G} \right) - A \sin \theta & \left(\frac{\partial Q}{\partial H} \right) & \left(\frac{\partial Q}{\partial P} \right) & \left(\frac{\partial Q}{\partial H_N} \right) \\ -\left(\frac{\partial F}{\partial G} \right) & -\left(\frac{\partial F}{\partial H} \right) + \frac{1}{V^2} \left(\frac{\partial V}{\partial H} \right) \sin \theta & -\left(\frac{\partial F}{\partial P} \right) + \frac{1}{V^2} \left(\frac{\partial V}{\partial P} \right) \sin \theta & 0 \\ -\left(\frac{\partial Q_N}{\partial G} \right) & -\left(\frac{\partial Q_N}{\partial H} \right) & -\left(\frac{\partial Q_N}{\partial P} \right) & -\left(\frac{\partial Q_N}{\partial H_N} \right) \end{bmatrix} \quad (5')$$

Transport equations describing the water side flow in a single heat transfer tube and the sodium side flow which exchanges heat in counter-current fashion with the water side flow can be expressed by Eq.(1).

$$\frac{\partial (X/V)}{\partial t} + \frac{\partial J}{\partial z} = \Phi \quad (1)$$

A vector composed of system state variables **X**, its flux density **J** and its production rate Φ are as given in Table 1. For the water side flow the following conditions have been assumed as in the previous report.

- (1) One dimensional flow.
- (2) Homogeneous flow.
- (3) Equilibrium flow.

And for the sodium side flow, the following conditions have been assumed beside above (1)–(3) conditions.

- (4) Constant weight velocity.
- (5) Constant specific volume.
- (6) Constant heat transfer coefficient.

It should be noted here that using these assumptions we have omitted equations of mass and momentum for the sodium side flow from transport equations. Also, kinetic energy and potential energy have been neglected in the sodium side energy equation for the sake of simplification.

After obtaining small perturbation of Eq.(1) we can drive Eq.(2) by Laplace transformation.

$$\frac{d\delta J}{dz} = -s \delta \left(\frac{X}{V} \right) + \delta \Phi \quad (2)$$

In the following, we take up a vector **Y** composed of weight velocity **G**, enthalpy **H**, pressure **P** of water and enthalpy **H_N** of sodium as a set of independent system state variables. If coefficients **B**, **A_x** and **A_φ** are defined as given by Eqs. (3), (4) and (5), and Table 1 is considered, then each coefficient can be expressed respectively by Eqs.(3)', (4)' and (5)'.

$$\delta J = B \delta Y \quad (3)$$

$$\delta \left(\frac{X}{V} \right) = A_x \delta Y \quad (4)$$

$$\delta \Phi = A_\phi \delta Y \quad (5)$$

Table 1. Element of $\mathbf{X}$, $\mathbf{J}$ and Φ in conservation equation.

Fluid	Equation	$\mathbf{X}$	$\mathbf{J}$	Φ	
Water	Mass	1	G	0	$Q = \frac{4}{d} \alpha (T_a - T)$
	Energy	$H - APV + AG^2V^2/2g$	$G(H + AG^2V^2/2)$	$Q - AG \sin \Theta$	$F = \frac{1}{d} \lambda G^2V/2g$
	Momentum	GV/g	$P + G^2V/g$	$-F - \frac{1}{V} \sin \Theta$	
Sodium	Energy	H_N	$-G_N H_N$	$-Q_N$	$Q_N = \frac{4}{d_e} \alpha_N (T_N - T_b)$

$$\left. \begin{aligned} \left(\frac{\partial Q}{\partial H}\right) &= f_{aa} \left\{ \frac{Q}{\alpha} \left(\frac{\partial \alpha}{\partial G} \right) \right\} \\ \left(\frac{\partial Q}{\partial H}\right) &= f_{aa} \left\{ \frac{Q}{\alpha} \left(\frac{\partial \alpha}{\partial H} \right) - \frac{4\alpha}{d} \left(\frac{\partial T}{\partial H} \right) \right\} \\ \left(\frac{\partial Q}{\partial P}\right) &= f_{aa} \left\{ \frac{Q}{\alpha} \left(\frac{\partial \alpha}{\partial P} \right) - \frac{4\alpha}{d} \left(\frac{\partial T}{\partial P} \right) \right\} \\ \left(\frac{\partial Q}{\partial H_N}\right) &= f_{ab} \left\{ \frac{4\alpha_N}{d} \left(\frac{\partial T_N}{\partial H_N} \right) \right\} \end{aligned} \right\} \text{----- (6)}$$

In Eq.(5)', elements related to Q and Q_N can be expressed as Eqs.(6) and (7) by introducing four parameters $f_{aa} - f_{bb}$. These $f_{aa} - f_{bb}$ are parameters related to the heating condition and are given in Table 2 together with other parameters for model A and model B.

Also, elements related to F can be expressed as follows in Eq.(8).

$$\left. \begin{aligned} \left(\frac{\partial F}{\partial G}\right) &= \frac{F}{G} \left\{ \frac{G}{\lambda} \left(\frac{\partial \lambda}{\partial G} \right) + 2 \right\} \\ \left(\frac{\partial F}{\partial H}\right) &= \frac{F}{H} \left\{ \frac{H}{\lambda} \left(\frac{\partial \lambda}{\partial H} \right) + \frac{H}{V} \left(\frac{\partial V}{\partial H} \right) \right\} \\ \left(\frac{\partial F}{\partial P}\right) &= \frac{F}{\lambda} \left\{ \frac{P}{\lambda} \left(\frac{\partial \lambda}{\partial P} \right) + \frac{P}{V} \left(\frac{\partial V}{\partial P} \right) \right\} \end{aligned} \right\} \text{----- (8)}$$

By rewriting Eq.(2) into one related to $\bar{\delta J}$ by using Eqs.(3), (4) and (5) we obtain Eq.(9). Here, $\mathbf{A}$ is as defined by Eq. (10).

$$\frac{d\bar{\delta J}}{dz} = \mathbf{A} \mathbf{B}^{-1} \bar{\delta J} \text{----- (9)}$$

$$\mathbf{A} = -S \mathbf{A}_x + \mathbf{A} \phi \text{----- (10)}$$

Defining a matrix transfer function $\mathbf{D}$ which describes the propagation of disturbances $\bar{\delta J}$ and rewriting Eq.(9) into a relation of $\mathbf{D}$ we obtain Eq.(12).

$$\bar{\delta J} = \mathbf{D} \bar{\delta J}_1 \text{----- (11)}$$

$$\frac{d\mathbf{D}}{dz} = \mathbf{A} \mathbf{B}^{-1} \mathbf{D} \text{----- (12)}$$

Here, though we have obtained Eq.(12) paying attention to the matrix transfer function $\mathbf{D}$ concerning $\bar{\delta J}$, we will obtain another equation, (14), if attention is given to the matrix transfer function $\mathbf{C}$ concerning $\bar{\delta Y}$ which is defined in Eq.(13).

$$\bar{\delta Y} = \mathbf{C} \bar{\delta Y}_1 \text{----- (13)}$$

$$\frac{d\mathbf{D}}{dz} = \mathbf{B}^{-1} \mathbf{A} - \frac{d\mathbf{B}}{dz} \text{----- (14)}$$

Integration of transfer functions can be carried out either by Eq.(12) or (14), but for the following reason Eq.(12) has been adopted as the basic formula for the analysis code which will be described later on. That is, in both Eqs.(12) and (14), there exist such elements that bring about discontinuity to matrix coefficients at the initiating points of boiling and superheating.

$$\left. \begin{aligned} \left(\frac{\partial Q_N}{\partial G}\right) &= f_{ba} \frac{d}{de} \left\{ \frac{Q}{\alpha} \left(\frac{\partial \alpha}{\partial G} \right) \right\} \\ \left(\frac{\partial Q_N}{\partial H}\right) &= f_{ba} \frac{d}{de} \left\{ \frac{Q}{\alpha} \left(\frac{\partial \alpha}{\partial H} \right) - \frac{4\alpha}{d} \left(\frac{\partial T}{\partial H} \right) \right\} \\ \left(\frac{\partial Q_N}{\partial P}\right) &= f_{ba} \frac{d}{de} \left\{ \frac{Q}{\alpha} \left(\frac{\partial \alpha}{\partial P} \right) - \frac{4\alpha}{d} \left(\frac{\partial T}{\partial P} \right) \right\} \\ \left(\frac{\partial Q_N}{\partial H_N}\right) &= f_{bb} \left\{ \frac{4\alpha_N}{de} \left(\frac{\partial T_N}{\partial H_N} \right) \right\} \end{aligned} \right\} \text{----- (7)}$$

In the case of Eq.(12), that discontinuity appears in the form of a step function and so no particular consideration need be taken in value analysis, whereas in the case of Eq.(14) the discontinuity of coefficients develops to such extent that it appears as a delta function and special treatment becomes necessary in value analysis (that is, adding of supplementary terms describing transition of the initiating points.).

Matrix transfer functions $\mathbf{C}$ and $\mathbf{D}$ have a relation expressed by Eq.(15), and by using this equation these functions can be converted to each other.

$$\mathbf{C} = \mathbf{B}^{-1} \mathbf{D} \mathbf{B}_1 \text{----- (15)}$$

3.2 Boundary conditions

When evaluating the flow stability of a parallel channel system, boundary conditions to be imposed on the water side flow are expressed by Eqs.(16), (17) and (18).¹⁾ If it is assumed that no forcible effect causing oscillation comes from the sodium side, it is reasonable to set, as the boundary condition, the change of sodium inlet enthalpy at 0, and so Eq.(19) is given.

$$\delta H_1^* = 0 \text{----- (16)}$$

$$\delta P_1^* = 0 \text{----- (17)}$$

$$\delta P_1^* = 0 \text{----- (18)}$$

$$\delta H_{NE} = 0 \text{----- (19)}$$

First, if we cancel out all sodium side elements from matrix transfer function $\mathbf{C}$, considering Eq.(19) alone, then we will be able to obtain a 3×3 -type matrix transfer function related only to water side in place of the above 4×4 -type matrix transfer function. The 3×3 type matrix transfer function thus obtained can be treated in exactly the same way as the one for model A and model B which was described in the previous report. If this new function is represented by $\mathbf{C}$, $\mathbf{C}$ can be expressed by Eq.(20) by using element C_{ij} of row i , column j of the original 4×4 type matrix transfer function.

$$\mathbf{C} = \left[C_{ij} - C_{i4} \left(\frac{C_{4j}}{C_{44}} \right) \right] \text{----- (20)}$$

Also, corresponding to the transformation of transfer function by way of Eq.(20), elimination of sodium

side elements will necessarily be done from disturbances $\bar{S}Y$ and $\bar{S}J$ and coefficient B which relates $\bar{S}Y$ and $\bar{S}J$ to each other. Then, the boundary conditions (16)–(18) related to water side will be applied. All of these are conditions pertaining to total pressure or total enthalpy, and so we can treat with more ease if system state variables are transformed just as Eq.(21). Here, coefficient B^* denotes a matrix given by Eq.(22).

$$\bar{S}Y^* = \begin{bmatrix} \bar{S}G \\ \bar{S}H^* \\ \bar{S}P^* \end{bmatrix} = B^* \begin{bmatrix} \bar{S}G \\ \bar{S}H \\ \bar{S}P \end{bmatrix} \quad (21)$$

$$B^* = \begin{bmatrix} 1 & 0 & 0 \\ \frac{AGV^2}{9} & 1 + \frac{AG^2V}{9} \left(\frac{\partial V}{\partial H} \right) & \frac{AG^2V}{9} \left(\frac{\partial V}{\partial P} \right) \\ \frac{GV}{9} & \frac{G^2}{29} \left(\frac{\partial V}{\partial H} \right) & 1 + \frac{G^2}{29} \left(\frac{\partial V}{\partial P} \right) \end{bmatrix} \quad (22)$$

If matrix transfer function C^* related to disturbances $\bar{S}Y^*$ is defined by Eq.(23) C^* will be expressed by Eq.(24), consideration being given to Eqs.(13) and (21).

$$\bar{S}Y^* = C^* \bar{S}Y^* \quad (23)$$

$$C^* = B^* C B_1^{-1} \quad (24)$$

$$C_{31}^* = 0 \quad (25)$$

By applying Eqs.(16)–(18) for boundary conditions to Eq.(23) we will obtain Eq.(25).

This is a characteristic equation that determines modes for free oscillations. As in the previous report, the left side of this characteristic equation (25) is termed characteristic value. Since characteristic value C_{31}^* implies the response of total pressure at steam outlet to the oscillation of feed water weight velocity, this value is known to be related to the water side flow resistance.

The root of Eq.(25) for Laplace variable s is a complex number, and may be expressed as $s = \gamma + i\omega$. The real part γ represents the logarithmic amplification factor of proper oscillation and the imaginary part ω , the frequency of proper oscillation. This root is hereinafter referred to as eigenvalue. It is needless to mention that such a mode as allows the real part to become positive brings about unsteady phenomena.

Table 2. Parameters related to heating condition.

model A	model B	model C
$f_{aa} = f_{21}/(f_{21} + \alpha f_{22})$	$f_{aa} = (f_{21} + \alpha_N f_{11})/f_0$	$f_{aa} = (f_{21} + \alpha_N f_{11})/f_0$ $f_{ab} = \alpha/f_0$
		$f_{ba} = (\alpha/b)\alpha_N/f_0$ $f_{bb} = (f_{21} + \alpha f_{22})/f_0$

$$f_0 = f_{21} + \alpha_N f_{11} + \alpha(f_{22} + \alpha_N f_{12})$$

$$f_{11} = \mu \alpha \{ I_1(\mu a) K_0(\mu b) + K_1(\mu a) I_0(\mu b) \}$$

$$f_{21} = -\lambda \mu^2 a \{ I_1(\mu a) K_1(\mu b) - K_1(\mu a) I_1(\mu b) \}$$

$$\mu^2 = s/\kappa$$

$$f_{12} = -\frac{a}{\lambda} \{ I_0(\mu a) K_0(\mu b) - K_0(\mu a) I_0(\mu b) \}$$

$$f_{22} = \mu a \{ I_0(\mu a) K_1(\mu b) + K_0(\mu a) I_1(\mu b) \}$$

Fig. 3 Effect of heating condition or parameter f_{aa} .

3.3 Parameters related to heating conditions.

In the previous report there was introduced a non-dimensional parameter corresponding to each of the heating conditions of models A and B, and in the present report four non-dimensional parameters have been introduced so as to correspond to model C. As to the effects of the heating condition on flow stability we can give its estimate to some extent through a comparative study of these non-dimensional parameters, since they are representative of the heating condition of each models.

A heat transfer model is assumed which details as shown in Fig. 2, and behavior of a heat transfer tube is described by a nonstationary heat conduction equation. Then if we set up heat transfer correlations applicable to the inside and the outside of the tube, we can obtain four non-dimensional parameters for model C by the same calculation as given in the supplement of the previous report. The ultimate results are given in Table 2, calculations of the related equations being omitted here as they were given in the previous report. Parameters for models A and B that were obtained in the previous study are also given in the table.

Fig. 2 Heat transfer model for tube wall.

Comparison between models A and B.

Figure 3 shows an example of comparison of parameter f_{aa} related to the heating conditions of models A and B. This figure shows the values of parameter f_{aa} corresponding to $s = i\omega$ on the plane of complex number. As the conditions for comparison, Table 3 was used for specifications of the heat transfer tube, and approximate values of SG in the single phase zone were assumed for heat transfer coefficients α and α_N . Also, in the figure is shown the value of frequency ω in the form of non-dimensional number which is obtained by multiplying it by $\tau = (b-a)^2/4\kappa$ which represents a characteristic time related to heat conduction of the tube.

Table 3 Structural specification of SG.

Items	Data
Tube inner diameter	16.3 mm
Tube outer diameter	25.4 mm
Tube length	40.0 m
Tube inclination to the horizontal	9
Tube material	STBA 24
Equivalent diameter of sodium flow channel	73.0 mm

As can be seen from Fig. 3, difference between models A and B viewed from the standpoint of heating condition depends on frequency of the disturbance. Difference appears conspicuously in the case of a low frequency where $\tau\omega$ is small. However, with $\tau\omega = 1$ as the turning point, difference disappears for higher frequencies. In the above-cited example of analysis its characteristic time is $\tau = 0.52$ seconds, and $\tau\omega = 1$ corresponds to an oscillation cycle of 3.3 seconds. In Fig. 3, for reference sake, such a case where the heat flux is kept constant on the inner surface of the tube is marked with $\bullet$, while a case where temperature is kept constant is marked with $\circ$.

Comparison between models B and C.

Figure 4 gives an example of analysis of the four parameters of model C. As seen in Table 2, there is no difference between B and C so far as parameter f_{aa} is concerned. Accordingly, it is evident that characteristics of model C that differ from those of model B should be found in the thermal interaction between the water side and the sodium side to be described by f_{ab} and f_{ba} . The limit without which the effects of f_{ab} and f_{ba} can be more or less disregarded is $\tau\omega = 3$ (which corresponds to an oscillation cycle of 1.1 seconds). In areas where the disturbance frequency is lower, difference shows up between B and C in respect to heating condition.

So far as the heating condition for low frequency disturbance is concerned models A, B and C differ noticeably from each other, and it is obvious that such difference should influence the flow stability.

Fig. 4 Four parameters related to heating condition for model C.

3.4 Analysis code

CRESCENDO-II code has been compiled for the purpose of analyzing mainly the flow stability of the SG loop system. Figure 5 shows an outline of the code in flow-chart. This code is based on its predecessor, CRESCENDO-I, and is expanded by the addition of energy equation of the sodium side so as to be usable in the analysis of the heating condition for model C. Transfer functions in the code should be handled as explained in 3.1 and 3.2. In the basic formulas, model C is interpreted as including models A and B, so analysis concerning models A and B can be carried out by means of this new code.

The analysis code consists of the following four parts; steady state analysis, transfer function analysis, eigenvalue analysis and oscillation pattern analysis. In

any of these analyses, the same method as the one used in CRESCENDO-I is used.

Fig. 5 Outline of computer code CRESCENDO-II

4. Results of analysis

4.1 Conditions for analysis

In the previous report, the author showed analytical values of flow instability in respect to a 1 MW SG of PNC (Power Reactor and Nuclear Fuel Development Co.), which were found in a fairly good agreement with the actually measured values.⁵⁾ In the present study, a comparative study is carried out on what effects would be given to flow stability by the difference among models A, B and C in heating condition. Structural specifications of the SG set up before analysis are given in Table 3, and operational conditions of the SG in comparison, in Table 4.

Table 4 Operating condition of SG

Items	Data
Feed water temperature	220 °C
Feed water flow rate*	0.675 T/h
Exit steam pressure	130 at
Sodium inlet temperature	510 °C
Sodium flow rate*	variable

* flow rate per one tube

Fig. 6 Steady state performances.

Of the operational conditions given in Table 4, static characteristics of the SG are shown in Fig. 6 in the case when the sodium flow rate alone is changed.

Fig. 7 Eigenvalues of oscillation (model A)

Fig. 8 Eigenvalues of oscillation (model B)

Fig. 9 Eigenvalues of oscillation (model C).

The solid lines indicate the analytical results of water side temperature, and the broken lines, those of sodium side temperature, which constitutes the condition for analysis of flow stability to be discussed in the following.

4.2 Eigenvalue

In Figures 7 – 9 are shown on planes of complex number the analytical results of eigenvalues of oscillation. Each eigenvalue is made non-dimensional by multiplying by characteristic time ζ , which actually is a period needed for water to pass the channel. Modes are termed as in the previous report in the order of the size of imaginary part $\zeta\omega$, starting with the one of the smallest imaginary part as 1st mode (I), followed by 2nd mode (II), ... The eigenvalues of $\zeta\omega = 0$ appearing in Figs. 7 and 9 are termed 0-order mode. Signs a – d correspond to those in Fig. 6.

The eigenvalues of models A – C given in Figs. 7 – 9 correspond to one and the same static characteristic, and the effect of heating condition alone can be known from the comparison among Figs. 7 – 9.

(1) Difference between models A and B in heating condition influences up to 2nd mode, and modes of 3rd and higher receive almost no influence. This is due to the effect of the heat transfer tube explained in 3.3 (that is, deterioration of response to high frequency disturbances).

(2) Difference between models B and C in heating condition can influence up to 1st mode, and influence on those of higher modes can be disregarded. This is due to the effect of the pattern of proper oscillation of sodium side enthalpy described in 4.3 (that is, decrease of amplitude of oscillation in higher modes).

The above mentioned observation is not necessarily generalized from the quantitative viewpoint. Generally, it is considered that the lengths of several characteristic times governing SG performances (e.g. water side passage time, sodium side passage time, response time of the tube to heat transfer, etc.) are influential.

4.3 Oscillation patterns

As an example of analysis of proper oscillation pattern, the results of analysis of case c of model C are shown in Figs. 10 and 11. In these figures are illustrat-

ed abridged patterns of proper oscillation as observed over a period of half cycle of weight velocity oscillation δG , enthalpy oscillation δH and pressure oscillation δP in water side and enthalpy oscillation δH_N in sodium side. The oscillation patterns for the remaining half cycle can be obtained by reversing the shown patterns from + to –.

Characteristics of the proper oscillation patterns of model C mostly agree with those of model B described in the previous report. Some of the important ones are described in the following.

- (1) δG is a stationary wave, and with the boiling zone as a boundary the one in the preheating zone assumes a reversed phase in relation to the one in the superheated zone.
- (2) δH and δH_N are travelling waves and the number of degree of its mode agrees with the number of peaks of δH existing inside the channel.
- (3) δP is a stationary wave, and over the whole channel it appears with nearly the same phase.
- (4) The higher is the degree of the mode, the smaller becomes the amplitude of oscillation of δH_N .

Characteristics mentioned in (4) have such an effect that model C approaches model B in respect to the oscillation of the high degree modes.

Fig. 10 Oscillation pattern of model C (1st mode oscillation).

Fig. 11 Oscillation pattern of model C (2nd mode oscillation).

4.4 Flow impedance

In studies published heretofore⁶⁾, the idea of flow resistance has been often introduced when discussing flow instability phenomena. In this paragraph the author introduces the idea of flow impedance which is developed from the flow resistance, and discuss the flow stability from the viewpoint of kinetics while showing its relationship with the transfer functions.

The energy flux density of a flow inside the tube is expressed by Eq.(26) if Table 1 is considered.

$$J_2 = GH^* = G(H - APV) + AGVP^* \quad (26)$$

The first term on the right side of the equation represents internal energy flux density, and the second term, kinetic energy flux density. Now, let us evaluate how much energy is to be supplied from the outside of the system for allowing stationary oscillation to continue, on the assumption that the condition of Eq.(17) is put aside from the boundary conditions. If consideration is limited up to 2nd order perturbations, Eq.(27) is obtained for kinetic energy flux density.

$$\delta(AGVP^*)_1 = AP_1^* \delta(GV)_1 + A(GV)_1 \delta P_1^* + A \delta P_1^* \delta(GV)_1 \quad (27)$$

Here, energy flux density is evaluated at the feed water inlet. If we take the time average, giving consideration to the fact that the oscillation is a stationary one, only the third term remains. And if we disregard the variation in specific volume at the feed water inlet, the mean value of necessary energy flux density supply, δK_1 , is eventually expressed by Eq.(28).

$$\delta K_1 = AV_1 \phi \delta P_1^* \delta G_1 dt / \phi dt \quad (28)$$

At the outlet, the boundary condition expressed by Eq.(18) is imposed, and there is no supply of kinetic energy.

By imposing boundary conditions (16) and (18) on transfer function C^* defined by Eq.(23) we can obtain Eq.(29) which gives the relationship between δP_1^* and δG_1 .

$$\delta P_1^* = -C_{31}^* / C_{33}^* \delta G_1 = Z \delta G_1 \quad (29)$$

Here, Z is a coefficient of such a nature as to be termed flow impedance, while real part Z_R denotes resistance, and imaginary part Z_I , reactance.

By substituting Eq.(29) into (28) we obtain Eq.(30) and from this it can be known that δK_1 is related only to resistance.

$$\delta K_1 = AV_1 Z_R |\delta G_1|^2 \quad (30)$$

Equation (30) gives a kinetic interpretation of flow stability and is consistent with the idea of previous flow resistance. In other words, when resistance Z_R is positive, supply of kinetic energy from the outside of the system becomes indispensable for making stationary oscillation continue, and so self-excited oscillation can not occur. However, if Z_R becomes negative, it becomes possible for the system to give kinetic energy to outside while maintaining stationary oscillation. This is a necessary condition for self-excited oscillation to occur. Generally, however, as there exists reactance Z_I besides the resistance Z_R , $Z_R < 0$ is not necessarily a sufficient condition for the occurrence of self-excited oscillation. But so far as such a case of $\omega = 0$ is concerned $Z_R < 0$ (so-called negative resistance) becomes the necessary and sufficient condition for the occurrence of a flow excursion as $Z_I = 0$ holds in all cases.

Figures 12 — 15 show the results of analysis of resistance Z_R in cases a — d as against ω . Z is made non-dimensional by multiplying by $g / G_1 V_1$, and by this process flow resistance is converted to the coefficient of pressure drop that is caused at an imaginary orifice placed at the feed water inlet. For reference, it should be noted that from such evaluation of Z_R like this it can be known that flow instability arising in model C in cases a and b and in models A, B and C in case d (refer to Figs. 7 — 9) can be corrected by providing an actual orifice at the feed water inlet, whose pressure drop coefficient cancels negative Z_R .

Fig. 12 Flow resistance (case a)

Fig. 13 Flow resistance (case b).

Fig. 14 Flow resistance (case c).

To summarize, by comparing the effects of heating condition on flow stability from the viewpoint of flow resistance, we can clarify even better the difference among models in respect to low frequency oscillation. Particularly, the difference of flow resistance is extremely large at $\tau\omega = 0$, and from this we could understand that selection of a proper model for heating condition is imperative for evaluating flow stability.

5. Conclusions

Based on the basic consideration of flow instability phenomena as well as the theoretical analysis of the propagation of thermal-hydrodynamic disturbance in a water flow, a new analysis method of phenomenon was proposed in the previous report in which the propagation of multi-dimensional disturbances is described in terms of matrix transfer functions. Based on this new method and by extending the content of matrix transfer function to the sodium flow, a new analysis code has been developed in the present study which enables analysis of flow stability in heating condition of the heat exchanger type. Further, this code has been applied to the three representative models whose heating conditions differ from each other, thereby performing an analytical investigation into the effects of heating condition on flow stability. In the following are given some of the results of the above study which will serve as a guideline in analysis of flow stability of an SG.

- (1) As models for SG two models, B and C, are useful. B corresponds to the parallel tube system in a SG and C, to the parallel loop system or parallel module system of SG's. These two models differ greatly in their stability under low mode oscillation.
- (2) In case the existing codes that are conditioned on model A are used for SG flow stability analysis, no proper evaluation of the stability can be made under

Fig. 15 Flow resistance (case d).

low mode oscillation.

- (3) Difference in model does not exert any influence on stability if the oscillation is restricted to that of high mode.

Also, by supplementing a kinetic observation on flow stability by the idea of flow impedance the following interpretations are obtained for the phenomena.

- (1) That the resistance, which is defined as the real part of flow impedance, becomes negative is a necessary condition for the occurrence of instability.
- (2) For the occurrence of flow excursion, the condition given in (1) is the necessary and sufficient condition.
- (3) The flow impedance is related to the matrix transfer function by means of Eq.(29).

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6. Reference

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