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Diffusion of Bubbles in Two-Phase Flow*
(The 1st Report, On the Method for the Measurement
of the Diffusivity of the Bubble)

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Measuring the spread of tracer bubbles from a point source located at the center of an 80 mm. dia. pipe, we obtained the information on the diffusion of bubbles in a fully developed turbulent air-water two-phase flow. The features of the measuring technique in this apparatus are as follows: First, the local void fraction is measured with a fiber glass probe based on the principle of light refraction. Second, we used a sampling probe based on the principle of the hot-wire anemometer probe, by which we were able to discriminate bubbles between helium(tracer) and air. As a result, we conclude that the diffusivity of the bubble is smaller than the eddy diffusivity of momentum and it increases with an increasing value of void fraction.

1. Introduction

Knowledge of the behavior of discrete particles in a turbulent flow is of great interest because it offers a clue to the analysis of heat and mass transfer problems in a two-phase flow. Such a flow is often encountered in the technical applications such as chemical reactors, boilers, and heat exchangers. Tchen⁽¹⁾, Flidlander⁽²⁾, and Soo⁽³⁾ investigated theoretically the behavior of solid particles in a turbulent flow. Kada⁽⁴⁾, Soo⁽⁵⁾⁽⁶⁾, and Hanratty⁽⁷⁾ carried out precise experiments. However, in the case of the air-liquid two-phase flow system, there are very few reports concerning the behavior of bubbles because it is difficult to carry out the experiments in such a state in which the radius of bubbles should be kept uniform and held in the small size region to facilitate the analysis. Rouhiainen⁽⁸⁾, Namie et al.⁽⁹⁾⁽¹⁰⁾ studied with regard to the transport of liquid drops in the mist flow regime. In the case of bubbly flow regime, though there are many theoretical and experimental studies⁽¹¹⁾ with regard to the slip velocity and the drag coefficient of bubbles, there are few reports concerning the random motion of bubbles in a turbulent flow from the view point of statistical formulation. Kurihara⁽¹²⁾ and recently Serizawa et al.⁽¹³⁾⁻⁽¹⁵⁾ carried out the experiments with regard to the bubble diffusion from a nozzle. The

nozzle adopted by Serizawa et al., in order to generate the tracer bubble, produces so many bubbles at a time that the diffusion of bubbles is affected considerably by the interference of tracer bubbles ejected from the nozzle. Okamoto et al.⁽¹⁶⁾ measured the diffusivity of bubbles with radius of about 0.03 mm by the method of flow visualization with hydrogen bubbles. According to the experiment, it is reported, the bubble diffusivity is analogous to the eddy diffusivity of momentum. In order to investigate the behavior of the bubble in a vertical upward flow approaching the wall of a duct, Sekoguchi et al.⁽¹⁷⁾ carried out an experiment in a 50×25 mm vertical rectangular duct, changing the size of bubbles, and obtained the ratio of adherence of the bubble to the wall. It was found that the ratio of adherence is greatly affected by the bubble size. This experiment was conducted with a relatively narrow duct in order to find the effect of the wall. Bankoff⁽¹⁸⁾, Koide⁽¹⁹⁾, Aoki et al.⁽²⁰⁾ assume that the diffusivity of the bubble which is necessary for the analysis of void fraction distribution in a vertical pipe is equal to the eddy diffusivity of momentum in a single-phase turbulent flow.

We have conducted a fundamental study on the diffusion of bubbles in a turbulent air-water two-phase flow, and defined the diffusivity of bubbles. We have established a measuring technique of the diffusivity of bubbles by a point source method with helium gas bubbles as tracers, and shown some experimental results.

2. Diffusivity of bubbles

From the statistical point of view, it is necessary to introduce the concept of diffusivity as a method of expressing the behavior of bubbles in a flow. Let us assume that n is the mean number of tracer

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bubbles in a unit volume in an isotropic and homogeneous turbulent flow. At stationary state tracer bubbles are distributed uniformly in the flow. In the case when the flow is not stationary at the beginning, we wait until the final stationary state is realized. Considering the plane on which x_1 is constant, the flow density of tracer bubbles is expressed by J_1 , and we assume that J_1 is proportional to the density gradient of tracer bubbles. Therefore,

$$J_1 = -D_B \frac{dn}{dx_1} \quad (1)$$

where J_1 is the number of tracer bubbles which cross x_1 in unit time and unit area number/sm², D_B is the diffusivity of bubbles m²/s. The equation of diffusion which describes how the number of tracer bubbles in unit volume and unit time behaves is given by

$$\frac{\partial n}{\partial t} = D_B \frac{\partial^2 n}{\partial x_1^2} \quad (2)$$

As is shown in Fig.1, when tracer bubbles are ejected from a fixed nozzle into the fluid at a certain number per second, the bubble density n at a point $A(x_1, x_2, x_3)$ which is x_1 m above the nozzle, coincides with the arrival probability P of the tracer bubbles. Therefore, at quasi-stationary state, Eq.(2) can be written as follows:

$$U_B \frac{\partial P}{\partial x_1} = D_B \frac{\partial^2 P}{\partial x_1^2} \quad (3)$$

where U_B is the mean bubble ascending velocity m/s. The solution of the above differential equation is given as that of heat conduction of a point source in an infinite medium⁽²¹⁾.

$$\left. \begin{aligned} P &= S/4\pi R D_B \exp[-U_B(R-x_1)/2D_B] \\ -D_B 4\pi R^2 \frac{\partial P}{\partial R} &= S \quad R \rightarrow 0 \\ R &= \sqrt{x_1^2 + x_2^2 + x_3^2} \end{aligned} \right\} \quad (4)$$

From Eq.(4)

$$\left. \begin{aligned} \ln PR - \ln \alpha &= -\beta(R-x_1) \\ \alpha &= S/(4\pi D_B), \quad \beta = U_B/2D_B \end{aligned} \right\} \quad (5)$$

where S is the intensity of ejection of tracer bubbles at a point source m³/s. When $R/x_1 \rightarrow 1$

$$P \approx S/(4\pi x_1 D_B) \exp[-U_B R^2/(4D_B x_1)] \quad (6)$$

This is a Gaussian probability distribution. Dipole moment $\bar{r}_m^2$, and variance $\bar{x}_m^2$ in the x_2 or x_3 direction are in the following relation:

$$\bar{r}_m^2 = 2\bar{x}_m^2 = \int_0^\infty r^2 P dr / \left(\int_0^\infty r P dr \right) = \frac{4D_B x_1}{U_B} \quad (7)$$

Fig.1 Physical model of bubble diffusion

when $x_1/U_B = t$ is large

$$D_B = \frac{1}{2} U_B \frac{\Delta \bar{x}_m^2}{\Delta x_1} \quad (8)$$

Equation(8) describes the relation between the bubble diffusivity D_B and the variance $\bar{x}_m^2$ of the probability distribution curve P .

Now, the motion of bubbles in a fluid at an arbitrary time t and t' is not independent contrary to the molecular diffusion of a gas. There exists velocity correlation and the diffusivity is a function of time. In a liquid it is known that in sufficiently long time the diffusivity coincides with the result of Eq.(6), and therefore it is possible to measure the eddy diffusivity with Eqs.(5) or (6)⁽²²⁾⁻⁽²⁴⁾.

However, as to a two-phase flow, the velocity correlation of bubbles which is affected by the turbulence of liquid is not yet clear. Therefore, in this report, we defined the diffusivity of bubbles with Eq.(8). By measuring the variance at several points, and from the gradient $\Delta \bar{x}_m^2 / \Delta x_1$, we obtained the diffusivity of bubbles.

3. Experimental apparatus and measuring method

Schematic diagram of the experimental apparatus is shown in Fig.2. The length of the measuring pipe is 4.11 m long from the void generator, the inner diameter of the pipe is 80 mm ϕ , and the pipe is made of transparent acrylic resin. Experiment was conducted in the bubbly regime of air-water two-phase flow. Water is sent from a water

1 Sampling probe, 2 Nozzle, 3 Probe for tracer bubble, 4 Optical fiber glass probe, 5 He gas tank, 6 Bridge circuit, 7 D.C. amplifier, 8 Synchroscope, 9 Electric counter, 10 Instruments for measuring void fraction, 11 Electric counter, 12 Orifice for measuring He gas flow rate, 13 Air compressor, 14 Surge tank, 15 Orifice for measuring air flow rate, 16 Void generator, 17 Water tank, 18 Pump, 19 Orifice for measuring water flow rate, 20 Camera

Fig.2 Schematic diagram of experimental apparatus

tank 17 by a pump 18 to the measuring pipe via an orifice 19. Water flows up vertically, and at the top of the pipe, water is separated from air and returns to the tank. Air is compressed by a compressor 13, via a surge-tank 14 and orifice 15, sent to a void generator 16, and mixed with water at the lower end of the measuring pipe. For the measurement of void fraction in the pipe, an optical fiber glass probe was used at 3.89 m from the bottom of the pipe. The optical fiber glass probe⁽²¹⁾ is made of a bundle of fine glass fibers, and the outer diameter of the probe is 0.5 mm ϕ . The tip is fixed with a wedge shaped prism with an angle of 90 degrees 21. Utilizing the difference of refraction between water and air, the existence of a bubble is indicated by an intermittent light signal. Details of the probe are shown in Fig.3(a).

Tracer bubbles are sent from a helium gas tank 5, and the pressure is adjusted by a valve, and then sent to a nozzle 3 which is designed to produce a signal bubble. The flow rate of water, which is to carry the helium bubble to the measuring pipe, is measured by an orifice 12, and the velocity of water from the nozzle 2 is regulated by a valve to be equal to the water velocity in the measuring pipe.

Helium gas bubbles with a constant diameter from the nozzle 2 are detected by a platinum probe utilizing electric resistance, and the number of bubbles is measured by an electric counter 11. The helium gas bubble is ejected into the measuring pipe

21 Wedge shaped prism, 22 Glass fiber bundle (30 $\mu\phi$, 200 elements), 23 Incident light guide probe, 24 Returning light guide probe

(a) Fiber glass probe

25 Electric lead wire, 26 Silver wire, 27 Brass ring, 28 Stainless wire 0.5 mm ϕ dia., 29 Phosphor-bronze wire 0.1 mm ϕ dia., 30 Platinum wire 0.005 mm ϕ dia.

Fig.3 (b) Sampling probe

one by one from the nozzle 2 with water through a funnel and a feeding pipe. A capillary tube is connected to the measuring pipe, and the mean diameter of the bubble is suitably measured.

The nozzle 2 is made of stainless-steel with an inner diameter 3.6 mm ϕ , outer diameter 4 mm ϕ , and is bent at right angles at the center of the measuring pipe, and the length of the nozzle in the axis of the pipe is 60 mm. The nozzles 2 are installed at two places 2.86 m and 3.16 m apart from the inlet of the pipe. The helium tracer bubbles ejected from the nozzle 2 ascend irregularly with a group of air bubbles.

Sampling probes 1 are attached at three places with an interval of 0.1 m, and 3.445 m apart from the inlet. Details of the sampling probe are shown in Fig.3(b). The principle of measurement is the same as the heat conduction cell. At the tip of a silver wire 26 soldered to a lead wire 25, a platinum resistance wire with 5 $\mu\phi$ dia. is stretched, and connected to a bridge circuit 6. The bridge circuit is balanced when the platinum wire is in the liquid. When the bubble which passes through the ring 27 touches the platinum wire, the resistance changes according to the difference of cooling effect of gases, and we can discriminate between helium and air. The signal is amplified by a d.c. amplifier 7, measured by an electric counter 9, and

Signal waves at the bottom of the photograph are of air bubbles.

Two signal waves at the middle of the photograph are of He bubbles.

Fig.4 Signal waves of bubble Measured by the sampling probe 20 ms/div, 1 v/div

Fig.5 Photograph of void signal waves measured by a fiber glass probe 20 ms/div, 2 v/div

observed by a synchroscope 8. By traversing the sampling probe 1 in a radial direction, the bubble arrival distribution curve was obtained, and the variance was calculated by statistical treatment. Then the diffusivity of bubbles was obtained.

A sample of signal wave form of bubbles is shown in Fig.4. A large number of waves at the bottom of the photograph is that of air bubbles, and the two waves at the middle of the photograph are helium bubbles. In Fig.5 void signal waves measured by the fiber glass probe are shown.

The mean ascending velocity of tracer bubbles U_b is obtained by superimposing the figures by a camera 20 and a stroboscope on a film and by measuring the vertical travel distance. It is reported that in a two-phase flow there exists a large variation of slip velocity by the interference of bubbles, but there is no adequate method at present to measure the ascending velocity of tracer bubbles, and we have decided to use the ascending velocity of a tracer bubble in a single-phase flow as U_b .

4. Experimental results and discussion

4.1 Error estimation

The brass ring 27 attached to the sampling probe is used in order to eliminate the difficulty of judging the helium bubble signal wave from incomplete signal waves of air bubbles which sweep over a part of the platinum wire. Many types of rings were manufactured for trial and experiments were conducted, and as a result the ring shown in Fig.3(b) was finally adopted which had good performance in judging between air and helium bubbles. However, there are some problems in using this ring. First, it disturbs the flow field. Second, the size of the ring determines the class of dispersion, and at a place where x_1 is small the variance becomes larger than the true value when the ring is too large. When the ring diameter is equal to the bubble diameter, it is ideal to catch the bubbles when the center of the ring coincides with that of the bubble, but in reality it often occurs that the center of the bubble does not coincide with that of the ring, and the ring catches a part of the bubble. In order to know the extent of error by the ring,

Fig.6 Effect of the ring of a sampling probe

the probability distribution of a bubble in a single-phase flow is compared for the cases when the ring is attached and not-attached. The result is shown in Fig.6. From this we can see that the existence of the ring affects the shape of probability distribution curve, but the error of variance is smaller than 10%. Experiments conducted for the investigation of reproducibility, the runs 4-3 and 8-5 coincide satisfactorily within the measuring error.

4.2 Bubble diffusivity

Fig.7 shows an example of the probability density function of bubbles ejected from a fixed nozzle placed at the center of a pipe in a single-phase turbulent flow for various values of distance x_1 . In this case liquid velocity was 0.355 m/s and bubble ascending velocity was 0.68 m/s. Experiments were also conducted for the cases when the liquid velocities were 0.12 m/s and 0.28 m/s.

Diameter of tracer bubbles was 6 mm ϕ , the period of generation was around one second. The probe was moved radially at an interval of 1 mm to 2 mm, and the number of ejected bubbles at each position was 500. It is seen that the experimental points coincide satisfactorily with the Gaussian distribution curve. For the case $x_1 = 0.745$ m, when the variance becomes largest, the spread of bubbles is around one half of the radius R_0 of the pipe. At this point the velocity is 5% lower than the central liquid velocity. For the distribution curve with wider spread, it is considered that the effect of the wall becomes large. Therefore, experiments were conducted at the central part of the pipe where the flow was homogeneous and isotropic.

Fig.8 shows an example of measured

Fig.7 Probability density function of bubbles

Fig.8 Probability density function of bubbles

results of bubble probability density function for the two-phase flow. Void fraction is 1.25 %. The curves stretch rapidly with distance x_1 . The fact that the total bubble collection ratio is lower than in the case of single-phase flow even using the same probe can be explained as follows: the effective collection area of the ring becomes smaller as the bubble enters the ring obliquely by a random movement of the bubbles. (See Appendix.) Variance $\bar{X}_m^2$ was obtained from the probability distribution curve, and the relation with the axial distance from the nozzle x_1 is shown in Fig.9. As a bubble has a finite size, the variance does not become zero at the origin. Near the origin, the effect of the size of the sampling probe is evident. Therefore it is not adequate to obtain the diffusivity from the variance near the origin in this measuring method. Therefore in this experiment, from the variance at several points apart from the origin, the gradient to the x_1 axis was obtained and from Eq.(8) the bubble diffusivity was obtained.

In the two-phase flow the increase of variance is very rapid. In Fig.10 the relation between the bubble diffusivity and the superficial liquid Reynolds number ($R_{to} = U_m 2R_0/\nu$) is shown. U_m is the mean liquid velocity and ν is the liquid kinematic viscosity. With R_{to} the bubble diffusivity increases. In general, for the dimensional analysis of the diffusivity, the friction velocity ($U^* = \sqrt{\tau_w/\rho} = \sqrt{\lambda/8} U_m$) which characterizes the turbulence at the central part of the pipe is used, and with the pipe diameter ($2R_0 = D$), the nondimensional number $\varepsilon_M/U^* 2R_0$ is used. It is

Fig.9 Relationship between variance $\bar{X}_m^2$ and x_1

Fig.10 Relationship between bubble diffusivity and R_{to}

known that the value of $\varepsilon_M/U^* 2R_0$ is around 0.035.⁽²⁷⁾ τ_w is frictional stress at the wall, ρ is the liquid density, λ is the friction coefficient of the pipe, and ε_M is the eddy diffusivity of momentum. In our experiment $D_B/U^* 2R_0$ was 0.012 and it is rather smaller than the value of $\varepsilon_M/U^* 2R_0$. On the contrary, the experimental result of Okamoto is $D_B/U^* 2R_0 = 0.031$ and coincides well with the value of $\varepsilon_M/U^* 2R_0$. The reason for our experimental result not coinciding with the value of $\varepsilon_M/U^* 2R_0$ is surmised as follows: large bubbles, because of the existence of slip velocity, the mechanism of the diffusion of bubbles is different from that of small bubbles.

In Fig.11 the relation between the bubble diffusivity and void fraction is shown. It is seen that the bubble diffusivity increases rapidly with an increase of void fraction. In Fig.12 the void fraction in the radial direction as measured by the fiber glass probe is shown at the time of the measurement of the bubble density, and it is seen that the void fraction is constant in the range of r/R_0 values smaller than 0.7, and it seems that there is no influence of the gradient of density on the probability distribution. In order to apply our measuring method to the range of high void fractions, it is necessary to improve the discrimination technique between tracer - and air bubbles, and also the measurement time.

5. Conclusions

In order to investigate the behavior of bubbles in a two-phase flow, we have de-

Fig.11 Relationship between bubble diffusivity and void fraction

Fig.12 Void fraction profile

Fig. 13

Fig. 14

fined the bubble diffusivity, and conducted experiments by the point source method.

(1) We made for trial a probe which is based on the principle that the thermal conductivity of tracer - and air bubbles is different. It was found that the probe has sufficient performance.

(2) It was found that the diffusivity of bubbles with 6 mm ϕ diameter in a single-phase flow increases with Reynolds number Re_0 . The non-dimensional bubble diffusivity $D_B/U*2R_0$ is smaller than $\epsilon_M/U*2R_0$ which is the non-dimensional number for the eddy diffusivity of momentum in the turbulent flow in a pipe.

(3) It was found that the bubble diffusivity in a two-phase flow increases with the void fraction.

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Appendix

In order to facilitate the explanation a ring is shown in Fig. 13, and we assume that the turbulent velocity v' is one-dimensional. It is essentially the same as in the actual three dimensional ring. The mean bubble ascending velocity is U_g , the transverse bubble turbulent velocity is v' , the angle of attack is θ , and the ring size is l .

Bubbles attack the ring from various directions, and the attack is a function of turbulence in a flow, and it can be expressed by a density function $f_v(\theta)$ of attack angle θ . The shape of function $f_v(\theta)$ is not clear, but we assume that the

Fig. 15

Fig. 16

distribution is as shown in Fig. 14(a) and (b). In general, turbulence is violent in a two-phase flow, the function $f_v(\theta)$ being considered to have the shape (b) with larger attack angle θ . The number of bubbles n_1 at an arbitrary radial position $r=r_1$ is given by Eq. (9)

$$n_1 = 2 \int_0^{\pi/2} f_v(\theta) d\theta \quad (9)$$

Bubble number obtained by our experiment $n_{1 \text{ exp}}$ is given by Eq. (10) (Fig. 15).

$$n_{1 \text{ exp}} = 2 \int_0^{\pi/2} f_v(\theta) l \cos \theta d\theta \quad (10)$$

$$n_{1 \text{ exp}}/n_1 = \frac{\int_0^{\pi/2} f_v(\theta) l \cos \theta d\theta}{\int_0^{\pi/2} f_v(\theta) d\theta} = \theta = \text{const.} \quad (11)$$

We assume that $f_v(\theta)$ does not change in the range of radii of measurement. It means that the flow is uniform and homogeneous, but it may be allowed, as our experiment was measured at the central part of the pipe where the void fraction and liquid velocity distribution are flat. It is deduced from Eq. (11) that the shape (b) has a smaller value of θ . In the calculation of D_B , it becomes the same whether we use n_1 or $n_1 \times \theta$, as there is no effect on the shape of distribution in Fig. 16.

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