621.833.031

Paper No. 148-13

Statistical Analysis of Dynamic Loads on Spur Gear Teeth* (Experimental Study)

By Toshimi TOBE**, Keijin SATO*** and Nobuo TAKATSU****

Some experimental results on the relation between transmission errors and dynamic loads of spur gears are presented. Using a power circulating type gear tester, the dynamic loads on two pairs of gears, i.e. an accurate one and a rough one, were measured and their dependence on gear speed and tooth load was investigated. The dynamic loads on the accurate gear pair showed recurrent time variations over a wide range of gear speeds. On the other hand, significant randomness of dynamic load was observed with an increasing speed in the rough gear set. An analogue computer simulation was also performed under the experimental condition. Good agreement was achieved between the experimental results and the simulation qualitatively; and it seems to show the propriety of the statistical theory on the estimation of dynamic loads which has already been developed in the previous paper.

1. Introduction

The authors have investigated theoretically the method to estimate the dynamic loads of a gear system based on the results of evaluation of transmission error from the statistical viewpoint in the previous papers(1)-(3). It has been pointed out that the recurrent time variation of dynamic load per revolution begins to miss due to a random component contained in the transmission error with an increasing gear speed and the time variation becomes very random at higher speeds.

This study examines experimentally whether the above phenomenon occurs in an actual gear system or not.

2. Experimental set up

A power circulating type gear tester which had a shaft distance of 120 mm driven by 11 kW induction motor was used. Schematic diagram of the test rig is shown in Fig.1. A loading torque to test gears is applied by twisting a torsion bar which is calibrated by means of strain gages attached to it. The gear speed may be changed by means of a variable speed drive during run. The torsion bar's stiffness is about one-hundredth of the meshing stiffness of test gear teeth so as to insulate the vibration of test gears from that of master gears. A forced lubrication system supplies #200 turbine oil to both gear teeth and bearings.

Dynamic load factors were obtained by taking the ratio of dynamic root strain of a tooth to static one of it, which were

measured by means of strain gages mounted at the root of every third tooth of the driven gear using slip rings and a strain amplifier. A phototransistor, a light beam source and a circular disk having a cut out equal to contact length were used to trigger the oscilloscope and display the contact position along the line of action in tooth strain measurements. As a large number of samples were required in order to process measured tooth strains statistically, five successive samples were recorded in a picture using a multi-trace device.

3. Dimensions and accuracies of test gears

Three test gears were used in the experiment to examine the influence of gear errors upon dynamic loads, that is, a hobbed gear with face width of 8 mm, which is termed Gear "A", and two Maag ground gears with face widths of 8 mm and 10 mm, both of which are termed Gear "B".

Two pairs of test gears were composed of these three gears; Pair I denotes a pair of Gear "B"s and Pair II denotes another pair of Gear "A" and one of Gear "B" with face width of 10 mm. Test gears' dimensions are shown in Fig.2 and Table 1. The maximum value of the equivalent torsional stiffness of test gear teeth is

Fig.1 Schematic diagram of test rig

* Received 12th September, 1974
 ** Professor, Faculty of Engineering, Tohoku University, Sendai.
 *** Assistant Professor, Faculty of Engineering, Utsunomiya University.
 **** Assistant, Faculty of Engineering, Tohoku University.

6.84×10^6 kg·cm and the natural frequency considering effective masses of additional masses and bearings and so on is 1573.3 Hz.

Total running period of the gear tester was about 6 hours; in which the time spent for the measurement was the last one hour.

In order to examine the degree of wear of tooth surfaces, tooth profile errors before and after the run were measured

Fig.2 Dimensions of test gears

Table 1 Dimensions of test gears

	Gear "A"	Gear "B"
tooth profile	standard	
module	4 mm	
pressure angle	20°	
number of teeth	30	
normal backlash	0.25 mm	
material	S45C	SNM22
	normalized	case hardened
finish	hobbed	Maag ground

Fig.3 Tooth profiles and lead errors of test gears before and after run

and the results are shown in Fig.3. The lead errors are also shown on the left hand in these figures. The tooth profiles of both Gear "B"s remain unchanged after the run.

Consequently, so far as the Pair I is concerned its static transmission error, which is measured by single flank meshing tester and is regarded as over all accuracy of the pair, may not be changed distinctly after the run. On the other hand, it is evident that the tooth profile of Gear "A" measured after the run differs from the one before run, showing a considerable wear of tooth surface. Based on the above considerations, the transmission errors measured after the run were employed for succeeding analysis.

Let's evaluate the transmission error by means of the method developed in the previous paper(1). The transmission error of Pair I was recorded during two revolutions, which is shown in Fig.4, and 580 sample points per revolution were obtained; then the total number of sample points was 1160 and the values of n and m were chosen to be 35 and 270, respectively. For the 0.1% significant level, a harmonic component detected is as follows

$$\hat{\sigma}_0^2 \Big|_{\lambda=(2\pi \times 30)/580} \approx 0.8 \mu\text{m}^2 \quad (1)$$

which suggests the presence of a harmonic component with one period per mesh of the amplitude of about $0.9 \mu\text{m}$. The consistent estimate of the random component remained after the removal of the periodic component is shown in Fig.6.(*)

The transmission error of Pair II was also recorded during two revolutions, which is shown in Fig.5, and 670 sample points per revolution were obtained; so the total number of sample points, m and n were 1340, 35 and 270, respectively. A harmonic component detected is as follows;

$$\hat{\sigma}_0^2 \Big|_{\lambda=(2\pi \times 30)/670} \approx 324 \mu\text{m}^2 \quad (2)$$

The result suggests the presence of a harmonic component which has one period per mesh with the amplitude of about $18 \mu\text{m}$.

Fig.4 Transmission error of Pair I measured by a single flank meshing tester

Fig.5 Transmission error of Pair II

(*) The reader should refer to References (1) and (5) for details about the method of the selection of m , n and the consistent estimate, and so on.

The consistent estimate of the random component contained there is shown in Fig.7 by circular marks. The solid line in the figure shows an approximate expression of measured consistent estimate, which is represented with an autoregressive model in the following form

$$\frac{d^2 W}{ds^2} + 0.06 \frac{dW}{ds} + 0.074 W = u(s) \quad (3)$$

$$\langle u(s_1) u(s_2) \rangle = D' \delta(s_1 - s_2)$$

$$\langle W(s_1) W(s_2) \rangle = R''(s_1 - s_2)$$

where $D' \approx 0.24$.

4. Tooth strain measurements

Static and dynamic tooth strains in Pair I were measured under normal tooth load of 65.7 kg/cm, and those of Pair II were measured under 65.7 kg/cm and 42 kg/cm. Results are shown in Figs.8 and 9. Only 5 samples of about 20 measurements under same condition are shown in a photograph of Fig.9, in which a segment of a line at the bottom shows a contact length.

The results on Pair I, shown in Fig.9 (1), indicate that 5 samples are almost same in each photograph at any speed, that is, the tooth strain shows recurrent time variation per one revolution over wide range of gear speeds. Fig.9(2) shows some typical examples on Pair II under normal tooth load of 42 kg/cm. In this case, the recurrent time variation of tooth strain begins to miss in the neighbourhood of 3100 rpm and misses completely above 4700 rpm. At higher speeds an entire loss of mesh is observed showing notable randomness of its time variation. These results differ from the results on Pair I distinctly.

Fig.6 A consistent estimate of Pair I

Fig.7 A consistent estimate of Pair II (circular mark) and its approximate expression (solid line)

Another example on Pair II under normal tooth load of 65.7 kg/cm is shown in Fig.9 (3) where the recurrent time variation begins to miss in the neighbourhood of 2500 rpm, and both the shape and the magnitude of tooth strain become very random with an increasing speed in the same manner as in the preceding example.

Maximum values of dynamic loads during one mesh were evaluated based on the dynamic tooth strain measurements and the effect of non-dimensional gear speed on them is shown in Figs.10, 11 and 12. In Figs.11 and 12 the mean value of the maximum dynamic load is signified by a circular mark, "o", and the vertical slender lines on either side of the mark indicate one standard deviation.

5. Comparison of experiment with analogue computer simulation

Analogue computer simulation under experimental conditions was performed regarding the gear system as a vibrating system with one degree of freedom, in which the shaft stiffness was neglected(2). The differential equation and the method of selection of parameters and so on are the same as in the previous paper(2) except some values of constants, and only an outline is described here. Contact length was obtained by a statical tooth strain measurement and the configuration of the time variation of mesh stiffness was assumed according to H.Opitz(4). A damping coefficient was estimated to be 0.15 from the system's damped vibration.

The static transmission error which is regarded as an exciter of the vibrating system was given based on the analysis of measured error in Chapt.3. In case of Pair I, however, the transmission error was considerably small as shown in Fig.4 or in its estimation Fig.6 and eq.(1); and it is doubtful whether such a small error still exists under loaded condition or not. Then the simulation on Pair I was performed without any error. On the other hand the simulation on Pair II was performed assuming a harmonic component of error which has one period per mesh with amplitude of 18 μ m and a random one expressed by eq.(3) to exist.

Fig.8 Static tooth strain of Pair I, normal tooth load: 65.7 kg/cm

The results of the simulations are shown in Fig.10 by a solid line and in Figs.11 and 12 by mark "x". Some examples of sample functions are shown in Figs.13 and 14. In Figs.11 and 12, the mark "x" signifies the mean value of the maximum dy-

namical load which is a sample average of 40 statistically independent samples and the vertical slender lines on either side of the mark indicate one standard deviation. The chain dotted line shows the maximum value among those maximum dynamic loads ob-

Fig.9 Measured dynamic tooth strains

Fig.10 Experimental results (circular mark) and computed results (solid line) of the maximum dynamic load on Pair I, normal tooth load: 65.7 kg/cm.

Fig.11 Experimental and computed results on Pair II, normal tooth load: 42 kg/cm, ϕ : mean value and variance of experimental results, $\times$: mean and variance of computed results, dotted chain line: computed results in a case of the random component being neglected, dotted line: curve joining the experimental values roughly.

Fig. 12 Experimental and computed results on Pair II, normal tooth load: 65.7 kg/cm, ϕ : mean value and variance of experimental results, $\times$: mean and variance of computed results, dotted chain line: computed results in a case of the random component being neglected, dotted line: curve joining the experimental values roughly.

tained when the random component of error is not considered, since it was observed that at some gear speeds the magnitude of dynamic load was different with every tooth or an entire loss of mesh occurred in this case.

It seems that in case of Pair I the experimental results agree with the simulation very well as shown in Fig. 10. In case of Pair II the results of the simulation almost agree with the result in the previous reports⁽²⁾⁽³⁾ which is shown by a chain dotted line in Figs. 11 and 12. That is, it seems that the mean of the maximum dynamic load remains at similar level to the one in the deterministic process where the random component is not considered at any speed and the variance increases with an increase in speed. The experimental results also show a similar tendency to the simulation and this fact indicates that a proposed mathematical model of gear sys-

Fig. 13 Examples of sample functions in the simulation on Pair II, normal tooth load: 42 kg/cm, (1)(2)(3): random component being considered, (4): random component being neglected, (5): spring constant

Fig. 14 Examples of sample functions in the simulation on Pair II, normal tooth load: 65.7 kg/cm, (1)(2)(3): random component being considered, (4): random component being neglected, (5): spring constant

tem's dynamics and transmission error are close to the actual case approximately.

6. Conclusion

An experimental investigation on the dynamic load of spur gear was carried out in order to examine the propriety of the statistical theory developed in the previous paper using two pairs of test gears. An analogue computer simulation under experimental condition was performed, too.

The main results obtained are summarized as follows:

(1) Experimental results on a pair of Maag ground gears with high accuracy under normal tooth load of 65.7 kg/cm agreed well with the analogue simulation and dynamic load shows recurrent time variation per revolution over a wide range of gear speeds.

(2) Another pair of test gears which is composed of a hobbled gear and a Maag ground gear having a larger transmission error was examined under normal load of both 42 kg/cm and 65.7 kg/cm. The results show that the recurrent time variation begins to miss with an increasing gear speed and at higher speeds an entire loss of mesh was also observed showing notable randomness of the time variation of dynamic load.

These results show good agreement with

analogue simulations qualitatively; and it seems that the propriety of the statistical theory on the estimation of dynamic load is proved.

Acknowledgement

The authors wish to express their most sincere thanks to Professor T. Tsuchikawa, Professor S. Takanashi, Assistant Professor M. Kato and the members of the Machine Shop and Laboratory of Machine Elements of Department of Precision Engineering of Tohoku University for their valuable discussions and supports.

References

- (1) Tobe, T. and Sato, K., Preprint of Japan Soc. Mech. Engrs. (in Japanese), Hitachi (1971-11), p.147.
- (2) Tobe, T. and Sato, K., Preprint of Japan Soc. Mech. Engrs. (in Japanese), No. 730-7 (1973-5), p.171.
- (3) Tobe, T., et al., Preprint of Japan Soc. Mech. Engrs. (in Japanese), Yamanashi (1973-11), p.186.
- (4) Opitz, H., JSME 1967 Semi-Int. Symp., Gearing, p.199.
- (5) Priestley, M. B., J. Roy. Statist. Soc., B, 26, (1964), p.123.