532.522:532.57

Paper No. 151-17

Effects of Swirling Flow Component on Discharge Coefficients of Constriction Meters (in Case of Orifice)*

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Discharge coefficients of pipe orifices in swirling flow were measured and the results were discussed theoretically.

Differential pressure across the orifices is affected by the swirling component and its amount is found to be a function of two parameters; the rate of constriction of orifice opening and the intensity of swirling component. In a small orifice opening the differential pressure is decreased by the swirling component, and in a larger orifice opening it is increased.

Relative change of the differential pressure was calculated theoretically by using a potential flow model.

Distributions of velocities and pressures were measured and changes of energy and momentum of the flow through the orifices were discussed.

1. Introduction

For accurate measurements of flow quantities by constriction meters, both the inlet and outlet sections should be sufficiently long to ensure steady flow without disturbances due to asymmetrical layout or the insertion of obstructions, such as elbows and valves. The requisite lengths are indicated in the code of JIS.⁽¹⁾ The constriction meters in practical use, however, are often installed in pipe lines with insufficient straight lengths owing to limited spaces. A pipe line bent in a three-dimensional form by two or more elbows produces a strong swirling flow, which extends over a considerably longer distance than is expected hitherto. The swirling flow component will alter the readings of the measuring pressure heads in the constriction meters.

Lugt⁽²⁾ studied experimentally the effects of the swirling component on discharge coefficients of constriction meters within a wide range of swirl intensities. But the flow patterns before and after the meters were not examined and the swirl intensity used was that expressed by guide vane angles in his swirl generator and the results will not be applicable directly to the other cases.

In the present study, orifices were used as constriction meters and a change of their discharge coefficients together with that of flow patterns before and after the orifices due to swirl flow components was measured. The results can be expressed in a general form with the use of swirl flow

intensities defined by the ratio of angular momentum flux to axial one. The change of the discharge coefficients can be calculated theoretically on an assumed simple flow model.

2. Nomenclature

- c : intensity of free vortex motion
- D : diameter of pipe
- d_0 : diameter of orifice opening
- E : energy flux, Eq. (5)
- f_α : rate of change of discharge coefficient due to swirl component, Eq. (15)
- M : axial momentum flux, Eq. (4)
- P : pressure
- Q : rate of discharge
- R : radius of pipe
- r : radial distance
- r_0 : radius of orifice opening
- Re : Reynolds number, $= v_m D / \nu$
- v_m : mean velocity, $= Q / (\pi/4) D^2$
- v_z : axial velocity
- v_θ : swirl velocity
- x : dimensionless expression of forced vortex core, $= \delta / R$
- α : discharge coefficient
- β : diametral ratio, $= d_0 / D$
- δ : radius of forced vortex core
- ϕ : rate of swirl, Eq. (6)
- ν : kinematic viscosity
- ρ : density
- ω : angular velocity of forced vortex core
- Ω : swirl intensity, (dimensionless expression of Ω^* , Eq. (3)')
- Ω^* : angular momentum flux, Eq. (3)

3. Analytical considerations

3.1. Discharge coefficients

A swirling flow approaching an orifice is assumed to be fully developed as is seen in a section sufficiently downstream from

* Received 10th December, 1975.

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the pipe entrance at which the swirl motion begins. In this case the profile of velocity curves is governed by only a single parameter, the swirl intensity in that section.⁽³⁻⁵⁾ If the swirl motion is strong and $\Omega > 0.2$, the swirling velocity component shows a combined vortex type; the central part is a forced vortex type with smaller axial velocities and the outer part is a free vortex one having larger axial velocities. If the swirling motion is weak, a forced vortex type motion with uniform axial velocities dominates the whole pipe section.

Hereafter, the change of discharge coefficients due to the swirl component is calculated on the assumption of the former type of velocity distribution. The results, however, cover well those calculated on the flow in the latter case. Velocity distributions assumed are as follows

$$\left. \begin{aligned} v_\theta &= r\omega, \quad v_z = 0 & 0 < r < \delta, \quad (0 < r < \delta') \\ v_\theta &= c/r, \quad v_z = v_z & \delta < r < R, \quad (\delta' < r < r_0) \end{aligned} \right\} \quad (1)$$

in which the ranges given in the brackets are available for the throat region of the passage. From Eq. (1) the rate of discharge can be written as

$$Q = \pi(R^2 - \delta^2)v_z \quad (2)$$

Flow of angular momentum flux through a cross section is given by

$$\Omega^* = 2\pi\rho \int_0^R v_\theta v_z r^2 dr = 2\pi\rho v_z \int_\delta^R \frac{c}{r} r^2 dr = \rho Q c \quad (3)$$

Divided by the mean axial momentum flux $\rho Q v_m$, Ω^* can be expressed dimensionlessly as

Fig. 1 Assumed velocity distributions

Fig. 2 Model of flow passing through the orifice

$$\Omega = 2\pi\rho \int_0^R v_\theta v_z r^2 dr / (\rho Q v_m R) = \Omega^* / (\rho Q v_m R) \quad (3)'$$

This value can be used as expression of swirl intensity of flow. Flow of axial momentum flux and energy flux through a section can be calculated by

$$M = 2\pi\rho \int_0^R v_z^2 r dr \quad (4)$$

$$E = 2\pi\rho \int_0^R \left(\frac{v_\theta^2 + v_z^2}{2} + \frac{P}{\rho} \right) v_z r dr \quad (5)$$

The radius of a forced vortex core x expressed dimensionlessly can be assumed by the following equation which was obtained by Shogenji and Shimoyama⁶ on the principle of minimum kinetic energy of flow

$$g^2 = 2x^2 / (1 - x^2) (1 - x^2 + 2x^2 \ln x) \quad (6)$$

where

$$g = V_\theta / V_m \quad (6)'$$

(R' is radius of the flow passage under consideration)

From Eqs. (3) and (6)

$$g = V_\theta / V_m = \Omega^* \pi R'^2 / (R' Q^2 \rho) = \Omega^* / (\rho Q V_m R') \quad (7)$$

From this result it is seen that the swirl rate g is given by the ratio of two momentum fluxes, i.e., angular momentum flux/axial momentum flux.

A model of flow across an orifice is shown in Fig. 2. In the flow upstream and inside of the orifice, all losses are assumed to be negligibly small and a potential flow model except the core region is applied, since the flow in that region is approximately in an accelerated state. Downstream of the orifice, separated flows accompanied with eddy motions dominate the channel section and a considerable energy loss must be assumed in the calculation. In this region symmetry of the flow becomes exceedingly poor as is shown in Fig. 2, details of which will be discussed later. The swirl rate or swirl angle will be changed when the flow enters the orifice opening. If the diameter of flow section is reduced from D to d_0 ($=\beta D$), the axial momentum of flow is increased by $1/\beta^2$, but the angular momentum remains unchanged. Thus, the value of the swirl rate in the orifice opening becomes

$$\begin{aligned} g' &= \Omega^* / [(\rho Q v_m / \beta^2) R \beta] \\ &= \beta (\Omega^* / \rho Q v_m R) = \beta g \quad (8) \end{aligned}$$

In this case the vortex core ratio decreases from x to x' ($=\delta'/r_0$).

By use of Bernoulli's equation the relationship of pressure between the sections ① and ② is derived as

$$P_2 = P_1 + \frac{\rho}{2} (v_1^2 - v_2^2) \quad (9)$$

where

$$v_1^2 = v_{z1}^2 + v_{\theta1}^2, \quad v_2^2 = v_{z2}^2 + v_{\theta2}^2$$

The continuity relation gives the following relationship

$$v_{z1}^2 - v_{z2}^2 = v_{z1}^2 \left\{ 1 - \left(\frac{R^2 - \delta^2}{r_0^2 - \delta'^2} \right)^2 \right\}$$

Put the pressures at the boundary of the free and forced vortex regions in the sections ① and ② as P_{C1} and P_{C2} respectively, then the following relationship is obtained from Eqs. (1) and (9)

$$P_{C2} = P_{C1} + \frac{\rho}{2} \left[v_{z1}^2 \left\{ 1 - \left(\frac{R^2 - \delta^2}{r_0^2 - \delta'^2} \right)^2 \right\} + c^2 \left(\frac{1}{\delta^2} - \frac{1}{\delta'^2} \right) \right] \quad (10)$$

Measurable pressures on the channel wall, P_{W1} , will be elevated over the pressure on the circumference of the forced vortex core, P_C , on account of the centrifugal force due to the swirling motion, the relationships of which are given by

$$P_{C1} = P_{W1} - \rho \int_{\delta}^R \frac{v_{\theta}^2}{r} dr, \quad P_{C2} = P_{W2} - \rho \int_{\delta'}^{r_0} \frac{v_{\theta}^2}{r} dr$$

With these results, Eq. (10) gives the following expressions for P_{W1} and P_{W2} .

$$\left. \begin{aligned} P_{W2} &= P_{C1} + \frac{\rho}{2} \left[v_{z1}^2 \left\{ 1 - \left(\frac{R^2 - \delta^2}{r_0^2 - \delta'^2} \right)^2 \right\} + c^2 \left(\frac{1}{\delta^2} - \frac{1}{\delta'^2} \right) \right] + \frac{\rho c^2}{2} \left(\frac{1}{\delta'^2} - \frac{1}{R^2} \right) \\ P_{W1} &= P_{C1} + \frac{\rho c^2}{2} \left(\frac{1}{\delta^2} - \frac{1}{R^2} \right) \end{aligned} \right\} \quad (11)$$

Due to sudden enlargement of flow passage at the orifice outlet a considerable amount of swirling flow momentum will be lost and the swirling motion is much weakened there. Consequently for an approximate calculation, the relative change of static pressures toward the duct wall against the static pressure P_{W2} can be neglected and

$$P_{W2} \doteq P_{W1}$$

When the differential pressure of an orifice or nozzle is measured by corner taps, the pressure difference is to be $\Delta P = P_{W1} - P_{W2}$ which will be expressed by use of Eq. (11) as

$$\begin{aligned} (P_{W1} - P_{W2}) / \left(\frac{\rho v_{m2}^2}{2} \right) &= \frac{1}{1-x} \left\{ \frac{1}{\beta^4} \left(\frac{1-x^2}{1-x'^2} \right) - 1 \right\} \\ &+ \Omega^2 \left(\frac{1}{\beta^2} - 1 \right) \quad (12) \end{aligned}$$

If the swirling component is absent, $\Omega=0$ and $x=x'=0$, Eq. (12) reduces to

$$(P_{W1} - P_{W2}) / \left(\frac{\rho v_{m2}^2}{2} \right) = \frac{1}{\beta^4} - 1 \quad (12)'$$

With these pressure differences the rate of discharge Q through the constriction meter can be calculated by

$$Q = \alpha \frac{\pi}{4} d^2 \sqrt{2g(P_{W1} - P_{W2})/\gamma} \quad (13)$$

where α is a discharge coefficient and is given by for Eq. (12)

$$\alpha = \sqrt{1 / \left[\frac{1}{1-x^2} \left\{ \frac{1}{\beta^4} \left(\frac{1-x^2}{1-x'^2} \right)^2 - 1 \right\} + \Omega^2 \left(\frac{1}{\beta^2} - 1 \right) \right]} \quad (14)$$

and for Eq. (12)'

$$\alpha = 1 / \sqrt{\frac{1}{\beta^4} - 1} \quad (14)'$$

Neglecting the change of contraction rate of jets at the vena contracta in swirling flow, the relative change of the discharge coefficients due to a swirling component is given by

$$\begin{aligned} f_a &= \frac{\alpha - \alpha_n}{\alpha_n} \\ &= \left\{ \frac{\sqrt{\frac{1}{\beta^4} - 1}}{\frac{1}{1-x^2} \left\{ \frac{1}{\beta^4} \left(\frac{1-x^2}{1-x'^2} \right)^2 - 1 \right\} + \Omega^2 \left(\frac{1}{\beta^2} - 1 \right)} - 1 \right\} \quad (15) \end{aligned}$$

The calculated values of Eq. (15) are plotted against Ω in Fig. 3, where β is taken as a parameter. Tendency of the curves is seen to be changed much by the parameter β . When $\beta < 0.5$, f_a increases firstly with Ω to attain maximum and decreases beyond it but when $\beta > 0.6$, f_a decreases monotonously with an increase of Ω .

3.2. Asymmetry of wall pressures

If swirling motion is not too weak, a fairly good axisymmetry of swirling flow is observed before an orifice plate but the symmetry is greatly lost by passing out of the orifice opening, showing a remarkable uneven distribution of wall pressures along the circumference of channel sections. On the basis of a visual experiment, the flow pattern after the orifice can be expressed by a model in Fig. 2. Provided that the size of the forced vortex core in the orifice is maintained unaltered up to the downstream region of the orifice, the difference between maximum and minimum wall pressures measured along the circumference of a section can be calculated by

$$\begin{aligned} \Delta P_{\max} &= \rho \int_{\delta'}^{D-\delta'} \frac{v_{\theta}^2}{r} dr \\ &= \frac{\rho c^2}{2} \left(\frac{1}{\delta'^2} - \frac{1}{(D-\delta')^2} \right) \end{aligned}$$

If $D \gg \delta'$, a dimensionless expression of this becomes

$$\Delta P_{\max} / \left(\frac{\rho v_{m2}^2}{2} \right) \doteq \Omega^2 / \beta^2 x'^2 \quad (16)$$

Fig. 3 Rate of change of discharge coefficient due to swirling motion, calculated results from Eq. (15)

4. Experimental apparatus

A general arrangement of the experimental equipment is shown in Fig. 4. Water in a reservoir was pumped up to a swirl generating tank, in which the swirl component of prescribed intensity was given by a rotating impeller. The water flowing out of the swirl generating tank was introduced into a straight brass tube having inside diameter 104.25mm, length 11470mm (110D) and hydraulically smooth surface. Test orifices were set up 40D downstream from the pipe inlet and followed by the same brass pipe of 70D length. This inlet length ensures a fully developed swirling flow at the orifice entrance, where the velocity distributions are only governed by Ω and are independent of the swirl generating devices, since the effects of the swirl generating devices will die out in a length of $15D^{(7)}$. Flow rate was obtained by a weighing method. The pressure differential across the orifices was measured with two methods; use of the pressure difference between two corners on each side of the orifice plate, being called corner taps, and the pressure difference between two taps one diameter upstream and half diameter downstream on the pipe wall, being called radius taps. The diametral ratios of orifices were 0.4, 0.6, and 0.8, respectively. Velocity and pressure distributions within a duct were measured across the sections $S_1, S_2, \dots, S_7$ by means of a cylindrical Pitot tube (3.98mm in diameter). As a wall pressure in a section a mean value of pressures over the circumference was used except for the purpose of discussing the uneven pressure distribution over there.

Fig. 4 Experimental apparatus

Fig. 5 Relationship between discharge coefficient and swirl intensity

5. Experimental results and discussions

5.1. Change of discharge coefficients due to swirling component

With the measured pressures P_{w1} and P_{w2} , the values of discharge coefficients α calculated by Eq.(13) are plotted against the swirl intensity Ω in Fig. 5, in which values of different Reynolds numbers are also plotted but the effects of Reynolds number within the range of $4 \times 10^4 \sim 10 \times 10^5$ are seen to be negligible. When β is small, $\beta=0.4$ and 0.6 , the values of α by corner taps and radius taps do not show any appreciable difference, but when β is increased to 0.8 , the difference becomes remarkable; for example when Ω is 0.5 , α by corner taps is larger than that by radius taps by 9% . The cause of this difference can be explained by examining the wall pressures along the

Fig. 6 Rate of change of discharge coefficient due to swirling motion, experimental results

ducts as is seen in the next section.

Using the results in Fig. 5, the rate of change of the discharge coefficient f_α can be obtained as shown in Fig. 6 in which the calculated values by Eq. (15) are also plotted in broken lines. The calculated curves show a similar tendency to the experiments. In the same figure Lugt's results are also shown against the same swirl intensities, which are assumed from the dimensions of his experimental apparatus. The results show the same tendency.

When β is less than 0.6, f_α increases firstly to attain a maximum and decreases with an increase of Ω . When β is greater than 0.8, however, f_α decreases monotonously as Ω increases.

5.2. Pressure distributions on walls

Distributions of wall pressures $(P-P_1)/(1/2\rho v_m^2)$ before and after the orifices are shown in Fig. 7, in which the pressure at the entrance of the orifices P_1 is taken as a base. Due to the action of centrifugal forces, the swirling component in flow forcibly reduces the separated region behind the orifice plates and brings about an increase of pressure recovery there. This becomes increasingly important for orifices

Fig. 7 Pressure distributions up- and downstream regions of orifice (pressure at the entrance of orifice P_1 is taken as a base)

with smaller diametral ratios β , since the percentage of the separated region behind the orifices increases with a decrease of β .

When $\beta=0.8$, the pressures at sections half the pipe diameter downstream, $L/D=0.5$, and one diameter upstream, $L/D=-1$, are seen to be almost independent of the swirl intensities, and the discharge coefficients for $\beta=0.8$ based on radius taps remain unchanged irrespective of Ω as is seen in Figs. 5 and 6. Pressure curves downstream of the orifice plates exhibit a minimum for each value of Ω which corresponds to the vena contracta of jets from the orifices. Distances of the minimum pressure points measured from orifice plates are plotted in Fig. 8. The points approach the orifice plates as Ω is increased and β is decreased. When $\beta=0.8$ the position is nearly constant at $L/D \approx 0.3$.

5.3. Uneven wall pressure

In swirling flow the eccentricity of swirling center there causes a considerable uneven pressure distribution on the duct circumference.⁽⁸⁾ The eccentricity reaches a maximum at sections $L/D \sim 3D$ downstream from orifice plates and yields a great uneven wall pressure distribution there. Some of the experimental results for $\Omega=0.482$ and $\beta=0.6$ are shown in Fig. 9. The magnitude and direction of the maximum wall pressure $P_{\max}$ for each section change from section to section. To examine the unevenness of the pressure on the duct periphery, the differences between the maximum and minimum pressures on each section, $\Delta P_{\max} = P_{\max} - P_{\min}$, are considered, the results of which are plotted dimensionlessly in Fig. 10.

Though the plots show some scattering, the maximum pressure difference $\Delta P_{\max}/(\Omega^2/\beta^2 x'^2)$ attains a maximum at about $L/D = 1.5$, irrespective of diametral ratio β or swirl intensity Ω . It follows from these results that $\Delta P_{\max}$ becomes larger as Ω is increased or β is decreased. From the results in Fig. 9, the eccentricity of the swirl center is expected to change along the duct. This is confirmed by the photographs in Fig. 11, in which the locus of the swirl center is visualized by injected air. Deviation of the swirl center from the pipe axis becomes maximum at about L/D

Fig. 8 Positions of minimum wall pressure out of orifice (vena contracta)

~3D downstream from the orifice and in this case the forced vortex core of flow must be in contact with the pipe wall as seen in Fig. 2. Within a short distance after that section, an occurrence of a vortex breakdown, conjectured from dispersion of air bubbles, can be seen.

5.4. Velocity distributions

As an example the distributions of axial and circumferential velocity components before and after an orifice are shown in Figs. 12 and 13. It may be noticed that the effects of passage contraction of the orifice on flow are limited almost to the downstream region and at sections s_1 and s_2 in the upstream region the effects can not be seen. Due to a contraction of passage, concaved profiles of the axial velocity component in the center become less remarkable in the downstream region of the orifice (compare velocity profiles at sections s_1 and s_2 with those at sections s_3 and s_4).

The broken line in Fig. 12 exhibits an axial velocity distribution at section s_4 in a non-swirling flow. Comparison of these results reveals that the swirling flow component makes the separated zone considerably small. Unsymmetric flow conditions along the duct can be detected from the velocity profiles. As the flow goes downstream from the orifice, the distortion of velocity profiles increases firstly, reaches a maximum and then reduces gradually to a negligible extent, showing a symmetrical flow pattern.

Velocity profiles of the swirling component also receive a sudden change when the flow passes through the orifices. A symmetric flow with a free-forced vortex type profile in the upstream region of the orifices retains its form till the section s_4 ($L/D=1.61$) situating in the downstream

Fig. 9 Eccentricity of wall pressure distribution ($\Omega=0.482, \beta=0.6$), (length of each arrow shows the pressure difference between mean wall pressure on the periphery and local wall pressure)

region but the symmetry is considerably lost. At section s_5 this flow pattern is changed abruptly to a forced vortex type, with which the swirling motion decays till the last. In this process the symmetry of flow is recovered gradually. The abrupt change of flow mentioned above will correspond to a phenomenon so called the vortex breakdown.

On the assumption of an axisymmetric flow, pressure distributions on one section can be calculated by

$$P = P_w - \rho \int_r^R (v_\theta^2/r) dr \quad (17)$$

where P_w is a pressure on the wall. Within the region of the sections $s_4 \sim s_6$, the pressures rise higher as the flow goes downstream. A remarkable pressure rise between the sections s_4 and s_5 will be due to the

Fig. 10 Variation of unevenness of wall pressure along pipe axis, Eq. 16

$$\beta=0.8, \Omega=0.40, Re=1.0 \times 10^5$$

$$\beta=0.6, \Omega=0.48, Re=1.0 \times 10^5$$

$$\beta=0.4, \Omega=0.51, Re=1.0 \times 10^5$$

Fig. 11 Locus of swirl center

vortex breakdown.

5.5. Momentum and energy fluxes

With the experimental results, the flow of axial momentum can be calculated by Eq. (4) as shown in Fig. 15. The values expressed dimensionlessly reduce rapidly to unity near the orifices and the rate of reduction is seen to be increased by swirling components. Angular momentum fluxes across a duct section can be calculated by Eq. (3), and their variations along the duct axis are shown in Fig. 16, where Ω_0^* is the swirling intensity at the inlet section. The values of Ω^*/Ω_0^* decay substantially within the hatched region, the rate of the decay being dependent only on β but not on Ω_0^* . From the results in Fig. 16 the following relationship can be obtained as shown in Fig. 17.

$$\Omega^*/\Omega_0^* \approx \beta$$

Namely, the angular momentum flux of flow is reduced to a value equal to β times the initial angular momentum when the flow passes through an orifice of diametral ratio β . The energy flux defined by Eq. (5) also reduces in that process. The amount of reduction, namely, the flow energy loss can be estimated by taking the difference of energy flows between the inlet and outlet

Fig. 12 Axial velocity distributions before and after the orifice plate, $Re=1 \times 10^5$, $\beta=0.6$, $\Omega=0.8$, (s_1, s_2 : upstream side sections, $s_3, \dots, s_7$: downstream side sections)

Fig. 13 Circumferential velocity distributions before and after the orifice plate

Fig. 14 Pressure distributions within each section

Fig. 15 Variation of axial momentum flux of flow downstream of orifice

Fig. 16 Variation of angular momentum flux along pipe axis by passing through the orifice (substantial change occurs within the hatched region)

Fig. 17 Rate of decay of angular momentum flux by passing through the orifice

sections of the orifice, $\Delta E = E_0 - E_1$. In this case, as the value at the outlet section E_1 , the value at the section s_7 is taken. The results are plotted against the swirl intensity Ω in Fig. 18, where ΔE_0 is the energy difference in a non-swirling flow. The relative value of the energy loss $\Delta E_\Omega / \Delta E_0$ increases with an increase of Ω , when the diametral ratios β are 0.6 and 0.8. When $\beta=0.4$, however, $\Delta E_\Omega / \Delta E_0$ decreases as Ω increases. These phenomena can be explained as follows; the swirling flow component has two opposite effects on the mechanism of loss. The swirling flow component on one hand makes the separated region behind the orifice small and brings about an increase of pressure recovery there but on the other hand the component itself elevates the kinetic energy of flow, which brings about a greater loss in course of flow. Combination of these two effects makes different pictures in energy loss.

Fig. 18 Loss of energy of flow by passing through the orifice (relative change of ΔE_Ω , with swirl component, to ΔE_0 , without swirl component)

6. Conclusions

1. The discharge coefficient of an orifice is increased or decreased by a swirling flow component, the magnitude of which is governed by two factors Ω and β (Fig. 6).
2. When a fluid with a swirling velocity component goes through an orifice, velocity and energy of the flow receive a remarkable change. Magnitude and distribution profile of the swirling velocity are changed abruptly at about section $1.5D$ downstream from the orifice where a vortex breakdown occurs.

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