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Laminar Heat Transfer Downstream of a Sudden Enlargement of a Duct*

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The effect of buoyancy force on laminar heat transfer rate downstream of a sudden enlargement of a two-dimensional duct is numerically discussed for a variety of velocity profiles at entrance to the enlarged part.

It is clarified that variation of physical properties or of Grashof number with change of wall heat flux has a considerable influence on convective heat transfer as well as flow pattern, and that the Nusselt number in a separated region is closely connected with the velocity profiles at entrance to the enlarged part.

1. Introduction

Convective heat and mass transfer concerned with a flow separation occurring in many industrial equipments has so far been studied by many investigators. For example mass transfer from the wall associated with two-dimensional reattaching jet flow is reported by Kumada et al.^{(1)~(3)}, and heat transfer in the wake of a bluff body is studied by Igarashi et al.^{(4)~(5)}. Meanwhile, as for the inner flow configuration the experimental investigations are reported by Krall & Sparrow⁽⁶⁾ on an orifice in circular pipe, and by Filetti & Kays⁽⁷⁾ or Seki et al.⁽⁸⁾ on a two-dimensional enlargement of flat duct. However most of these previous works are the results carried out in turbulent flow field, therefore, it can be said that the heat transfer mechanism with separation has not completely been clarified due to a complexity of turbulent heat transport mechanism.

Roache & Mueller⁽⁹⁾ studied a laminar separated flow for a backstep configuration by means of numerical analysis. They mainly discussed compressible or incompressible flow field but insufficiently heat transfer in these fields. Recently, Seki et al.⁽¹⁰⁾ experimented the heat transfer rate downstream of a two-dimensional enlargement of flat duct by using transformer oil and suggested an availability of the method of numerical analysis proposed by Gosman et al.⁽¹¹⁾.

In the present study, the effect of buoyancy force on laminar heat transfer rate downstream of a two-dimensional enlargement of flat duct is examined by numerical analysis, in which fluid properties except for a specific heat are assumed to be dependent on temperature, for a variety of velocity profiles at entrance to the enlarged part.

Nomenclature

C_p : specific heat
 g : gravitational acceleration
 H : step height
 Gr : Grashof number, $g\beta_0 L^4 q_w / \nu_0^2 \lambda_0$
 L : entrance height
 Nu_{max} : maximum Nusselt number, $\alpha_{max} L / \lambda_0$
 Nu_{mean} : mean Nusselt number in a separated region, $\frac{1}{x_R} \int_0^{x_R} Nu_{local} dx$
 Pr : Prandtl number, $C_p \mu_0 / \lambda_0$
 q_w : heat flux from wall
 Re : Reynolds number, $u_0 L / \nu_0$
 t : temperature
 T : non-dimensional temperature, $\lambda_0 (t - t_0) / q_w L$
 u_0 : mean value of entrance velocity
 U, V : non-dimensional velocities in X - and Y -directions, respectively, u/u_0 , v/u_0
 x_R : stall length
 X : non-dimensional distance downstream from the step, x/L
 Y : non-dimensional distance normal to duct wall, y/L

Greek symbols

α : heat transfer coefficient, $q_w / (t_w - t_0)$
 β^* : non-dimensional thermal expansion, β / β_0
 δ : boundary layer thickness of entrance velocity profile
 λ^* : non-dimensional thermal conductivity, λ / λ_0
 μ^* : non-dimensional viscosity, μ / μ_0
 ν^* : non-dimensional kinetic viscosity, ν / ν_0
 ρ^* : non-dimensional density, ρ / ρ_0
 Ψ : non-dimensional stream-function, $\Psi / \rho_0 \mu_0 L$
 Ω : non-dimensional vorticity, $\omega L / u_0$

Subscripts

w : value at the wall
 o : value of entrance fluid

2. Analysis

Figure 1 shows schematically the configuration of a two-dimensional enlargement

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of flat duct performed in the present computations. A general expression of the governing equations in terms of the dimensionless quantities can be devised by introducing Ψ and vorticity Ω ;

$$\alpha \left\{ \frac{\partial}{\partial X} \left(\phi \frac{\partial \Psi}{\partial Y} \right) - \frac{\partial}{\partial Y} \left(\phi \frac{\partial \Psi}{\partial X} \right) \right\} - \frac{\partial}{\partial X} \left\{ b \frac{\partial}{\partial X} (c\phi) \right\} - \frac{\partial}{\partial Y} \left\{ b \frac{\partial}{\partial Y} (c\phi) \right\} + d = 0 \quad (1)$$

where ϕ is a general dependent variable representing either Ψ , Ω or T . The terms of a , b , c and d are listed in the following table, where Res^* denotes the terms associated with higher order derivatives of μ^* , U and V with respect to X or Y . In the present analysis the mesh size is employed as 23 and 17 in the X - and Y -directions, respectively. Then, the mesh distribution in both directions is set to be non-uniform so as to particularly be narrow in the vicinity of the wall and the step edge. The boundary conditions are assumed as the heat flux on the duct wall being uniform and the step wall adiabatic.

Fig.1 Schematic configuration of flow field

ϕ	a	b	c	d
Ω	1	$\frac{1}{Re}$	μ^*	$\frac{Gr}{Re^2} \frac{\partial}{\partial X} (\rho^* \beta^* T) + \frac{\partial}{\partial Y} \left(\frac{U^2 + V^2}{2} \right) \frac{\partial \rho^*}{\partial X} - \frac{\partial}{\partial X} \left(\frac{U^2 + V^2}{2} \right) \frac{\partial \rho^*}{\partial Y} + Res^*$
Ψ	0	$\frac{1}{\rho^*}$	1	$-\Omega$
T	1	$\frac{\lambda^*}{RePr}$	1	0

Table. The functions a , b , c and d .

The velocity profiles at entrance to an enlarged part are given by Blasius' solution⁽¹²⁾. As for the viscosity-temperature relationship of fluid, Sutherland's formula⁽¹³⁾ is employed for air, and the following form is approximated for water within the range of $1^\circ\text{C} \leq t \leq 90^\circ\text{C}$;

$$\mu = [(5673 - 0.04687 t^{1.85}) / (t + 28.76) - 14.45] \times 10^{-6} \quad [\text{kgs/m}^2] \quad (2)$$

The iterative procedure used to solve Eq.(1) is repeated until the following condition is satisfied at every grid node.

$$|\phi^{(n)} - \phi^{(n-1)}| / |\phi^{(n)}| \leq 5 \times 10^{-4} \quad (3)$$

where the superscripts (n) and $(n-1)$ indicate the values at the (n) th and $(n-1)$ th iterations, respectively. Though the numerical solutions are obtained for several values of H/L , Re , δ/L and Gr , the results for $H/L = 0.1$ are representatively discussed in the following chapter.

4. Results and Discussions

The predicted flow patterns for water whose Prandtl number is 7.07 are shown in Fig.2, where the solid and the dashed lines indicate the contours of a stream-function Ψ and temperature T , respectively. The downstream part beyond a distance of 9 step-heights is skipped out from the present diagrams. Figure 2(a) shows the result for $Gr \approx 0$, corresponding to small heat flux from the wall, when a variation of fluid properties and a buoyancy force should be negligible. From this result it is recognized that the contours of temperature in the reattached region are likely to be pressed to the vicinity of the wall, therefore, the wall temperature in that region becomes low. Further, it is well observed that the recirculating flow acts as a useful medium carrying high-temperature fluid near the wall toward the dividing streamline. As a result the appearance of a steep temperature gradient near the dividing streamline means that the heat from the wall is forced to be transferred across the dividing streamline. On the other hand, Fig.2(b) shows the result for $Gr = 7 \times 10^3$. In this case, the recirculating motion becomes strong due to a decreasing of fluid viscosity with an increasing temperature. It follows that the energy transported by the recirculating flow from the wall to the dividing streamline increases; therefore, the temperature contours have more sinuous trend than those shown in Fig.2(a). Consequently, the decreasing fluid viscosity with an increasing temperature accelerates the recirculating flow and invites an increase of heat transfer rate on the wall.

Figure 3 shows the results for air. In this case, attention should be paid to a weak recirculating motion as can be seen in

Fig.2 Contours of stream-function Ψ and temperature T for water, $Re = 600$, $Pr = 7.07$, $H/L = 0.1$

Fig.3 Contours of stream-function ψ and temperature T for air, $Re = 1000$, $Pr = 0.71$, $H/L = 0.1$

Fig.3(b) for a Grashof number of 6.5×10^2 . Such a trend is contrary to that obtained for water. The reason is in the fact that the viscosity of air increases with an increasing temperature. In other words, the re-circulating motion becomes hard due to the increased viscosity. Therefore, the temperature contours in Fig.3(b) show a less sinuous trend than that in Fig.3(a). However, the heat transfer rate of air may also be influenced by the increasing thermal conductivity of air with an increasing Grashof number. These variations will become more evident by the representation as shown in Fig.4. In this figure, the ordinate means the maximum Nusselt number divided by that for $Gr \approx 0$ and the parameter of boundary layer thickness at entrance δ is divided by step height H so as to make non-dimensional expression. It is noticed that the effect of Grashof number on Nu_{max} of water is distinguishably large in case of low Reynolds number of $\delta/H = 1.0$ as compared with $\delta/H = 0.3$. However for air, the same behavior is not seen in Fig.4. The Maximum Nusselt number increases with an increasing Grashof number after a first slight decrease. This decreasing trend of Nu_{max} may be due to a weak recirculating motion associated with an increasing Grashof number as discussed in Fig.3. After this, Nu_{max} gradually increases owing to the variation of thermal conductivity of air. These behaviors of Nu_{max} introduce a correlation with δ/L in Fig.5.

Fig.4 Variation of Nu_{max} with Gr

Figure 5 shows Nu_{max} against δ/L of water for $Gr = 5 \times 10^3$ and of air for $Gr = 6.5 \times 10^2$ comparing with the values for $Gr \approx 0$, where Nu_{max} for $\delta/L = 0.5$ are obtained under the condition of parabolic profile at entrance. Nu_{max} of water has a tendency to increase with an increasing Grashof number regardless of Reynolds number or δ/L , as shown by dashed lines. On the contrary, those of air for $Gr \approx 0$ become larger than those for $Gr = 6.5 \times 10^2$ as $\delta/L = 0.03$. This trend of Nu_{max} could also be understood from the results for air shown in Fig.4. An interesting point to be noted in Fig.5 is that Nu_{max} has each peak value at $\delta/L \approx 0.03$ and shows a little uneven decrease at $\delta/L \approx 0.07$. Such a tendency might be similar to that for stall length x_R .

Figure 6 shows a variation of x_R against δ/L , in which x_R has minimal values at both $\delta/L = 0.03 \sim 0.04$ and $\delta/L = 0.07 \sim 0.08$.

Fig.5 Variation of Nu_{max} with δ/L

Fig.6 Variation of stall length with δ/L

Fig.7 Variation of Nu_{mean} with δ/L

These remarkable values of δ/L correspond to those of δ/L in Fig.5, in which a noticeable change of Nu_{max} is indicated. The behavior of x_R might result from the variations of both the decreasing rate of fluid velocity on the center-line and the shear stress in the free shear layer, which are dependent on δ/L . In Fig.6 the values of x_R for water with buoyancy force as indicated in dashed line are larger than those without buoyancy force. This behavior is opposed to that of air as explained in Fig.2. It seems that a considerable change in x_R occurs in a situation of large δ/L which corresponds to the weak recirculating motion with small shear stress in the free shear layer.

Finally, the values of mean Nusselt number in the separated region Nu_{mean} are presented against δ/L in Fig.7. In general, the results of Nu_{mean} show a similar tendency to those of Nu_{max} shown in Fig.5. However, the increasing rate of Nu_{mean} of water with an increasing Grashof number is smaller than that of Nu_{max} in contrast with the results for air. This may be probably be caused by the change of recirculating motion resulted from the variation of fluid viscosity with temperature, as afore-mentioned in Figs.2 and 3.

4. Summary and conclusions

The effects of Gr , Re and δ/L on laminar

heat transfer rate downstream of a sudden enlargement of a two-dimensional duct are numerically discussed. Fluid properties except for a specific heat are assumed to be dependent on temperature in the present study, and the results predicted for water of $Pr = 7.07$ and air of $Pr = 0.71$ are discussed.

1 The variation of fluid viscosity with temperature influences considerably the flow patterns and heat transfer rate, therefore, the dependence of fluid viscosity on temperature should be taken into consideration to analyze the heat transfer on the wall downstream of a sudden enlargement of flat duct.

2 The heat transfer rate in the separated and reattached regions is considerably influenced by the velocity profile at entrance to an enlarged part, and has a peak value at $\delta/L \approx 0.03$ in the present study.

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