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On an Application of the Rolamites to Non-uniform Motion Mechanisms*

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By means of pairing the guides and rollers of rolamites with each other, a new mechanism with multi-degree of freedom whose inputs and outputs are displacements and/or forces has been derived. The mechanism has been called a compound rolamite, and an analysis of the numbers of the elements and quantities determining the mechanism, a construction of the fundamental structure and a determination of the necessary band-generated forces for the prescribed output function have been carried out with some examples of its application given.

1. Introduction

Most of the researches on the rolamite (1)~(5) in these several years were on the basic rolamite which consists of two parallel guides, two rollers and a band; and the mechanism has been considered useful mainly as a force-generator and as a frictionless device.

However, connecting the guides and guides or rollers of two or more basic rolamites with revolute pairs and fixing at least one of the guides or the center-axes of rollers to the frame, a new mechanism with multi-degree of freedom whose inputs and outputs are the displacements and forces of the guides and/or rollers can be obtained.

This mechanism is called a compound rolamite in the present paper and its design problems have been discussed to make it applicable to a non-uniform motion mechanism.

2. Numbers of elements and variables in the compound rolamites

2.1 Numbers of inputs and outputs

In the compound rolamites, those basic rolamites whose guides or axes of rollers are fixed to the frame shall be called fixed rolamites, and the others shall be called moving rolamites. These basic rolamites are symbolically illustrated as shown in Fig.1, where $\circ$ indicates elastic bands and — indicates both of guides and rollers. When guides and rollers have to be distinguished, G and R shall be affixed to —. Thus Fig.2(a) describes the mechanism shown in Fig.2(b).

There exist generally three kinds of variables in compound rolamites; they are the absolute displacements of rollers and

guides, the relative displacements between rollers and guides in the moving rolamites, and the external forces acting on rollers and guides. These variables can be adopted as the inputs and outputs of the mechanisms. The relative displacements between rollers and guides in each basic rolamite shall be simply called relative displacements in the present paper.

Because the displacements of the guides and rollers which are connected with each other take always the same values, they shall be regarded as one guide or one roller and their displacements shall be counted as one variable. Moreover the rollers shall be counted by the number of clusters of rollers. Now, let N_g be the number of guides and rollers and let N_b be the number of absolute displacements existing in a compound rolamite, then the next equation will hold, because N_b is equal to the number obtained by subtracting the number of the fixed guides or rollers from N_g .

$$N_b = N_g - 1 \quad \dots\dots\dots (1)$$

Similarly, those relative displacements which take always the same value with each other on account of structural constraints shall be regarded as one variable, and let N_r be the number of relative displacements.

(a) Fixed rolamite (b) Moving rolamite
Fig.1 Symbolic expressions of basic rolamites

Fig.2 Example of compound rolamite

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ments existing in a compound rolamite. Moreover, let N_c be the number of moving rolamites whose relative displacements are mutually different, and the next equation will hold.

$$N_{b_2} = N_c \quad \dots\dots\dots (2)$$

Considering as one variable the external forces or torques acting upon the guides and rollers which are connected with each other, the maximum number of forces N_{b_3} is equal to the number of moving guides and rollers.

$$N_{b_3} = N_a - 1 \quad \dots\dots\dots (3)$$

Therefore, the total number N_b of the variables which can be adopted as the inputs and outputs becomes as follows.

$$N_b = N_{b_1} + N_{b_2} + N_{b_3} = 2N_a + N_c - 2 \quad \dots\dots\dots (4)$$

2.2 Freedom of mechanism

The number N_d of the equations which relate the absolute and relative displacements with each other is equal to N_c , and the number N_{d_2} of the equations which describe the balancing of the band-generated, external, inertial and frictional forces is equal to N_{b_3} .

$$N_{d_1} = N_c, \quad N_{d_2} = N_{b_3} = N_a - 1 \quad \dots\dots\dots (5)$$

Therefore, the total number N_d of equations which hold among the variables may become as follows.

$$N_d = N_{d_1} + N_{d_2} = N_a + N_c - 1 \quad \dots\dots\dots (6)$$

In a compound rolamite, the whole position of the mechanism may be determined when the values of some variables are given. The number of these variables shall be called the degrees of freedom of the compound rolamite. Denoting the degrees of freedom by M , the next equation holds.

$$M = N_b - N_d = N_a - 1 \quad \dots\dots\dots (7)$$

Denoting the numbers of the fixed and moving rolamites by N_1 and N_2 , and denoting the total number of the basic rolamites by N , the equations and domains of values of the numbers mentioned above may be summarized as shown in Table 1.

2.3 Determination of fundamental structure

The number of the basic rolamites shall be evaluated and the connection of the rola-

mites, that is a fundamental structure of a compound rolamite, shall be determined in order to obtain a mechanism with the prescribed degrees of freedom.

Substituting the given degrees of freedom M into Eq.(7), the number of guides and rollers may be obtained.

$$N_a = M + 1 \quad \dots\dots\dots (8)$$

Then, the number of basic rolamites N shall be assumed from Table 1 so that the next equation may hold.

$$N \geq M \quad \dots\dots\dots (9)$$

By the way, there exists only one band in each basic rolamite. Therefore, putting n_i as the number of guides and rollers to which i bands are directly connected, the following equations hold.

$$1 \leq i \leq N, \quad N_a = \sum_{i=1}^N n_i, \quad 2N = \sum_{i=1}^N i n_i \quad \dots\dots\dots (10)$$

Accordingly, substituting the values of N_a and N of Eqs.(8) and (9) into Eq.(10), n_i can be evaluated and the fundamental structures of the mechanisms may be determined.

As an example, the compound rolamites with $M=3$ shall be derived. From Eqs.(8) and (9), $N_a=4$ and $N \geq 3$ are obtained. Thus, the case $N=3$ shall be considered here, and the next equations may be derived from Eq.(10).

$$n_1 + n_2 + n_3 = 4, \quad n_1 + 2n_2 + 3n_3 = 6 \quad \dots\dots\dots (11)$$

The solutions of the above equations become as follows.

$$(a) \quad n_1 = 3, \quad n_2 = 0, \quad n_3 = 1$$

$$(b) \quad n_1 = 2, \quad n_2 = 2, \quad n_3 = 0$$

Therefore, the basic rolamites are known to be connected with each other as shown in Fig.3.

Fixing one of the guides or the center-axes of rollers to the frame, the fundamental structures of the mechanisms may be determined. Figures 4(a) and 4(b) show the structures respectively derived from the solutions (a) and (b) of Eq.(11). Moreover, distinguishing the guides and rollers in Fig. 4, the definite structures may be obtained.

3. Application of compound rolamites to non-uniform motion mechanisms

Table 1 Numerical relations of compound rolamites

Item	Number	Equation	Domain	
			When $N_2=0$	When $N_2 \geq 1$
Basic rolamites	N		$N \geq 1$	$N \geq 2$
Fixed rolamites	N_1		$N_1 = N$	$1 \leq N_1 \leq N-1$
Moving rolamites	N_2	$N_2 = N - N_1$	$N_2 = 0$	$1 \leq N_2 \leq N-1$
Guides and rollers	N_a		$2 \leq N_a \leq N+1$	$3 \leq N_a \leq N+1$
Variables capable to be inputs and outputs	N_b	$N_b = 2N_a + N_c - 2$	$2 \leq N_b \leq 2N$	$5 \leq N_b \leq 2N + N_2$
Absolute displacements	N_{b1}	$N_{b1} = N_a - 1$	$1 \leq N_{b1} \leq N$	$2 \leq N_{b1} \leq N$
Relative displacements	N_{b2}	$N_{b2} = N_c$	$N_{b2} = 0$	$1 \leq N_{b2} \leq N_2$
External forces	N_{b3}	$N_{b3} = N_a - 1$	$1 \leq N_{b3} \leq N$	$2 \leq N_{b3} \leq N$
Moving rolamites with different displacements	N_c		$N_c = 0$	$1 \leq N_c \leq N_2$
Equations of inputs and outputs	N_d	$N_d = N_a + N_c - 1$	$1 \leq N_d \leq N$	$3 \leq N_d \leq N + N_2$
Equations of displacements	N_{d1}	$N_{d1} = N_c$	$N_{d1} = 0$	$1 \leq N_{d1} \leq N_2$
Equations of forces	N_{d2}	$N_{d2} = N_a - 1$	$1 \leq N_{d2} \leq N$	$2 \leq N_{d2} \leq N$
Degrees of freedom	M	$M = N_a - 1$	$1 \leq M \leq N$	$2 \leq M \leq N$

As an example, the mechanism with two basic rolamites shown in Fig.2 shall be applied to a non-uniform motion mechanism whose input and output are the displacements x_2 and x_1 of the moving guide and the roller respectively.

3.1 Equation of motion

The relation between inputs and outputs of compound rolamites may be generally described by means of the equations of balancing of the band-generated, external, inertial and frictional forces. Therefore, the band-generated forces necessary for the generation of a given function should be derived from the equations of motion of the mechanisms.

In the mechanism shown in Fig.2, the masses of rollers, the moments of inertia of rollers, the effective radii of rollers, the mass of connecting link and the mass of moving guide are expressed respectively by m_{ij} , J_{ij} , R_{ij} , m_1 and m_2 , where $i=1, 2$ and $j=1, 2$. $i=1$ and $i=2$ are suffixed respectively to the fixed and moving rolamites. The effective radius of roller means (diameter of roller + thickness of band)/2.

Denoting the band-generated forces of the fixed and moving rolamites by $f_1(x_1)$ and $f_2(x_{12})$ respectively, the equations of motions of the roller and moving guide may become Eqs.(12) and (13) respectively.

$$(m_{eq1} + m_1)\ddot{x}_1 + c_1\dot{x}_1 + m_{eq2}\ddot{x}_{12} + m_r\ddot{x}_2 + c_2\dot{x}_{12} = f_1(x_1) + f_2(x_{12}) + F_1 \quad (12)$$

$$m_2\ddot{x}_2 + c_3\dot{x}_2 = -f_2(x_{12}) + F_2 \quad (13)$$

In which,

$$m_{eqi} = m_{i1} + m_{i2} + \frac{J_{i1}}{R_{i1}^2} + \frac{J_{i2}}{R_{i2}^2} \quad (i=1, 2) \quad (14)$$

$$m_r = m_{21} + m_{22} \quad (15)$$

$$x_{12} = x_1 - x_2 \quad (16)$$

Fig.3 Connections of basic rolamites

Fig.4 Compound rolamites with three degrees of freedom

F_1 and F_2 are external forces acting respectively on the roller and the moving guide. c_1 , c_2 and c_3 are respectively the coefficients of viscosity of rollers of the fixed and moving rolamites and of the moving guide.

Therefore, describing x_1 and x_2 as functions of time and substituting them into Eqs.(12) and (13), the band-generated forces $f_1\{x_1(t)\}$ and $f_2\{x_{12}(t)\}$ may be determined so that the relation between the input and output may satisfy the prescribed function.

3.2 Stability of mechanism

In the determination of the band-generated forces, it must be examined whether the balance of forces is stable or not. There exists a potential field in the rolamite. The potential may vary in its magnitude in the direction of the motion of rollers on account of variation of the strain-energy of band. The potential $U(x_1, x_2)$ of the mechanism shown in Fig.2 may be expressed as follows.

$$-\frac{\partial U}{\partial x_1} = f_1(x_1) + f_2(x_{12}) \quad (17)$$

Therefore it can be determined by the sign of $\partial^2 U / \partial x_1^2$ whether the mechanism may operate stably or not.

$$\frac{\partial^2 U}{\partial x_1^2} = -\frac{\partial f_1(x_1)}{\partial x_1} - \frac{\partial f_2(x_{12})}{\partial x_1} \quad (18)$$

$$= -\frac{\partial f_1(x_1)}{\partial t} \frac{1}{\dot{x}_1} - \frac{\partial f_2(x_{12})}{\partial t} \frac{1}{\dot{x}_{12}} \quad (19)$$

Consider the case where the mechanism operates at such a low speed that the rates of variation of external, inertial and viscous forces are adequately small in comparison with those of the band-generated forces, and the following equation may be derived from Eq.(12).

$$\frac{\partial f_1(x_1)}{\partial t} + \frac{\partial f_2(x_{12})}{\partial t} = 0 \quad (20)$$

Substituting Eqs.(16) and (20) into Eq.(19), the next equation is obtained.

$$\frac{\partial^2 U}{\partial x_1^2} = \frac{\partial f_1(x_1)}{\partial t} \frac{\dot{x}_2}{\dot{x}_1 \dot{x}_{12}} \quad (21)$$

Accordingly, the condition for stability becomes as shown in Table 2. In the case of $\dot{x}_2=0$, however, the condition of $\partial^2 U / \partial x_1^2 > 0$ must be searched from Eq.(19), and, in the case of $dx_1/dx_2=0$ or 1, the state of variation of U must be examined according to the signs of $\dot{x}_1$, $\ddot{x}_1$, $\dot{x}_{12}$ and $\ddot{x}_{12}$ in the neighborhood of $dx_1/dx_2=0$ or 1.

3.3 Determination of band-generated forces

The band-generated forces shall be determined so that the input and output of the mechanism may satisfy the prescribed function.

When the displacement equations of $x_1(t)$ and $x_2(t)$ are given, the output displacement x_1 and the relative displacement x_{12} are expressed as functions of time, and the dimensions of the guides, rollers and bands are determined in consideration of the domains of x_1 and x_{12} , the conditions of external forces and the strengths of materials.

Then, the band-generated forces shall be determined as functions of time. In this determination, it must be considered that the forces are uniquely determined in relation to the displacements of rollers and that the conditions for stability mentioned in the above section are satisfied. Therefore the band-generated forces are determined in different ways according to whether x_1, x_2 and x_{12} are monotone functions of time or not.

(i) Band-generated forces in the case where all of x_1, x_2 and x_{12} are monotone functions of time.

In this case, if the band-generated forces f_1 and f_2 are single-valued functions of time, it is obvious that f_1 and f_2 are uniquely determined with respect to the displacement of rollers. Therefore either f_1 or f_2 can be arbitrarily determined as a single-valued function of time under the condition of $\partial^2 U / \partial x_1^2 > 0$ or Table 2, and the other force may be determined from Eq. (12).

However, with consideration of the case where the mechanism may suffer external disturbances, the value of $\partial^2 U / \partial x_1^2$ shall be made here as great as possible.

The next equations may be obtained from Eq. (21).

$$f_1(x_1(t)) = \int U_{x_1}'' \frac{\dot{x}_1 \dot{x}_{12}}{\dot{x}_2} dt \dots\dots\dots (22)$$

$$U_{x_1}'' = \frac{\partial^2 U}{\partial x_1^2} > 0 \dots\dots\dots (23)$$

Integrating Eq. (22) in the domain of $t_0 \leq t \leq t_1$, the next equation is obtained.

$$|f_1(x_1(t_1)) - f_1(x_1(t_0))| \geq (U_{x_1}'')_{\min} \left| \int_{t_0}^{t_1} \frac{\dot{x}_1 \dot{x}_{12}}{\dot{x}_2} dt \right| \dots\dots\dots (24)$$

In which $(U_{x_1}'')_{\min}$ is the minimum value of U_{x_1}'' in $t_0 \leq t \leq t_1$ and the equality may hold when U_{x_1}'' is a constant.

The maximum value of the left side of Eq. (24) may be determined by the radii of rollers and the rigidity of band, and the value of integral in the right side may be determined by the displacement curve. Therefore the maximum value of $(U_{x_1}'')_{\min}$ for the given displacement curve may be evaluated. Thus, U_{x_1}'' shall be put as follows.

$$U_{x_1}'' = (U_{x_1}'')_{\min} = \max = \text{const.} \dots\dots\dots (25)$$

Now, $f_1(x_1(t))$ may be obtained from Eq. (22), and $f_2(x_{12}(t))$ may be obtained from Eq. (12). Therefore the profiles of bands can be determined by the method mentioned in the reference (5).

(ii) Band-generated forces in the case where all of x_1, x_2 and x_{12} are not monotone functions of time.

Table 2 Condition for stability

$\frac{dx_1}{dx_2}$	$\frac{dx_{12}}{dx_2}$	When $\dot{x}_2 > 0$		When $\dot{x}_2 < 0$	
		$\partial f_1 / \partial t$	$\partial f_2 / \partial t$	$\partial f_1 / \partial t$	$\partial f_2 / \partial t$
$\frac{dx_1}{dx_2} < 0$	$\frac{dx_{12}}{dx_2} < -1$	+	-	-	+
$0 < \frac{dx_1}{dx_2} < 1$	$-1 < \frac{dx_{12}}{dx_2} < 0$	-	+	+	-
$\frac{dx_1}{dx_2} > 0$	$\frac{dx_{12}}{dx_2} > 0$	+	-	-	+

As an example, the case shall be considered where $x_1(t)$ and $x_2(t)$ are monotonous but $x_{12}(t)$ has two extremals x_a, x_b ($x_b > x_a$) as shown in Fig.5.

If f_1 is given as an arbitrary single-valued function of time, it can be uniquely determined for the given displacement of rollers. However, even if f_2 is given as a single-valued function of time $g(t)$, $f_2 = g(t(x_{12}))$ becomes a triple-valued function of x_{12} in the domain of $x_a \leq x_{12} \leq x_b$ which means that the value of the band-generated force f_2 cannot be uniquely determined for the given value of the displacement x_{12} . Thus, the force f_2 in the domain $x_a \leq x_{12} \leq x_b$ shall be beforehand given as an arbitrary single-valued function $f_2(x_{12})$. In the domains of $t < t_a$ and $t > t_b$, f_2 can be given as an arbitrary function of time because $x_{12}(t)$ is monotonous in these domains.

Therefore f_2 in the domain between the extremals or in the whole domain shall be firstly determined as a function of $x_{12}(t)$ under the condition of Table 2, and then $f_1(x_1(t))$ shall be determined from Eq. (12).

To other cases where all of x_1, x_2 and x_{12} are not monotonous, similar methods to the above example may be applicable. That is, dividing the domains of x_1, x_2 and x_{12} into the subdomains whose boundary points are extremals, both $f_1(x_1)$ and $f_2(x_{12})$ may be determined under the condition of $\partial^2 U / \partial x_1^2 > 0$ as single-valued functions in each subdomain.

However, if $x_1(t)$ and $x_{12}(t)$ have extremals x_a, x_b and x_c, x_d and there exists a common domain between $t_a \leq t \leq t_b$ and $t_c \leq t \leq t_d$ which are time-intervals corresponding to $x_a \leq x_{12} \leq x_b$ and $x_c \leq x_{12} \leq x_d$, it may be proved that $f_1(x_1)$ and $f_2(x_{12})$ cannot be simultaneously single-valued functions. Therefore, such displacement relations cannot be realized with the mechanism shown in Fig.2.

4. Examples of application of compound rolamite

4.1 Non-uniform motion mechanism

The displacements of the mechanism of Fig.2 shall be given as follows.

Input displacement:

$$x_2 = 8t \text{ (mm)} \dots\dots\dots (26)$$

Output displacement:

$$\text{When } 0 \leq x_2 \leq 10 \text{ mm,} \\ x_1 = 0$$

Fig.5 Relative displacement $x_{12}(t)$ with two extremals

$$\left. \begin{array}{l} \text{When } 10 < x_2 < 90 \text{ mm,} \\ x_1 = 40 \left\{ \frac{x_2 - 10}{80} - \frac{1}{2\pi} \sin \frac{2\pi}{80} (x_2 - 10) \right\} \text{ (mm)} \\ \text{When } 90 \leq x_2 \leq 100 \text{ mm,} \\ x_1 = 40 \text{ (mm)} \end{array} \right\} \dots\dots\dots(27)$$

This displacement relation shall be realized by the mechanism shown in Fig.6 whose fixed and moving rolamites have the same dimensions as mentioned below.

Distance between guides : 45 mm

Diameters of rollers : 30 mm

Thickness of band : 0.1 mm

Maximum width of band : 76 mm

The equivalent masses of rollers of the fixed and moving rolamites are $m_{eq1} = m_{eq2} = 1.54 \text{ g cm}^{-1} \text{ s}^2$, the mass of the connecting link is $m_l = 0.177 \text{ g cm}^{-1} \text{ s}^2$, and the modulus of longitudinal elasticity of bands is $2.1 \times 10^6 \text{ Kg cm}^{-2}$.

Describing the displacements x_1 and x_{12} as functions of time, $f_1\{x_1(t)\}$ is obtained from Eq.(22), and then $f_2\{x_{12}(t)\}$ is obtained from Eq.(12).

The band-generated force of the fixed rolamite :

$$\left. \begin{array}{l} \text{When } x_1 = 0, \\ -30 \leq f_1 \leq 0 \text{ (g)} \\ \text{When } 0 < x_1 < 40 \text{ mm,} \\ f_1 = -3(4t-5) + \frac{30}{\pi} \sin \frac{\pi}{10} (4t-5) - 30 \text{ (g)} \\ x_1 = 4t-5 - \frac{20}{\pi} \sin \frac{\pi}{20} (4t-5) \text{ (mm)} \\ 1.25 < t < 11.25 \text{ (s)} \\ \text{When } x_1 = 40 \text{ mm,} \\ -180 \leq f_1 \leq -150 \text{ (g)} \end{array} \right\} \dots\dots\dots(28)$$

The band-generated force of the moving rolamite :

$$\left. \begin{array}{l} \text{When } -10 \leq x_{12} \leq 0 \text{ mm,} \\ f_2 = -3x_{12} + 6.36 \text{ (g)} \\ \text{When } -50 < x_{12} < -10 \text{ mm,} \\ f_2 = 3(4t-5) - \frac{30}{\pi} \sin \frac{\pi}{10} (4t-5) + 37 \\ + 0.817 \sin \frac{\pi}{20} (4t-5) \\ - 0.64 \cos \frac{\pi}{20} (4t-5) \text{ (g)} \end{array} \right\}$$

Fig.6 Experimental equipment

$$\left. \begin{array}{l} x_{12} = -(4t-5) - \frac{20}{\pi} \sin \frac{\pi}{20} (4t-5) - 10 \text{ (mm)} \\ 1.25 < t < 11.25 \text{ (s)} \\ \text{When } -60 \leq x_{12} \leq -50 \text{ mm,} \\ f_2 = -3x_{12} + 6.36 \text{ (g)} \end{array} \right\} \dots\dots\dots(29)$$

In the derivation of Eqs.(28) and (29), the viscous coefficients were put as $c_1 = c_2 = 0.08 \text{ g mm}^{-1} \text{ s}$, Coulomb's friction was considered an external force of $F_l = -7 \text{ g}$, and $U_{x_1} = 12 \text{ g mm}^{-1}$ which is the result of calculation of Eqs.(24) and (25).

The force-displacement curves for each basic rolamite have become as shown in Fig. 7. The profiles of bands for these curves may be determined by using the method mentioned in the reference (5). Figures 7 and 8 show the band-generated forces and the output displacement whose experimental results are of the mechanism shown in Fig.6.

4.2 Gripping device of manipulator

As the compound rolamite can be utilized as a non-uniform motion mechanism with the force-controlling function, the rolamite shall be applied here to a gripping device of a manipulator as shown in Fig.9 which can generate automatically the prescribed optimum gripping forces in accordance with the dimensions of objects.

Putting the force at the center of roller O caused by the inertial forces of the linkage and the friction as $F(x_1, \dot{x}_1, \ddot{x}_1)$ and denoting the gripping force by P , the equation of motion of the roller becomes as follows from Eq.(12).

(i) Before the object is gripped,

$$f_1(x_1) + f_2(x_1 - at) + F(x_1, \dot{x}_1, \ddot{x}_1) = (m_{eq1} + m_{eq2} + m_l)\ddot{x}_1 + c_1\dot{x}_1 + c_2(\dot{x}_1 - a) \dots\dots(30)$$

Fig.7 Force-displacement curves

Fig.8 Output-input displacement curve

(ii) After the object is gripped,

$$f_1(x_1) + f_2(x_1 - at) = g(x_1)P - c_2 a, \quad x_1 = \text{const.} \quad (31)$$

In which $g(x_1)$ denotes the ratio of the force caused at the center O of the roller by the gripping force P to the force P , and $a = \dot{x}_2 = \text{constant}$.

When the values of parameters of the device are given, $F(x_1, \dot{x}_1, \ddot{x}_1)$ and $g(x_1)$ may be evaluated. Therefore the necessary band-generated forces can be determined from Eqs.(30) and (31) by applying the method mentioned in Chapter 3 if the relation between the gripping forces and the dimensions of objects is given.

As an example, the case shall be considered where cylindrical objects with the same length but with different diameters D are to be gripped. Let the maximum weight and diameter of objects be 400 g and 60 mm, and let the parameters of the linkage be as shown in Table 3. Then the band-generated forces $f_1(x_1)$ and $f_2(x_{12})$ become respectively the solid and broken lines for the necessary gripping force shown by the chain line in Fig.10, where the driving speed and distance of the moving guide are put respectively as $\dot{x}_2 = a = 10 \text{ mm/s}$ and $\Delta x_2 = 150 \text{ mm}$.

An experimental device has been actually manufactured, whose band-generated forces and the gripping force are as plotted in Fig.10. An example of the output displacement x_1 and the gripping force P is shown in Fig.11 in relation to the input displacement.

4.3 Weighing machine with two sensibilities

In Fig.12, a pulley with radius of R' is attached concentrically to one of the rollers with radius of R , and a scale is hung with a flexible band wound around the pulley. Denoting the total weight of the scale and

Table 3 Parameters of gripping linkage

Link	Length mm	Weight g	Moment of inertia g mm^2
a	$\overline{OA} = 210.0$	313.0	3.45
b	$\overline{AB} = 55.9$	84.2	
c	$\overline{BC} = 59.9$	214.9	
d	$\overline{BD} = 93.1$	134.0	
DD'	$\overline{DD'} = 200.0$		12.72

Fig.9 Gripping device of manipulator

the object by W , the balance of forces may be expressed as follows.

$$f_1(x_1) + f_2(x_{12}) + (R'/R)W = 0 \quad (32)$$

If the left side of the above equation is a linear function of x_1 , the displacement of the rollers becomes proportional to the weight W . However the band-generated forces shall be given here as follows.

$$\begin{aligned} f_1(x_1) &= -k_1 x_1 \\ f_2(x_{12}) &= \begin{cases} -k_2 x_{12} & (x_{12} \leq x_s) \\ k_3 x_{12} - (k_2 + k_3)x_s & (x_{12} > x_s) \end{cases} \end{aligned} \quad (33)$$

In which $k_i (i=1,2,3)$ are positive constants and $k_1 > k_3$. The values of k_i and x_s are arbitrarily given.

The relation between the displacement x_1 of the roller and the weight W becomes as shown in Fig.13. x_{10} , x_{1s} , and W_s in the figure may be obtained as follows from Eqs. (32) and (33).

$$x_{10} \equiv (x_1)_{W=0} = k_2 x_{12} / (k_1 + k_2) \quad (34)$$

$$x_{1s} \equiv (x_1)_{x_{12}=x_s} = x_s + x_s \quad (35)$$

$$W_s \equiv (W)_{x_1=x_{1s}} = (R/R') \{ k_1 x_{12} + (k_1 + k_2)x_s \} \quad (36)$$

When the moving guide is fixed at a position of x_2 , the derivative of Eq.(32) with respect to W may become as follows.

When $W < W_s$,

$$\frac{dx_1}{dW} = \frac{R'}{R} \frac{1}{k_1 + k_2} \equiv S_L \quad (37)$$

When $W > W_s$,

$$\frac{dx_1}{dW} = \frac{R'}{R} \frac{1}{k_1 - k_3} \equiv S_H \quad (38)$$

Fig.10 Gripping force and band-generated forces

Fig.11 Output displacement and gripping force (Diameter of object: $D=40 \text{ mm}$)

Because $k_1 > 0$ and $k_1 > k_3$ in Eq. (33), $S_H > S_L$ may hold, which means the sensibility in the range of $W > W_s$ is higher than the one in the range of $W < W_s$. The changing point of the sensibility $W = W_s$ can be varied by shifting the position x_2 of the moving guide, as is known from Eq. (36).

In Fig. 12, the scale S_I graduated on the moving guide according to Eq. (38) is combined with the needle N_I fixed on the connecting link, and the scale S_{II} graduated on the fixed guide according to Eq. (36) is combined with the needle N_{II} fixed on the moving guide. Therefore, if the needle N_I points to the graduation w on the scale S_I when the needle N_{II} is fixed at the graduation W_0 on the scale S_{II} , for example, the weight may be read as $W = W_0 + w$.

A weighing machine with the following specification has been manufactured for experiment.

Weighing range : 0 ~ 350 g

Lower sensibility : 0.2 mm/g

Higher sensibility : 1 mm/g

Using the rolamites shown in Fig. 6, the maximum band-generated force becomes $|f|_{\max} = 360$ g which is determined from the limit of elasticity of bands. As the next equation holds, R' is assumed equal to R .

$$\frac{R'}{R} = \frac{|f_1(x_1) + f_2(x_{12})|}{W} \leq \frac{|f|_{\max} \cdot 360}{350}$$

From Eqs. (37) and (38), the next equations are obtained.

$$k_1 + k_2 = 5, \quad k_1 - k_3 = 1$$

Therefore, if $k_1 = 3$ g/mm, $k_2 = k_3 = 2$ g/mm.

x_s in Eq. (33) shall be determined so that the maximum weight $W_{\max} = 350$ g may be able to exist in the range of the lower sensibility.

Fig. 12 Weighing machine with two ranges of sensibilities

Fig. 13 Weight-displacement curve

Thus, putting $x_s = 75$ mm, the necessary band-generated forces may become as follows.

$$f_1(x_1) = -3x_1$$

$$f_2(x_{12}) = \begin{cases} -2x_{12} & (x_{12} \leq 75 \text{ mm}) \\ 2x_{12} - 300 & (x_{12} > 75 \text{ mm}) \end{cases}$$

The bands which satisfy the above equations have been manufactured. The actual forces generated by the bands are shown in Fig. 14. Examples of the relation between the displacement of roller x_1 and the weight W are shown in Fig. 15 for the cases where $W_s = 0$ and 100 g.

When the same object was weighed repeatedly, the maximum deviation of the reading on the scale was 0.2 mm, which corresponds to 0.2 g in weight or 0.057 % of the full range.

5. Conclusions

Connecting two or more basic rolamites, mechanisms with multi-degree of freedom whose inputs and outputs are displacements and/or forces have been developed. The mechanisms have been called here compound rolamites, and their design problems have been discussed with examples of their applications given. The main results are summarized as follows.

(1) There exist three kinds of quantities in the inputs and outputs of the compound rolamites; they are the absolute displacements of guides and rollers, the relative

Fig. 14 Band-generated forces

Fig. 15 Weight-displacement curves at $W_s = 0$ and 100 g

displacements of rollers to the guides in each basic rolamite, and the external forces acting on the guides and rollers. Therefore the compound rolamites may be applied to non-uniform motion mechanisms with force-controlling function.

(2) The relations among the numbers of rollers, guides, input and output quantities and degrees of freedom have been formulated, and the domains where the number of each quantity may exist have been tabulated. The number of basic rolamites and their arrangements have been also determined so that the mechanism might have the prescribed degrees of freedom.

(3) In a compound rolamite, the relation between the inputs and outputs is determined by the balancing of the band-generated, inertial, external and frictional forces. Therefore the band-generated forces necessary to obtain the prescribed relation among the inputs and outputs must be determined by means of the equations of motion with consideration of the conditions where the mechanism is stable and the band-generated forces are single-valued functions of the relative displacements of rollers. An example of the determination of the above forces has been shown on a mechanism consisting of two basic rolamites.

(4) The compound rolamite with two basic rolamites has been applied to a non-uniform motion mechanism with a single-degree of freedom, to a gripping device of manipulator which generates automatically the optimum gripping forces in accordance with the dimensions of objects, and to a weighing machine with two ranges of sensibilities whose boundary can be arbitrarily varied. The experiments for these applications show that the necessary functions between the input and output have been obtained and that the compound rolamite can give a new non-uniform motion mechanism with force-controlling function.

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