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Frictional Resistance of a Rotating Disk in Polymer Solutions* (Laminar and Transitional Region from Laminar to Turbulent Flow)

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The coefficient of moment c_f of polymer solutions which have viscoelastic properties in the laminar and transitional region from the laminar to the turbulent flow, have been measured by a new type torque pick-up. It is shown that c_f of the dilute polymer solutions under the laminar flow condition increases only slightly compared with Newtonian fluids, but in the particular limiting case of the polymer solutions in which more viscoelastic effects occur there is some reduction compared with c_f of Ostwald de-Waele power-law fluid. In the transition region, the variations of c_f with Reynolds number become gentle compared with Newtonian fluids.

1. Introduction

The reduction of frictional drag in turbulent flow due to the presence of long-chain polymer additives in liquids has been actively researched in recent years. But, there are very few studies on an unusual transition region in dilute polymer solutions where the process of transition from laminar to turbulent flow is not the same as that of Newtonian fluids, and it seems to be that T. Hasegawa, et al.'s (1) study is the only experimental study on the transition recognized in circular pipe flow.

On the other hand, the frictional resistance of a rotating disk in the dilute polymer solutions at the laminar flow region was analyzed by Y. Mochimaru, et al. (2) and L. Elliott (3). Y. Mochimaru, et al. showed that the coefficient of moment of a rotating disk in this region increases only slightly with an increase of the viscoelastic effect compared with that of Newtonian fluids. By the analogous analysis, L. Elliott obtained the result which is in contradiction with Y. Mochimaru, et al.'s result.

D.D. Kale, et al. (4) obtained the similar solution of the coefficient of moment in the laminar region using the 2nd order fluid in which the variable coefficients are expressed by the three dimensional power-law model. Under the condition of purely viscous Ostwald de-Waele power-law model, the comparison of the experimental result with the theoretical one is indicated, and it becomes clear that the reduction of the coefficient of moment in viscoelastic fluids is recognized. Although it became clear through those studies that the behavior of the coefficient of moment in viscoelastic fluids at laminar and transition regions is very different from

that of Newtonian fluids, it can be considered that there is not enough of the experimental data on these regions. Thus, the purpose of this paper is to clarify the effect of non-Newtonian viscous and viscoelastic effects on the coefficient of moment of a rotating disk in various concentrated polymer solutions at the laminar and transition regions by the torque measurements using a new type torque pick-up.

2. Nomenclature

- a : radius of disk
- c_f : coefficient of moment in one side of disk $\equiv 3M/(\rho a^2 \omega^2)$
- c_f' : coefficient of moment defined by Eq. (21)
- d : diameter of disk
- M : frictional moment acting on one side of disk
- n : exponent in viscous stress power-law
- Re : Reynolds number $(\equiv a^2 \omega / \nu)$
- Re_g : generalized Reynolds number
- v_r : radial component of velocity distribution in boundary-layer
- v_θ : tangential component of velocity distribution in boundary-layer
- δ : thickness of boundary-layer
- ρ : fluid density
- μ_{psu} : apparent viscosity
- ν : kinematic viscosity
- τ_r : radial component of disk wall shearing stress
- τ_θ : tangential component of disk wall shearing stress
- ω : angular velocity of disk
- r, θ, z : cylindrical coordinates

3. Experimental apparatus and procedure

Figure 1 shows the experimental apparatus. The volume of liquids in the tank ⑤ was about 1650 liters, and the disk was rotated by D.C. motor ①. The rotational speeds of the disk were measured by the magnetic transducer ②, and the frictional moment of the disk was measured by the torque pick-up of strain gauge type.

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Figure 2 is a detail drawing of the torque pick-up provided on the shaft. In Fig. 2, four phosphor bronze plates of 0.4 mm in thickness are soldered with the boss made of brass. Four strain gauges are stuck on those plates, and for waterproofing, the gap is filled with insulator material and then the cylindrical part is wrapped with the insulator tape.

①	DC. motor
②	Magnetic Transducer
③	Slipring
④	Rotating Disk
⑤	Tank
A	Part of Torque Pick-up

Fig. 1. Experimental Apparatus

	l	d_0	d_1	h
①	14	4	22	3.5
②	14	4	25	3.5
③	14	4	28	3.5

Fig. 2. Detail drawing of part of torque pick-up

The torque pick-up was calibrated by applying a known static torque to the shaft. From the result of the testing, it was found that the relations between the torque and the strain in the three torque pick-up shafts built on a trial gave essentially straight lines and a high accuracy of the measurement was obtained.

The disks used in this study were a 250 mm dia and a 350 mm dia aluminum smooth ones of 2 mm in thickness. The liquids tested were water and aqueous solutions of polyethylene oxide (Peo 10), polyacrylamide (Separan AP 30) and hydroxy ethyl cellulose (HEC AL-5000). Water was used as a Newtonian fluid. The concentrations and the physical constants of the fluids tested are given in Tables 1 (a) ~ (b). The kinematic viscosity, ν in Table 1 (a) was obtained with Ubeloude viscometer, and the rheological parameters, n and μ_{psu} , in Table 1 (b) were determined by the flow curves which were obtained with a capillary and a rotational viscometers.

4. Results and discussion

4.1 Newtonian fluids

The experimental results obtained with water are presented in Fig. 3. The straight lines ① and ③ in Fig. 3 are von Kármán's laminar and turbulent theoretical results⁽⁵⁾, respectively. It is clearly seen that the agreement between the theoretical result and the experimental data of the coefficient of moment, C_f is excellent in the range of laminar boundary-layer flows, and the numerical value of Reynolds number at which the transition occurs (critical Reynolds number), is obtained as being approximately $Re_c = 2.8 \times 10^5$. This value agrees with the result of

Table 1. Properties of test fluids

(a) Dilute solutions

	Concentration	Temperature $^{\circ}\text{C}$	$\nu \text{ cm}^2/\text{s}$
PEO 10	5 ppm	6.5	1.43×10^{-2}
	20	5.5	1.44
	50	8.7	1.37
	100	7.5	1.51
Separan AP 30	0.6 ppm	7.0	1.42×10^{-2}
	2	6.1	1.47
	5	6.0	1.50
	10	6.5	1.47
	20	6.0	1.53
	50	6.0	1.61

(b) Concentrated solutions

	Concentration	Temperature $^{\circ}\text{C}$	n	$\mu_{psu} \left(\frac{\text{dyn} \cdot \text{s}}{\text{cm}^2} \right)$
HEC AL-5000	100 ppm	8.8	1.00	1.33×10^{-2}
	500 ppm	8.5	1.09	2.18×10^{-2}
	0.2%	9.2	1.16	8.05×10^{-2}
	0.5%	10.0	1.27	5.46×10^{-1}
PEO 10	0.1%	12.0	1.14	3.08×10^{-2}
	0.3%	7.2	1.19	1.78×10^{-1}
Separan AP 30	500 ppm	6.2	1.47	2.33×10^{-1}
	0.1%	7.0	1.81	2.48
	0.3%	6.0	2.10	4.16×10

N. Gregory, et al.⁽⁶⁾ which were calculated using the transition position found at the number of rotational speeds with the china-cray technique.

It is seen that Ito's theoretical result⁽⁷⁾ (the curve ② in Fig. 3) calculated from the critical Reynolds number is in very good agreement with the experimental data.

4.2 Non-Newtonian fluids

4.2.1 Effect of non-Newtonian viscosity

By neglecting the effect of viscoelasticity and thixotropy in the solutions, Y. Tomita⁽⁸⁾ analyzed the coefficient of moment of non-Newtonian viscous fluids, such as Ostwald de-Waele power-law fluid, for the laminar and turbulent boundary-layer flows along the rotating disk surface using the integral momentum boundary-layer equations. But, there still remains the question about the assumption of the velocity distributions in the turbulent boundary-layer flow, and it seems that his result is not sufficient for the experimental data on the range from the laminar flow to the turbulent flow, because the correlation of the coefficient of moment between the laminar and turbulent boundary-layer flows has not been given.

In addition, the theoretical result of coefficient of moment for the transition region in the non-Newtonian viscous fluids has not been obtained. Therefore, in the present paper, the coefficient of moment of the turbulent flow region in non-Newtonian viscous fluids is analyzed in order to obtain the theoretical result in the transition region.

Fig. 3 C_f - Re , (Water)

If the effects of viscoelasticity and thixotropy in the solutions can be neglected, the boundary-layer momentum equations in radial and tangential directions are given by

$$\int_0^\delta v_\theta^2 dz - \frac{d}{dr} \left(r \int_0^\delta v_r^2 dz \right) = \frac{r \tau_r}{\rho} \quad \dots\dots\dots (1),$$

$$\frac{d}{dr} \left(r^2 \int_0^\delta v_\theta v_r dz \right) = -\frac{r^2 \tau_\theta}{\rho} \quad \dots\dots\dots (2).$$

In order to solve Eqs. (1) and (2), the velocity distributions of non-Newtonian fluids must be assumed, and it is convenient for assuming the velocity distributions in boundary-layer on the rotating disk to obtain the velocity profile in a circular pipe. D.W. Dodge, et al.⁽⁹⁾ obtained the experimental formula for the relation between the friction, f and the generalized Reynolds number $R_{*}' (\equiv D'^n u_m^{2-n} \rho / K 8^{n-1})$, where, D is pipe diameter and u_m is mean velocity. The rheological equation is $\tau = K \dot{\gamma}^{n'}$ as

$$f = a' (R_{*}')^{-b'} \quad \dots\dots\dots (3).$$

Hence, in the present study, the experimental data a' and b' in Eq. (3) are approximated as

$$a' = 0.0779n^{-0.1}, \quad b' = (3n+17)/80 \quad \dots\dots\dots (4).$$

Table 2 lists the comparison between Eq. (4) and the experimental data obtained by D.W. Dodge, et al. It is seen that the two resembles each other very much, that is, accuracy better than 3.5% is obtained in the range of $n = 3.33 \sim 0.719$.

Substitution of Eq. (4) into Eq. (3) gives

$$f = 0.0779n^{-0.1} \times \left\{ \frac{2^{(2-1/n)}}{(n+3)^{1/n}} \frac{\rho D^{1/n} u_m^{2-1/n}}{\mu_{ps}^{1/n}} \right\}^{-(3n+17)/80} \quad (3.33 \geq n \geq 0.79) \quad \dots\dots\dots (5).$$

From Eq. (5), the expression for the ratio of the velocity at the distance from the pipe wall, y to the maximum velocity is obtained as follows:

$$\left(\frac{u}{u_{\max}} \right) = \left(\frac{y}{r_0} \right)^{(3n+17)/(17+129n-6n^2)} \quad \dots\dots\dots (6)$$

where, r_0 denotes the pipe radius.

Table 2 Comparison between Eq. (4) and D.W. Dodge, et al's experimental data

D.W. Dodge, et al's data ⁽⁹⁾ $\tau = K \dot{\gamma}^{n'}$			Values of Eq.(4) $\tau = \mu_{ps} \dot{\gamma}$				
n'	a'	b'	$n=1/n'$	$a'=0.0779n^{-0.1}$	Error %	$b'=\frac{3n+17}{80}$	Error %
0.2	0.0646	0.349	5.00	0.0663	2.6	0.400	-10
0.3	0.0685	0.325	3.33	0.0691	0.9	0.336	3.4
0.4	0.0712	0.307	2.50	0.0711	-0.2	0.306	-0.3
0.6	0.0740	0.281	1.67	0.0740	0	0.275	-2.1
0.8	0.0761	0.263	1.25	0.0762	0.1	0.259	-1.5
1.0	0.0779	0.250	1.00	0.0779	0	0.250	0
1.4	0.0804	0.231	0.71	0.0806	0.2	0.239	3.5
2.0	0.0826	0.213	0.50	0.0835	1.1	0.231	8.5

On the other hand, the disk wall shearing stress becomes

$$\tau_w = c \rho u_{\max}^2 \left(\frac{\mu_{psu}^{1/n}}{\rho u_{\max}^{2-1/n} r_0^{1/n}} \right)^{(3n+17)/80} \dots \dots \dots (7)$$

where,

$$c = 0.0779 n^{-0.1} \{ 2^{-(9n+181)/80} (n+3)^{(8n+17)/80n} \xi^{(17+129n-6n^2)/80n} \dots \dots \dots (8),$$

and ξ denotes the velocity ratio, $(u_n / u_{\max})$. If the numerical value of C in Newtonian fluids is calculated in case of $n = 1$, Eq. (8) becomes $C = 0.0230$, and this value is about 2.2% higher than the numerical value, $C = 0.0225$ obtained by von Kármán.

By using Eq. (6), the velocity distributions in the boundary-layer along the rotating disk surface are assumed as

$$v_\theta = \omega r \left\{ 1 - \left(\frac{z}{\delta} \right)^{(3n+17)/(17+129n-6n^2)} \right\}, \quad v_r = \alpha \omega r \left(\frac{z}{\delta} \right)^{(3n+17)/(17+129n-6n^2)} \left(1 - \frac{z}{\delta} \right) \dots \dots \dots (9).$$

Based on the velocity distributions of Eq. (9), the velocity U relative to the surface would be

$$U = \sqrt{1+\alpha^2} \omega r \left(\frac{z}{\delta} \right)^{(3n+17)/(17+129n-6n^2)} \dots \dots \dots (10).$$

The components of the disk wall shearing stress are denoted as $\tau_r = \alpha \tau_0 / \sqrt{1+\alpha^2}$ and $\tau_\theta = -\tau_0 / \sqrt{1+\alpha^2}$, respectively by using the disk wall shearing stress, τ_0 . In this paper, it is assumed that the value of τ_0 agrees with τ_w of Eq. (7) under the condition of $u_{\max} = U$ in the boundary-layer. By substituting Eq. (9) and these wall shearing stress components into Eqs. (1) ~ (2), and then, by integrating, the following equations are obtained

$$k_1 \delta r^2 - \frac{d}{dr} (k_2 \alpha^2 r^2 \delta) = c \alpha r^3 (1+\alpha^2)^{(1/2)\{1-(2-1/n)(3n+17)/80\}} \left(\frac{\mu_{psu}^{1/n}}{\rho \omega^{2-1/n} r^{2-1/n} \delta^{1/n}} \right)^{(3n+17)/80} \dots \dots \dots (11)$$

$$\frac{d}{dr} (k_3 \alpha r^4 \delta) = c r^4 (1+\alpha^2)^{(1/2)\{1-(2-1/n)(3n+17)/80\}} \left(\frac{\mu_{psu}^{1/n}}{\rho \omega^{2-1/n} r^{2-1/n} \delta^{1/n}} \right)^{(3n+17)/80} \dots \dots \dots (12)$$

where,

$$k_1 = \left(\int_0^\delta v_\theta^2 dz \right) / \delta \omega^2 r^2, \quad k_2 = \left(\int_0^\delta v_r^2 dz \right) / \delta \alpha^2 \omega^2 r^2 \quad \text{and} \quad k_3 = \left(\int_0^\delta v_\theta v_r dz \right) / \delta \alpha \omega^2 r^2$$

If the thickness of boundary-layer, δ is denoted as $\delta = H r^{(17+49n-6n^2)/(83n+17)}$ and α is assumed as a constant value, α_0 , then, the values of H and α_0 are given as

$$H = \left\{ \frac{c(83n+17)}{k_3 \alpha_0 (85+381n-6n^2)} \right\}^{80n/(83n+17)} (1+\alpha_0^2)^{(17+49n-6n^2)/(2(83n+17))} \left(\frac{\mu_{psu}^{1/n}}{\rho \omega^{2-1/n}} \right)^{n(3n+17)/(83n+17)} \dots \dots \dots (13)$$

$$\alpha_0 = \left\{ \frac{k_1(83n+17)}{k^2(68+298n-6n^2) + k_3(85+381n-6n^2)} \right\}^{1/2} \dots \dots \dots (14)$$

Thus δ becomes

$$\delta = B r \left(\frac{\mu_{psu}^{1/n}}{\rho r^2 \omega^{2-1/n}} \right)^{n(3n+17)/(83n+17)} \dots \dots \dots (15),$$

where

$$B = \left\{ \frac{c(83n+17)(1+\alpha_0^2)^{(1/2)\{1-(2-1/n)(3n+17)/80\}}}{k_3 \alpha_0 (85+381n-6n^2)} \right\}^{80n/(83n+17)} \dots \dots \dots (16),$$

The frictional moment acting on one side of the disk M is

$$M = 2\pi \rho a^2 \left[\int_0^\delta v_\theta v_r dz \right]_{r=a} = 2\pi \rho a^5 \omega^2 B k_3 \alpha_0 \left(\frac{\mu_{psu}^{1/n}}{\rho a^2 \omega^{2-1/n}} \right)^{n(3n+17)/(83n+17)} \dots \dots \dots (17).$$

The coefficient of moment, c_f can be evaluated as

$$c_f = \frac{2M}{\rho a^5 \omega^2} = 4\pi \alpha_0 B k_3 R_*^{-n(3n+17)/(83n+17)} \dots \dots \dots (18)$$

For Newtonian fluids ($n=1$), Eq. (18) becomes $c_f=0.0741R_{e*}^{-1/5}$, and it shows an increase of 1.8% against Kármán's(5) formula $c_f=0.0728R_{e*}^{-1/5}$.

The graphs of α_0 and B against n are shown in Figs. 4 ~ 5.

From these results, it is seen that c_f of the non-Newtonian viscous fluids shows a decrease against c_f of Newtonian fluids with an increase of the non-Newtonian viscous effect, that is, an increase of n . Concerning c_f in the transition region, based on the experimental result(6) that the condition of the transitional flow on the rotating disk is a coexistence condition of the laminar and turbulent flows, Ito(7) obtained the formula of c_f in the transition region for Newtonian fluids. In the present paper, the author analyzes c_f in the transition region for the non-Newtonian viscous fluids in much the same manner as Ito's method.

The frictional moment M in the transition region is given as

$$M=2\pi \int_0^{r_c} r^2(-\tau_\theta)dr + 2\pi \left\{ \int_0^a r^2(-\tau_\theta')dr - \int_0^{r_c} r^2(-\tau_\theta')dr \right\} \quad \dots\dots\dots(19).$$

Where, r_c is the transition radius, τ_θ and τ_θ' are the tangential components of the disk wall shearing stress in the laminar and turbulent boundary-layer flows, respectively. By substituting Tomita's result(8) and Eq. (17) into Eq. (19), c_f in the transition region is given by

$$c_f=4\pi\beta R_{e*}^{-5/2} \left\{ R_{ec*}^{(3n+5)/(2(n+1))} + \frac{k_3\alpha_0}{\beta} (R_{e*}^p - R_{ec*}^p) \right\} \quad \dots\dots\dots(20)$$

where, $p=(5/2-n(3n+17)/(83n+17))$, β is a function of n and R_{ec*} is a generalized Reynolds number in which the characteristic dimension is r_c .

Fig. 4 α_0 - n

Y. Tomita correlated the coefficient of moment in laminar region, and showed that c_f is expressed by one straight line even if the value of n changes. It is given as

$$c_f' = \frac{1.953}{4\pi\beta} R_{e*}^{(n-1)/(2(n+1))} c_f \quad \dots\dots\dots(21).$$

Therefore, if c_f of Eqs. (18) and (20) is correlated with Eq. (21), the following are obtained:

for the turbulent flow region,

$$c_f' = 1.953 K_1 R_{e*}^{(n-1)/(2(n+1)) - n(3n+17)/(83n+17)} \quad \dots\dots\dots(22)$$

and for the transition region,

$$c_f' = 1.953 R_{e*}^{(n-1)/(2(n+1)) - 5/2} (K_1 R_{e*}^p - K_2) \quad (3.33 \geq n \geq 0.79) \quad \dots\dots\dots(23)$$

where,

$$K_1 = \alpha_0 B k_3 / \beta, \quad K_2 = \{ K_1 R_{ec*}^p - R_{ec*}^{(3n+5)/(2(n+1))} \}$$

A graph of K_1 against n is shown in Fig. 6.

Fig. 5 B - n

Fig. 6 K_1 - n

Based on this theoretical result, the experimental data on HEC, Peo and Separan solutions are shown in Figs. (7) ~ (9), respectively. From Fig. (7), it is clearly seen that the experimental data on HEC solutions in the laminar flow region agree with Tomita's formula, and the data on 500 ppm solutions in the transition region are significantly lower than those on Newtonian fluids, and the theoretical result of Eq. (23) calculated as $Re_{*} = 1.7 \times 10^5$ is in good agreement with the experimental data.

On the other hand, the experimental data on Peo 0.3% solutions show that, although c_f in the range of Re_{*} smaller than 1.4×10^5 agrees with Tomita's formula, the data deviate slightly upward with an increase of Re_{*} , and the variation of the experimental data in the transition region becomes gentle, compared with the data on HEC 500 ppm solutions.

In the case of Separan solutions (Fig. 9), a decreasing tendency of c_f is seen in the laminar flow region compared with Tomita's formula. From these experimental results, it becomes clear that there is some increase or decrease compared with Tomita's formula in which only non-Newtonian viscous effect is considered in c_f of the high concentrated polymer solutions in the laminar boundary-layer flow region.

Fig. 7 $c_f'-Re_{*}$ (HEC Solutions)

Fig. 8 $c_f'-Re_{*}$ (PEO Solutions)

4.2.2 Viscoelastic effect

If the concentration of the polymer solutions were dilute, it would be possible to assume that the non-Newtonian viscous effect in the solutions can be neglected. But, it seems that the viscoelastic effect in the solutions exists slightly. Thus, in order to clarify the effect it has a significant meaning to obtain here the experimental coefficient of moment data on the dilute polymer solutions in the concentration range from several ppm to several hundred ppm.

Table 1(a) shows the concentration range and the kinematic viscosity of Peo and Separan solutions used in this experiment. The experimental data of C_f are shown in Figs. 10 ~ 11.

It is seen in Figs. 10 ~ 11 that the variation of c_f in the laminar and transition regions can be classified into the following two patterns; (1) a pattern in which c_f agrees with Kármán's formula in the lower Reynolds number region, and increases slightly compared with that in the Reynolds number region less than $Re = 2.8 \times 10^5$ and has a tendency to increase against Re , and (2) a pattern in which c_f decreases monotonously according to an increase of Re and agrees with c_f in the saturated range of Toms' effect. As already described, c_f of the dilute polymer solutions which possess the viscoelastic properties in the laminar boundary-layer flow region has been analyzed by Y. Mochimaru, et al. (12) and L. Elliott (3). Y. Mochimaru, et al. obtained the result which is in contradiction with the analogous analysis by L. Elliott, and the experimental data agree qualitatively with the result obtained by Y. Mochimaru, et al. in which c_f of the dilute polymer solutions increases compared with Newtonian fluids. And then, it can be considered from Figs. 10 ~ 11 that c_f in the laminar boundary-layer flow region is summarised by using Reynolds and Weissenberg numbers. (10)

On the other hand, as the concentration of the solutions increases, the viscoelastic effect of the polymer solutions becomes even stronger with an increase of non-Newtonian effect. The viscoelastic effect depends upon the kind and molecular weight of the

Fig. 9 $c_f'-Re_{*}$ (Separan Solutions)

polymer additives; and if the solutions are of the same concentration, the effect will be stronger in order of Separan, Peo and HEC solutions.

From the experimental results in Figs. 7 ~ 8, it is seen that the variation of c_f in HEC and Peo solutions is similar to that in the dilute polymer solutions, and the viscoelastic effect contributes to the increase of c_f in the laminar boundary-layer flow region. But, in the case of Separan solutions in Fig. 9, the experimental data

of c_f are approximated by an envelope which slightly decreases compared with Tomita's formula. Although the similar behaviour of decreasing for c_f in the concentrated solutions is reported by D.D. Kale, et al.⁽⁴⁾, it has not been ascertained yet whether there is any problem of the limit in the application of the boundary-layer approximations or the phenomenon can be foreseen by the non-dimensional parameter representing the characteristic of the viscoelastic fluids flow.

Fig. 10 c_f - Re (Separan Solutions)

Fig. 11 c_f - Re (PEO Solutions)

(a)

(b)

(c)

Fig. 12 Power spectrum of dynamic change in frictional moment

4.3 Dynamic change in frictional moment

As is evident from Fig. 2, the frictional moment acting on the rotating disk has been measured directly in this study. Therefore, the experimental results of the dynamic change in the frictional moment are described in this section in order to consider the flow condition around the disk surface. The data obtained through the dynamic strain meter, were recorded by the data recorder, and were analyzed by the correlation analyzer. The fluids used in this experiment were water, air and Separan 5 ppm solutions whose kinematic viscosity is numerically almost equal to water. In order to avoid the errors of the measurement caused by the replacement of the torque pick-up shaft, the experimental data were measured using the same shaft. The natural frequency, ω_n of the free vibration was about 17 (Hz). Figures 12 (a) ~ (c) show the power spectrum of the dynamic change in the frictional moment.

It is clearly seen from Fig. 12 (a) that the power spectrum in the laminar boundary-layer flow region of Separan 5 ppm solutions is distributed in the frequency range of 100 (Hz) ~ 180 (Hz), and that there is a peak around about 48 (Hz). These results are very different from the case of water.

When Toms effect occurs and the variation of c_f in the transition region becomes gentle according to the increase of Re , the amplitude in dynamic change of frictional moment is very small (Figs. 12 (b) ~ (c)). This fact can be interpreted to show that the flow around the rotating disk surface in Separan 5 ppm has not been disturbed so much as in water, and that the viscoelastic effect of the polymer solutions suppresses the disturbance of the flow around the rotating disk surface, because there is little change of the kinematic viscosity.

5. Conclusions

Using a new type torque pick-up shaft, the frictional moments of the rotating disk in water and the polymer solutions were measured at the laminar and transition regions, and the effects of non-Newtonian viscosity and viscoelasticity on c_f were considered. The conclusions were obtained as follows:

- (1) The coefficients of moment, c_f for Ostwald de-Waele power-law fluid in the transition and turbulent regions were analyzed, and confirmed experimentally.
- (2) In the laminar region of the dilute polymer solutions in which the effect of non-Newtonian viscosity can be neglected, the experimental results of c_f agree qualitatively with the theoretical result obtained Y. Mochimaru, et al. (2), that is, c_f increases according to an increase of the viscoelastic effect compared with c_f of Newtonian fluids. However, in the case of the concentrated polymer solutions in which the non-Newtonian viscous and viscoelastic effects occur extremely, there is a case where c_f decreases compared with Tomita's formula (8).

(3) In the transition region of the polymer solutions, the variations of c_f against Re can be classified into two patterns, and these variations become gentle in either case compared with the case of Newtonian fluids.

(4) The dynamic change in the frictional moment acting on the rotating disk in water and the dilute Separan solutions whose kinematic viscosity is numerically almost equal to that of water, was measured, and it was found by comparing the power spectrum that the flow condition around the rotating disk surface very much differs between water and the dilute polymer solutions.

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