Dynamic Behavior of a Load Lifted by a Mobile Construction-type Crane*

[4th Report, The Study on Boom Hoist Motion, etc.]

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This paper deals with the behavior of a load lifted by a mobile crane during boom hoisting, and during boom hoisting and lowering which are executed at the same time.

The equations of motion are given in the form of Lagrange's equation, and the solutions are obtained by Runge-Kutta-Gill method.

In experiments the rope tension was measured with load cells, and displacements of the load and the boom point were measured with cameras and sound-wave.

In conclusion, the theoretical analysis is verified, since the theoretical values are in good accord with the experimental values.

1. Introduction

In the previous reports (1), (2), (3) the authors discussed the behavior of a load lifted by a mobile crane during swinging and load hoisting. This paper deals with the boom hoist motion and compound motions of boom hoist and load lowering.

The analysis of boom hoist motion has never been made in the world except on investigation of boom hoist motion in a pedestal crane by Лавриненко(4). But he does not investigate the behavior of a lifted load by a mobile crane. Neither does he take into consideration the elastic strain and the performance of damping vibrations in crane structures, nor does he refer to the rope tension. In this paper we consider booms, ropes and power lines as elastic bodies with viscous damping.

Usually, in a mobile crane, the power line originates from a diesel engine and branches into three lines of swing, load hoist and boom hoist. To analyze the combined motions, we take into account the energy distribution by each line reduction ratio.

In the actual operation, three motions of swing, load hoist and boom hoist are not executed at the same time, but let us assume that the three motions take place simultaneously. Thus we obtain general equations of motion. Then, by eliminating unrelated parts from the equations, we obtain individual equations for the boom hoist motion and compound motions of boom hoist and load lowering. Furthermore, we make numerical analysis by using the equations of motion.

Under the same conditions as used in the theoretical analysis, experiments are made with the use of a real crane, and the theoretical analysis will check the results.

Notations

A :	Cross-sectional area of rope
E :	Modulus of elasticity
e :	Internal viscous damping coefficient of rope
F :	Dissipation energy due to viscous damping
f :	Energy loss due to friction in bearings of drums or sheaves, etc.
g :	Acceleration of gravity
I :	Geometrical moment of inertia
J :	Moment of inertia
K :	Reduction ratio in power lines
k :	Spring constant
L :	Boom length
l :	Distance from boom point to lifted load
m, n :	Number of parts of rope
P_O :	Tensile force in load hoist rope

P_B :	Tensile force in boom suspension rope
P_n, P_t :	Components of P_O in the normal and tangential direction
p :	Load acting on bearings
Q :	Driving torque or braking torque
R :	Radius of pitch circle in drum or sheave
r :	Radius of bearing in drum or sheave
S :	Displacement of lifted load along load hoist line
T :	Kinetic energy
t :	Time
U :	Elastic strain energy
V :	Potential energy
v :	Velocity
W_O :	Weight of load
W_B :	Weight of boom
α :	Angle of rotation of engine output shaft, clutch, drum, sheave, etc.
β :	Deviation angle between the projection of load hoist line and that of the tangent to the locus of rotation on the horizontal plane.
δ :	Angle of delay of boom point from boom foot
η_B, ζ_B :	Maximum deflection of boom
θ :	Angle between load hoist rope and the vertical (angle of pendulum)
κ :	Viscous damping coefficient of boom
λ :	Elongation of rope
μ :	Coefficient of friction
ν :	Boom angle between boom and the horizontal
ρ :	Viscous damping coefficient of power lines
φ :	Angle of rotation

Suffixes

B :	Boom, boom hoist motion, brake
C :	Crane, clutch
E :	Diesel engine
H :	Load hoist motion
O :	Load
R :	Rope
S :	Swing motion

2. Calculation of Energy

As shown in Fig. 1, let us take as the origin O an intersection point of the central axis of rotation with the horizontal plane including the boom foot. The x and y axes are taken in the horizontal plane and z axis on the central axis of rotation. Furthermore, as the relative co-ordinate, let us take the boom foot as the origin O' and the central axis of the boom as ξ axis. The η and ζ axes are taken perpendicular to ξ axis. The angle φ between x axis and OO' is called "angle of rotation in a crane", the angle ν between ξ axis and the horizontal "boom angle", and the projection of the boom point on the x - y plane during swinging "locus of rotation".

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Fig. 1 Model of dynamic system of boom and lifted load in a mobile crane.

2-1 Energy of Booms

Let us assume that three motions of swing, load hoist and boom hoist take place at the same time. In this case the displacements η and ζ in the η and ζ directions, at distance ξ along the boom of length L from the boom foot, are

$$\eta = \eta_B \sin \frac{\pi}{L} \xi, \quad \eta_\nu = \xi \nu - \eta \quad (1)$$

$$\zeta = \zeta_B \left(1 - \cos \frac{\pi}{2L} \xi \right) \quad (2)$$

$$\zeta_\nu = (a + \xi \cos \nu) \varphi - \zeta$$

where η_B and ζ_B are the maximum deflections of the boom in the η and ζ axial directions, respectively. From Eqs. (1) and (2), when the weight of the boom is W_B , the kinetic energy T_B of the boom is

$$T_B = \frac{W_B}{2gL} \int_0^L (\dot{\eta}_\nu^2 + \dot{\zeta}_\nu^2) d\xi \quad (3)$$

and the potential energy V_B is

$$V_B = \frac{W_B}{L} \int_0^L \left(\xi \sin \nu - \eta_B \sin \frac{\pi}{L} \xi \cos \nu \right) d\xi \quad (4)$$

If the longitudinal modulus of elasticity is E_B and the geometric moments of inertia with respect to η and ζ axes are $I_{B\eta}$ and $I_{B\zeta}$, the elastic strain energy U_B is

$$U_B = \frac{E_B}{2} \int_0^L \left\{ I_{B\zeta} \left(\frac{d^2 \eta}{d\xi^2} \right)^2 + I_{B\eta} \left(\frac{d^2 \zeta}{d\xi^2} \right)^2 \right\} d\xi \quad (5)$$

Denoting by κ_η and κ_ζ the viscous damping coefficients on the η and ζ axes, the viscous damping energy F_B becomes

$$F_B = \frac{1}{2} (\kappa_\zeta \dot{\eta}_B^2 + \kappa_\eta \dot{\zeta}_B^2) \quad (6)$$

2.2 Energy of Ropes

As shown in Fig. 2, let α_i ($i = 1, 2, \dots, n$) be the angles of displacement of the boom point sheaves and the hook block sheaves; J_i the moments of inertia; R_i the radii of pitch circles; r_i the radii of bearings; and μ_i the coefficients of friction. Usually a mobile crane has the same sheaves at the boom point as ones

Fig. 2 Model of the motion of load hoist rope.

at the hook block. Therefore, taking $i = 1, 2, \dots, n$, we can equate as follows:

$$\left. \begin{aligned} &\text{when } i \text{ is even} \\ &R_i = R_p, \quad r_i = r_p, \quad \mu_i = \mu_p \\ &\text{when } i \text{ is odd} \\ &R_i = R_q, \quad r_i = r_q, \quad \mu_i = \mu_q \end{aligned} \right\} \quad (7)$$

Using Eq. (7), when the displacement of the load along the load hoist rope is ξ the elongation $\lambda_{i,i+1}$ of the hoist rope is

$$\left. \begin{aligned} \lambda_{i,1} &= R_{H1} \alpha_{H1} - R_p \alpha_1 \\ \lambda_{i,i+1} &= \left(\frac{R_p}{R_q} \right) \alpha_i - \left(\frac{R_q}{R_p} \right) \alpha_{i-1} - S, \quad (i = 1, 2, \dots, n) \end{aligned} \right\} \quad (8)$$

Here we assume that the upper notations in the parentheses represent the case where i is odd and the lower ones the case where i is even.

There are two kinds of booms in mobile cranes: one is a latticed boom and the other a telescopic box boom. This study applies to the former boom. As shown in Fig. 1, the suspension ropes consist of fixed parts (the number of parts of rope is m_2) and movable parts (the number of parts of rope is m_1), and the boom can be hoisted or lowered by winding the latter parts into or off the drum.

Denoting by a_B the length of the line connecting the boom foot with the gantry point and by Q_B the angle that the line makes with the horizontal, (see Fig. 1), the length l_B of the boom suspension rope is

$$l_B = \sqrt{a_B^2 + L^2 + 2a_B L \cos(\nu + Q_B)} \quad (9)$$

If l_{B10} denotes the length of the boom suspension rope in the initial condition ($\nu = \nu_0$) and R_B the radius of pitch circle of the boom hoist drum, the elongation of the rope is obtained as follows:

$$\lambda_B = -m_1 (l_{B10} - l_B) + R_B \alpha_{B1} \quad (10)$$

Now, let us denote by k_{B1} , k_{B2} the spring constants of fixed ropes (m_1) and movable ropes (m_2); l_{B1} , l_{B2} the lengths of ropes in the initial condition; A_{B1} , A_{B2} the cross-sectional areas, respectively. We obtain

$$\left. \begin{aligned} k_{B1} &= \frac{A_{B1} E_R m_1}{l_{B1} - (R_B / m_1) \alpha_{B1}} \\ k_{B2} &= \frac{1}{l_{B2}} A_{B2} E_R m_2, \quad k_B = \frac{k_{B1} k_{B2}}{k_{B1} + k_{B2}} \end{aligned} \right\} \quad (11)$$

where E_R is the modulus of elasticity in tension. Similarly, the spring constants $k_{i,i+1}$ of the load hoist rope can be represented as follows:

$$\left. \begin{aligned} k_{i1} &= \frac{A_H E_R}{l_H}, \quad k_{i,i+1} = k = \frac{A_H E_R}{l - S} \\ (i &= 1, 2, \dots, n) \end{aligned} \right\} \quad (12)$$

in which A_H is the cross-sectional area of the rope, E_R the modulus of elasticity in tension and l_H the distance from the boom point to the drum.

By using Eqs. (8), (10), (11) and (12), we obtain the kinetic energy T_R , the elastic strain energy U_R and the dissipation energy F_R due to viscous damping, of all the systems of ropes, as follows:

$$T_R = \frac{1}{2} \sum_{i=1}^n J_i \dot{\alpha}_i^2 \quad (13)$$

$$U_R = \frac{1}{2} \left(k_{\pi 1} \lambda_{\pi 1}^2 + \sum_{i=1}^n k \lambda_{i,i+1}^2 + k_B \lambda_B^2 \right) \quad (14)$$

$$F_R = \frac{e}{2} \left(k_{\pi 1} \dot{\lambda}_{\pi 1}^2 + \sum_{i=1}^n k \dot{\lambda}_{i,i+1}^2 + k_B \dot{\lambda}_B^2 \right) \quad (15)$$

where e is the coefficient of internal viscous damping in rope.

2.3 Energy of a lifted load

During swinging the boom point always lags behind the boom foot. The angle of delay δ [see Fig. 1] is, since $\delta \ll 1$,

$$\delta \approx \frac{\zeta_B}{a + L \cos \nu} \quad (16)$$

Hence, when θ is the angle that the load hoist rope makes with the vertical and β is the deviation angle [see Fig. 1], the location of a lifted load is represented as follows:

$$\left. \begin{aligned} x &= (a + L \cos \nu) \cos(\varphi - \delta) \\ &\quad + (l - s) \sin \theta \sin(\varphi - \delta + \beta) \\ y &= (a + L \cos \nu) \sin(\varphi - \delta) \\ &\quad - (l - s) \sin \theta \cos(\varphi - \delta + \beta) \\ z &= L \sin \nu - (l - s) \cos \theta \end{aligned} \right\} \quad (17)$$

where l is the distance from the boom point to the load and S is the displacement of the load along the hoist rope. [see Fig. 1] The kinetic energy of the load is

$$T_O = \frac{W_O}{2g} (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) \quad (18)$$

in which W_O is the weight of the load.

The potential energy V_O is

$$V_O = W_O z \quad (19)$$

2.4 Energy of power lines

Figure 3 shows a model of power lines. Let us denote the angular displacement of power lines as follows:

Fig. 3 Model of power lines in a mobile crane.

Diesel engine; α_0
Boom hoist clutch and boom hoist drum; $\alpha_{BI}, \alpha_{BII}$
Swing clutch and the final reduction of swing power line; α_{SI}, φ

Hoist clutch and hoist drum;

$\alpha_{HI}, \alpha_{HII}$

In addition, let k_{0I}, ρ_{0I} be the spring constant and the viscous damping coefficient, of power lines between diesel engine and each clutch; J_0, J_I, J_{II} the moments of inertia of diesel engine, clutches and drums; K_{0I}, K_{II} the reduction ratios of power lines between diesel engine and each clutch and of power lines between diesel engine and each drum. The suffixes B, S and H show the boom hoist, swing and hoist motion, respectively. The kinetic energy T_C of power lines, the elastic strain energy U_C due to torsion of the axis and the dissipation energy F_C due to viscous damping are obtained as follows:

$$\begin{aligned} T_C &= \frac{1}{2} \{ J_0 (K_{0I} \dot{\alpha}_0)^2 + J_{BI} (K_{BI} \dot{\alpha}_{BI})^2 \\ &\quad + J_{BII} \dot{\alpha}_{BII}^2 + J_{SI} (K_{SI} \dot{\alpha}_{SI})^2 + J_{SH} \dot{\alpha}_{SH}^2 \\ &\quad + J_{HI} (K_{HI} \dot{\alpha}_{HI})^2 + J_{HII} \dot{\alpha}_{HII}^2 \} \quad (20) \end{aligned}$$

$$\begin{aligned} U_C &= \frac{1}{2} \{ k_{B0I} (K_{B0I} \alpha_{B0I})^2 + k_{S0I} (K_{S0I} \alpha_{S0I})^2 \\ &\quad + k_{H0I} (K_{H0I} \alpha_{H0I})^2 \} \quad (21) \end{aligned}$$

$$\begin{aligned} F_C &= \frac{1}{2} \{ \rho_{B0I} (K_{B0I} \dot{\alpha}_{B0I})^2 + \rho_{S0I} (K_{S0I} \dot{\alpha}_{S0I})^2 \\ &\quad + \rho_{H0I} (K_{H0I} \dot{\alpha}_{H0I})^2 \} \quad (22) \end{aligned}$$

$$\left. \begin{aligned} \alpha_{B0I} &= \alpha_0 - \alpha_{BI}, & \alpha_{BI} &= \alpha_{BI} - \alpha_{BII}, \\ \alpha_{S0I} &= \alpha_0 - \alpha_{SI}, & \alpha_{SI} &= \alpha_{SI} - \varphi, \\ \alpha_{H0I} &= \alpha_0 - \alpha_{HI}, & \alpha_{HI} &= \alpha_{HI} - \alpha_{HII}, \\ \alpha_{B0II} &= \alpha_0 - \alpha_{BII}, & K \left\{ \frac{B}{S} \right\}_{0I} &= K \left\{ \frac{B}{S} \right\}_{0I} K \left\{ \frac{B}{S} \right\}_{II} \\ \alpha_{S0II} &= \alpha_0 - \varphi, & & \\ \alpha_{H0II} &= \alpha_0 - \alpha_{HII} & K_{0II}^2 &= K_{B0II}^2 + K_{S0II}^2 + K_{H0II}^2 \end{aligned} \right\} \quad (23)$$

in which the suffixes in the braces are assumed to represent one of them. Generally speaking, in a mobile crane, the values in Eq. (13) are smaller than those in Eq. (20), and can be neglected in the calculation of the total energy of the dynamic systems.

2.5 Total energy of the dynamic systems

The kinetic energy T , the potential energy V , the elastic strain energy U and the dissipation energy F due to viscous damping, of the dynamic systems, from Eqs. (3) ~ (6), (14), (15), (18) ~ (22), can be obtained as follows:

$$\left. \begin{aligned} T &= T_B + T_O + T_C, & V &= V_B + V_O \\ U &= U_B + U_C + U_R, & F &= F_B + F_C + F_R \end{aligned} \right\} \quad (24)$$

2.6 External forces acting on the dynamic systems

The output torque Q_E of the diesel engine, the transmitted torque Q_C of the clutch and the braking torque Q_B of the load hoist brake [Fig. 3] can be obtained from the following empirical formulas:

$$Q_E = D_1 \exp(-D_2 \dot{\alpha}_0) + D_3 \quad (25)$$

$$Q_C \left\{ \frac{B}{S} \right\} = D_4 \left\{ \frac{B}{S} \right\} \left[\frac{t}{t_m} \right] + D_5 \left\{ \frac{B}{S} \right\} \quad (26)$$

$$Q_B = D_6 \left[\frac{t}{t_n} \right] + D_7 \quad (27)$$

where $D_i (i=1, 2, \dots, 7)$ is a constant peculiar to each system and t_m and t_n are the times when the rise of torque ends. The upper notations in brackets mean $t \leq t_m, t_n$ and the lower ones $t > t_m, t_n$.

Now, let us obtain the energy loss due to friction, etc. The torque losses Q_{fB}, Q_{fS} and Q_{fH} due to Coulomb's friction in power lines between a diesel engine and each clutch, taking plus or minus sign into consideration, are assumed to be f_{BI}, f_{SI} and f_{HI} , respectively,

$$\left. \begin{aligned} f_{BI} &= Q_{fB} \text{sgn}(\dot{\alpha}_{BI}), & f_{SI} &= Q_{fS} \text{sgn}(\dot{\alpha}_{SI}) \\ f_{HI} &= Q_{fH} \text{sgn}(\dot{\alpha}_{HI}) \end{aligned} \right\} \quad (28)$$

Let r_B and r_S denote the radii of bearings of the boom hoist drum and the swing clutch, μ_B and μ_S the corresponding coefficients of friction and p_S the vertical load applied on bearings of the swing clutch. Then, the torque losses of the boom hoist drum, of the swing clutch and of the hoist drum are

$$\left. \begin{aligned} f_{BII} &= \mu_B r_{BK} \frac{\lambda_B}{m_1} \operatorname{sgn}(\dot{\alpha}_{BII}) \\ f_{SII} &= \mu_S r_{SpS} \operatorname{sgn}(\dot{\varphi}) \\ f_{HII} &= \mu_{HII} r_{HII} k_{HII,1} (R_{HII} \alpha_{HII} - R_p \alpha_1) \operatorname{sgn}(\dot{\alpha}_{HII}) \end{aligned} \right\} \dots\dots\dots (29)$$

Taking $\mu_p r_p = \mu_q r_q = \mu_r$, we obtain the torque losses of each sheaves $f_{\alpha 1}, f_{\alpha i} (i=2, 3, \dots, n)$:

$$\left. \begin{aligned} f_{\alpha 1} &= \frac{1}{\sqrt{2}} \mu_r \left(\sin \frac{\nu}{2} + \cos \frac{\nu}{2} \right) (k_{H,1} (R_{HII} \alpha_{HII} - R_p \alpha_1) + k (R_p \alpha_1 - R_q \alpha_2 - S)) \operatorname{sgn}(\dot{\alpha}_1) \\ f_{\alpha i} &= \mu_r k \left\{ \left(\frac{R_q}{R_p} \right) (\alpha_{i-1} - \alpha_{i+1}) - 2S \right\} \operatorname{sgn}(\dot{\alpha}_i) \\ &(i=2, 3, \dots, n) \end{aligned} \right\} \dots\dots\dots (30)$$

3. The equation of motion

The equation of motion can be represented in the form of Lagrange's equation as follows:

$$\left. \begin{aligned} \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} + \frac{\partial U}{\partial q_i} + \frac{\partial F}{\partial q_i} &= Q_i' - f_i' \\ q_i &= \alpha_0, \alpha_{BI}, \alpha_{SI}, \alpha_{HI}, \alpha_{BII}, \varphi, \theta, \beta, \nu, \eta_B, \zeta_B, S, \alpha_{HII}, \alpha_1, \alpha_2, \dots, \alpha_n \\ Q_i' &= Q_E', Q' \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^I \end{aligned} \right\} \dots\dots\dots (31)$$

Replacing $Q_E, Q_C, J_0, J_I, J_{II}, \rho, k, f, \alpha_{0I}, \alpha_I, \alpha_{II}$ and φ by the values on the axes of the boom hoist drum, of the hoist drum and of the center of rotation, and furthermore replacing these values by those on the axis of the hoist drum, $Q_E', Q_C', J_0', J_I', \dots$, we obtain

$$\left. \begin{aligned} Q_E' &= Q_E K_{H0II}, \quad Q_C' \begin{Bmatrix} B \\ S \\ H \end{Bmatrix} = Q_C \begin{Bmatrix} B \\ S \\ H \end{Bmatrix} K \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^I \Pi K_H \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}, \quad J_0' = J_0 K_{H0II}^2 \\ J' \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^I &= J \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^I K^2 \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^I \Pi K_H^2 \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}, \quad J' \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^{\Pi} = J \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^{\Pi} K_H^2 \begin{Bmatrix} B \\ S \\ H \end{Bmatrix} \\ \rho' \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^0 &= \rho \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^0 \begin{Bmatrix} I \\ \Pi \end{Bmatrix} K^2 \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^I \Pi K_H^2 \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}, \quad k' \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^0 = k \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^0 \begin{Bmatrix} I \\ \Pi \end{Bmatrix} K^2 \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^I \Pi K_H^2 \begin{Bmatrix} B \\ S \\ H \end{Bmatrix} \\ f' \begin{Bmatrix} B \\ S \\ H \end{Bmatrix} \begin{Bmatrix} I \\ \Pi \end{Bmatrix} &= f \begin{Bmatrix} B \\ S \\ H \end{Bmatrix} \begin{Bmatrix} I \\ \Pi \end{Bmatrix} K \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^I \Pi K_H \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}, \quad \alpha_{0I}' \begin{Bmatrix} B \\ S \\ H \end{Bmatrix} = \alpha_{0I} K_H \begin{Bmatrix} B \\ S \\ H \end{Bmatrix} \\ \alpha' \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^0 \begin{Bmatrix} I \\ \Pi \end{Bmatrix} &= \alpha \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^0 \begin{Bmatrix} I \\ \Pi \end{Bmatrix} K_H \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}, \quad \alpha' \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^I = \alpha \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^I K_H \begin{Bmatrix} B \\ S \\ H \end{Bmatrix} \\ \alpha' \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^{\Pi} &= \alpha \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^{\Pi} K_H \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}, \quad \alpha_{BII}' = \alpha_{BII} K_{HB}, \quad \varphi' = \varphi K_{HS} \\ Q' \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^I &= \mp Q_C' \begin{Bmatrix} B \\ S \\ H \end{Bmatrix} \operatorname{sgn} \left(\alpha \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^I \Pi \right) - f' \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^I, \quad K_H \begin{Bmatrix} B \\ S \\ H \end{Bmatrix} = K_{H0} \begin{Bmatrix} I \\ \Pi \end{Bmatrix} / \left(K \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}^0 \begin{Bmatrix} I \\ \Pi \end{Bmatrix} \right) \end{aligned} \right\} \dots\dots\dots (32)$$

where the sign $-$ in the braces means there are no applicable notations. Here, let us replace $Q_E', Q_C', J_0', \dots$ in Eq. (32) by

$Q_E, Q_C \begin{Bmatrix} B \\ S \\ H \end{Bmatrix}, J_0, \dots$ in Eq. (31), $f_i' = f_{\alpha i} (i=1, 2, \dots, n)$. For convenience of calculation, from Eq. (16), the following equations are introduced,

$$\left. \begin{aligned} \dot{\delta}_{\zeta B} &= \frac{1}{a + L \cos \nu}, \quad \dot{\delta}_{\nu} = \frac{L \zeta_B \sin \nu}{(a + L \cos \nu)^2}, \quad \ddot{\delta}_{\zeta B} = \ddot{\delta}_{\zeta B} = \frac{L \dot{\nu} \sin \nu}{(a + L \cos \nu)^2} \\ \dot{\delta}_{\nu} &= \ddot{\delta}_{\nu} = \frac{L (\ddot{\zeta}_B \sin \nu + \zeta_B \dot{\nu} \cos \nu)}{(a + L \cos \nu)^2} + \frac{2L^2 \zeta_B \dot{\nu} \sin^2 \nu}{(a + L \cos \nu)^3}, \quad \ddot{\delta}_0 = \ddot{\delta}_{\zeta B} \dot{\zeta}_B + \dot{\delta}_{\nu} \dot{\nu} \\ \ddot{\delta} &= \ddot{\delta}_{\zeta B} \ddot{\zeta}_B + \dot{\delta}_{\nu} \ddot{\nu} + \ddot{\delta}_0, \quad f(\delta) = -\frac{\partial T_0}{\partial \delta} \\ f'(\delta) &= \frac{d}{dt} \left(\frac{\partial T_0}{\partial \dot{\delta}} \right) = f'_{\dot{\zeta}} (\dot{\delta}) \dot{\varphi} + f'_{\dot{\beta}} (\dot{\delta}) \dot{\beta} + f'_{\dot{\theta}} (\dot{\delta}) \dot{\theta} + f'_{\dot{\nu}} (\dot{\delta}) \dot{\nu} + f'_{\dot{\zeta}_B} (\dot{\delta}) \dot{\zeta}_B + f'_{\dot{S}} (\dot{\delta}) \dot{S} - f'_0 (\dot{\delta}) \end{aligned} \right\} \dots\dots\dots (33)$$

Substituting Eqs. (24) ~ (30), (32) into Eq. (31), we obtain a non-linear simultaneous differential equation with respect to q_i . The variables of this equation can be taken in a wide range, so it is advantageous to solve it as a problem of initial values by Runge-Kutta-Gill method using a computer. To lower the order of the equation, we put as follows:

$$\left. \begin{aligned} \dot{\alpha}_0 &= \dot{\alpha}_0, \quad \dot{\alpha}_{BI} = \dot{\alpha}_{BI}, \quad \dot{\alpha}_{SI} = \dot{\alpha}_{SI}, \quad \dot{\alpha}_{HI} = \dot{\alpha}_{HI}, \quad \dot{\alpha}_{BII} = \dot{\alpha}_{BII}, \quad \dot{\varphi} = \dot{\varphi}, \\ \dot{\theta} &= \dot{\theta}, \quad \dot{\beta} = \dot{\beta}, \quad \dot{\nu} = \dot{\nu}, \quad \dot{\eta}_B = \dot{\eta}_B, \quad \dot{\zeta}_B = \dot{\zeta}_B, \quad \dot{S} = \dot{S} \\ \dot{\alpha}_{HII} &= \dot{\alpha}_{HII}, \quad \dot{\alpha}_i = \dot{\alpha}_i, \quad (i=1, 2, \dots, n) \quad \dot{\delta} = \dot{\delta} \end{aligned} \right\} \dots\dots\dots (34)$$

To express the equation in term of a matrix, we find the elements of matrix

$$\begin{aligned}
C_{1,1} &= J_0, \quad C_{1,14} = Q_E - k_{B0}\alpha_{B0I} - k_{S0}\alpha_{S0I} - k_{H0}\alpha_{H0I} - \rho_{B0}\hat{\alpha}_{B0I} - \rho_{S0}\hat{\alpha}_{S0I} - \rho_{H0}\hat{\alpha}_{H0I} \\
C_{2,2} &= J_{B1}, \quad C_{2,14} = Q_{B1} + k_{B0}\alpha_{B0I} + \rho_{B0}\hat{\alpha}_{B0I}, \quad C_{3,3} = J_{S1}, \quad C_{3,14} = Q_{S1} + k_{S0}\alpha_{S0I} + \rho_{S0}\hat{\alpha}_{S0I} \\
C_{4,4} &= J_{H1}, \quad C_{4,14} = Q_{H1} + k_{H0}\alpha_{H0I} + \rho_{H0}\hat{\alpha}_{H0I}, \quad C_{5,5} = J_{B1} \\
C_{5,14} &= Q_{B1} - \frac{R_{B1}k_B}{m_1} (\lambda_B + e\hat{\lambda}_B) - \frac{1}{2} \frac{\partial k_B}{\partial \alpha_{B1}} \lambda_{B^2} \\
C_{6,6} &= J_{S1} + \frac{W_B}{g} (a^2 + aL \cos \nu + \frac{L^2}{3} \cos^2 \nu) + f_{\hat{\beta}}'(\hat{\delta}), \quad C_{6,7} = f_{\hat{\theta}}'(\hat{\delta}), \quad C_{6,8} = f_{\hat{\beta}}'(\hat{\delta}) \\
C_{6,9} &= -\frac{W_B}{g} L \varphi \sin \nu \left(\frac{L}{3} \cos \nu + \frac{a}{2} \right) + f_{\hat{\nu}}'(\hat{\delta}) \\
C_{6,11} &= -\frac{W_B}{g} \left[\left(1 - \frac{2}{\pi} \right) a + \left\{ \frac{1}{2} - \frac{2}{\pi} \left(1 - \frac{2}{\pi} \right) \right\} L \cos \nu \right] + f_{\hat{\zeta}_B}'(\hat{\delta}), \quad C_{6,12} = f_{\hat{\zeta}}'(\hat{\delta}) \\
C_{6,14} &= Q_{S1} + \frac{W_B}{g} L \hat{\nu} (\varphi \hat{\nu} \cos \nu + 2 \hat{\varphi} \sin \nu) \left(\frac{L}{3} \cos \nu + \frac{a}{2} \right) + f_0'(\hat{\delta}) \\
C_{7,6} &= -(a + L \cos \nu) \cos \beta \cos \theta, \quad C_{7,7} = I - S \\
C_{7,9} &= -L \left(\sin \beta \cos \theta \sin \nu - \sin \theta \cos \nu - \frac{\zeta_B \cos \beta \cos \theta \sin \nu}{a + L \cos \nu} \right), \quad C_{7,11} = \cos \beta \cos \theta \\
C_{7,14} &= 2 \hat{S} \hat{\theta} + L \hat{\nu}^2 (\sin \theta \sin \nu + \sin \beta \cos \theta \cos \nu) + \{ (a + L \cos \nu) (\hat{\varphi} - \hat{\delta})^2 \sin \beta + (I - S) (\hat{\varphi} - \hat{\delta} + \hat{\beta})^2 \sin \theta - 2 L \hat{\nu} (\hat{\varphi} - \hat{\delta}) \cos \beta \sin \nu \} \cos \theta - g \sin \theta + (a + L \cos \nu) \hat{\delta}_0 \cos \beta \cos \theta \\
C_{8,6} &= (I - S) \sin \theta + (a + L \cos \nu) \sin \beta, \quad C_{8,8} = (I - S) \sin \theta \\
C_{8,9} &= -L \sin \nu \left[\cos \beta + \frac{\zeta_B}{a + L \cos \nu} \left\{ \sin \beta + \frac{(I - S) \sin \theta}{a + L \cos \nu} \right\} \right], \quad C_{8,11} = -\sin \beta - \frac{(I - S) \sin \theta}{a + L \cos \nu} \\
C_{8,14} &= 2 (\hat{\varphi} - \hat{\delta} + \hat{\beta}) \{ \hat{S} \sin \theta - (I - S) \hat{\theta} \cos \theta \} + 2 L (\hat{\varphi} - \hat{\delta}) \hat{\nu} \sin \beta \sin \nu + (a + L \cos \nu) (\hat{\varphi} - \hat{\delta})^2 \times \sin \beta + L \hat{\nu}^2 \cos \beta \cos \nu + \{ (a + L \cos \nu) \sin \beta + (I - S) \sin \theta \} \hat{\delta}_0 \\
C_{9,6} &= -L \left\{ \frac{W_B}{g} \left(\frac{a}{2} + \frac{L}{3} \cos \nu \right) \varphi + \frac{W_0}{g} (I - S) \cos \beta \sin \theta \right\} \sin \nu - \hat{\delta}_i f_{\hat{\beta}}'(\hat{\delta}) \\
C_{9,7} &= -\frac{W_0}{g} L (I - S) (\sin \beta \cos \theta \sin \nu - \sin \theta \cos \nu) - \hat{\delta} f_{\hat{\theta}}'(\hat{\delta}) \\
C_{9,8} &= -\frac{W_0}{g} L (I - S) \cos \beta \sin \theta \sin \nu - \hat{\delta}_i f_{\hat{\beta}}'(\hat{\delta}) \\
C_{9,9} &= L^2 \left[\frac{W_B}{3g} (1 + \varphi^2 \sin^2 \nu) + \frac{W_0}{g} \left\{ 1 + \frac{\zeta_B (I - S) \cos \beta \sin \theta \sin^2 \nu}{(a + L \cos \nu)^2} \right\} \right] - \hat{\delta}_i f_{\hat{\nu}}'(\hat{\delta}) \\
C_{9,10} &= -\frac{L W_B}{\pi g} \\
C_{9,11} &= -\frac{W_B}{2g} L \varphi \sin \nu \left\{ \frac{4}{\pi} \left(1 - \frac{2}{\pi} \right) - 1 \right\} + \frac{W_0}{g} \frac{L (I - S) \cos \beta \sin \theta \sin \nu}{a + L \cos \nu} - \hat{\delta}_i f_{\hat{\zeta}_B}'(\hat{\delta}) \\
C_{9,12} &= \frac{L W_0}{g} (\sin \beta \sin \theta \sin \nu + \cos \theta \cos \nu) - \hat{\delta}_i f_{\hat{\zeta}}'(\hat{\delta}) \\
C_{9,14} &= -\frac{W_B}{g} \left\{ \frac{L^2}{3} \varphi \hat{\nu} \sin \nu (\varphi \hat{\nu} \cos \nu + 2 \hat{\varphi} \sin \nu) + g \left(\frac{L}{2} \cos \nu + \frac{2}{\pi} \eta_B \sin \nu \right) \right\} \\
&\quad - \frac{L W_0}{g} \{ 2 \hat{S} \{ (\hat{\varphi} - \hat{\delta} + \hat{\beta}) \cos \beta \sin \theta \sin \nu + \hat{\theta} (\sin \beta \cos \theta \cos \nu - \sin \theta \cos \nu) \} \\
&\quad + (I - S) \{ (\hat{\varphi} - \hat{\delta} + \hat{\beta})^2 \sin \beta \sin \theta \sin \nu + \hat{\theta}^2 (\sin \beta \sin \theta \sin \nu + \cos \theta \cos \nu) \} \\
&\quad - 2 (I - S) (\hat{\varphi} - \hat{\delta} + \hat{\beta}) \hat{\theta} \cos \beta \cos \theta \sin \nu + (a + L \cos \nu) (\hat{\varphi} - \hat{\delta})^2 \sin \nu + g \cos \nu \\
&\quad + (I - S) \hat{\delta}_0 \cos \beta \sin \theta \sin \nu \} - \hat{\delta}_i f_0'(\hat{\delta}) - \frac{a B k_B L \sin(\nu + \Omega_B)}{\sqrt{\alpha_B^2 + L^2 + 2 a_B L \cos(\nu + \Omega_B)}} (\lambda_B + e \hat{\lambda}_B) \\
C_{10,9} &= -\frac{L W_B}{g \pi}, \quad C_{10,10} = \frac{W_B}{2g}, \quad C_{10,14} = -\frac{\pi^4 E_B I_{B\zeta}}{2 L^3} \eta_B - \kappa_{\zeta} \hat{\eta}_B + \frac{2}{\pi} W_B \cos \nu \\
C_{11,6} &= -\frac{W_B}{g} \left[a \left(1 - \frac{2}{\pi} \right) - \frac{L}{2} \cos \nu \left\{ \frac{4}{\pi} \left(1 - \frac{2}{\pi} \right) - 1 \right\} \right] - \hat{\delta} \zeta_B f_{\hat{\beta}}'(\hat{\delta}), \quad C_{11,7} = -\hat{\delta} \zeta_B f_{\hat{\theta}}'(\hat{\delta}) \\
C_{11,8} &= -\hat{\delta} \zeta_B f_{\hat{\beta}}'(\hat{\delta}), \quad C_{11,9} = -\frac{L W_B}{2g} \varphi \sin \nu \left\{ \frac{4}{\pi} \left(1 - \frac{2}{\pi} \right) - 1 \right\} - \hat{\delta} \zeta_B f_{\hat{\nu}}'(\hat{\delta}) \\
C_{11,11} &= \frac{W_B}{g} \left(\frac{3}{2} - \frac{4}{\pi} \right) - \hat{\delta} \zeta_B f_{\hat{\zeta}_B}'(\hat{\delta}), \quad C_{11,12} = -\hat{\delta} \zeta_B f_{\hat{\zeta}}'(\hat{\delta}) \\
C_{11,14} &= \frac{L W_B}{2g} \hat{\nu} (\varphi \hat{\nu} \cos \nu + 2 \hat{\varphi} \sin \nu) \left\{ \frac{4}{\pi} \left(1 - \frac{2}{\pi} \right) - 1 \right\} - \frac{\pi^4 E_B I_{B\eta}}{32 L^3} \zeta_B - \kappa_{\eta} \hat{\zeta}_B - \hat{\delta} \zeta_B f_0'(\hat{\delta})
\end{aligned}$$

$$\begin{aligned}
C_{12,6} &= -(a + L \cos \nu) \cos \beta \sin \theta \\
C_{12,9} &= -L \left(\sin \beta \sin \theta \sin \nu + \cos \theta \cos \nu - \frac{\zeta_B \cos \beta \sin \theta \sin \nu}{a + L \cos \nu} \right), \quad C_{12,11} = \cos \beta \sin \theta, \quad C_{12,12} = -1 \\
C_{12,14} &= (I - S) \{ \hat{\theta}^2 + (\hat{\varphi} - \hat{\delta} + \hat{\beta})^2 \sin^2 \theta \} + (a + L \cos \nu) (\hat{\varphi} - \hat{\delta})^2 \sin \beta \sin \theta \\
&\quad + L \hat{\nu}^2 (\sin \beta \sin \theta \cos \nu - \cos \theta \sin \nu) - 2 L \hat{\nu} (\hat{\varphi} - \hat{\delta}) \cos \beta \sin \theta \sin \nu + g \cos \theta \\
&\quad - (a + L \cos \nu) \hat{\delta}_0 \cos \beta \sin \theta - \frac{g}{W_0} \left[\sum_{i=1}^n \left\{ k(\lambda_{i,i+1} + e \hat{\lambda}_{i,i+1}) - \frac{1}{2} \frac{\partial k}{\partial S} \lambda_{i,i+1}^2 \right\} \right], \quad C_{13,13} = J_{H1} \\
C_{13,14} &= Q_{H1} - k_{H1} R_{H1} (\lambda_{H1} + e \hat{\lambda}_{H1})
\end{aligned} \tag{35}$$

from which we obtain the following matrix

$$\begin{pmatrix} C_{1,1} & 0 & 0 & 0 & 0 \\ 0 & C_{2,2} & 0 & 0 & 0 \\ 0 & 0 & C_{3,3} & 0 & 0 \\ 0 & 0 & 0 & C_{4,4} & 0 \\ 0 & 0 & 0 & 0 & C_{5,5} \end{pmatrix} \begin{pmatrix} \dot{\alpha}_0 \\ \dot{\alpha}_{B1} \\ \dot{\alpha}_{S1} \\ \dot{\alpha}_{H1} \\ \dot{\alpha}_{B1} \end{pmatrix} = \begin{pmatrix} C_{1,14} \\ C_{2,14} \\ C_{3,14} \\ C_{4,14} \\ C_{5,14} \end{pmatrix}$$

$$\begin{pmatrix} C_{6,6} & C_{6,7} & C_{6,8} & C_{6,9} & 0 & C_{6,11} & C_{6,12} & 0 \\ C_{7,6} & C_{7,7} & 0 & C_{7,9} & 0 & C_{7,11} & 0 & 0 \\ C_{8,6} & 0 & C_{8,8} & C_{8,9} & 0 & C_{8,11} & 0 & 0 \\ C_{9,6} & C_{9,7} & C_{9,8} & C_{9,9} & C_{9,10} & C_{9,11} & C_{9,12} & 0 \\ 0 & 0 & 0 & C_{10,9} & C_{10,10} & 0 & 0 & 0 \\ C_{11,6} & C_{11,7} & C_{11,8} & C_{11,9} & 0 & C_{11,11} & C_{11,12} & 0 \\ C_{12,6} & 0 & 0 & C_{12,9} & 0 & C_{12,11} & C_{12,12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{13,13} \end{pmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\beta} \\ \dot{\nu} \\ \dot{\eta}_B \\ \dot{\xi}_B \\ \dot{S} \\ \dot{\alpha}_{H1} \end{pmatrix} = \begin{pmatrix} C_{6,14} \\ C_{7,14} \\ C_{8,14} \\ C_{9,14} \\ C_{10,14} \\ C_{11,14} \\ C_{12,14} \\ C_{13,14} \end{pmatrix} \tag{36}$$

We proceed in the same way with the equation of motion of the load hoist rope. The elements of matrix are

$$\begin{aligned}
a_{1,1} &= - \left(1 + \frac{k}{k_{H1}} \right) e R_p^2, \quad a_{1,2} = \frac{k}{k_{H1}} e R_p R_q, \\
a_{1,n+1} &= -e R_{H1} R_p \hat{\alpha}_{H1} - \lambda_{H1} \{ R_p - \mu r \sin \nu \operatorname{sgn}(\hat{\alpha}_1) \} + \frac{k}{k_{H1}} [\lambda_{1,2} \{ R_p + \mu r \operatorname{sgn}(\hat{\alpha}_1) \} - e R_p \hat{S}] \\
a_{i,i-1} &= e R_p R_q, \quad a_{i,i+1} = e R_p R_q, \quad (i \neq n), \quad a_{i,i} = -2e \left(\frac{R_q^2}{R_p^2} \right) \\
a_{i,n+1} &= 2 \left(\frac{R_q^2}{R_p^2} \right) \alpha_i + \mu r \left\{ \left(\frac{R_q}{R_p} \right) (\alpha_{i-1} - \alpha_{i+1}) - 2S \right\} \operatorname{sgn}(\hat{\alpha}_i) - R_p R_q (\alpha_{i-1} + \alpha_{i+1}), \quad (i=2, 3, \dots, n)
\end{aligned} \tag{37}$$

from which the following matrix is obtained.

$$\begin{pmatrix} a_{1,1} & a_{1,2} & & & \\ a_{2,1} & a_{2,2} & a_{2,3} & & \\ & a_{3,2} & a_{3,3} & a_{3,4} & \\ & & & & a_{n-1,n-2} & a_{n-1,n-1} & a_{n-1,n} \\ 0 & & & & & a_{n,n-1} & a_{n,n} \end{pmatrix} \begin{pmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \\ \hat{\alpha}_3 \\ \vdots \\ \hat{\alpha}_{n-1} \\ \hat{\alpha}_n \end{pmatrix} = \begin{pmatrix} a_{1,n+1} \\ a_{2,n+1} \\ a_{3,n+1} \\ \vdots \\ a_{n-1,n+1} \\ a_{n,n+1} \end{pmatrix} \tag{38}$$

Equations (36) and (38) can be solved by using an "amendment sweep method⁽⁵⁾" by dividing the time of motion into intervals roughly or finely. The tensile force P_0 acting on the load hoist rope, the normal and tangential components P_n, P_t of P_0 to the locus of rotation and the tensile force P_B acting on the boom hoist rope are

$$\begin{aligned}
P_0 &= k \sum_{i=1}^n (\lambda_{i,i+1} + e \hat{\lambda}_{i,i+1}) \\
P_n &= P_0 \sin \theta \sin \beta, \quad P_t = P_0 \sin \theta \cos \beta \\
P_B &= k_B (\lambda_B + e \hat{\lambda}_B)
\end{aligned} \tag{39}$$

Equation (36) is obtained by assuming that three motions of swing, load hoist and boom hoist are done at the same time. If we express Eq. (36) as $C(i, i+1)$ we can obtain the equation of motion for simple boom hoist eliminating the rows and columns of $i=3, 4, 6, 8, 11, 13$, and the equation for compound motions of boom hoist and load lowering by eliminating the rows and columns of $i=3, 6, 8, 11$. For the case of a steady state after engagement of the clutch, the equation of motion for boom hoist can be obtained by eliminating the rows and columns of $i=2, 3, 4, 6, 8, 11, 13$ and by replacing $C_{5,5}$ and $C_{5,14}$ by

$$\left. \begin{aligned} C_{3,5} &= J_{B1} + J_{B2} \\ C_{5,14} &= -R_{B2}k_B(\lambda_B + e\lambda_B) - \frac{1}{2} \frac{\partial k_B}{\partial \alpha_{B2}} \lambda_B^2 \\ &\quad - f_{B1} - f_{B2} - \rho_{B0} \operatorname{sgn}(\dot{\alpha}_{B02}) + k_{B0}\alpha_{B02} \end{aligned} \right\} \dots (40)$$

4. Experiment

4.1 Testing conditions and method

A testing mobile crane was installed on the horizontal plane paved with asphalt concrete. The boom length was 15.24m and the boom angle was $53^\circ \sim 66^\circ$ (the working radius is 10.00 ~ 7.00m). The crane was operated in the dead calm under such conditions that the weight of the load was 1150kg, the number of parts of load hoist line was 3 and the governor lever of the diesel engine was held in a constant position. Table 1 shows the working conditions and the specifications of the testing crane. The load hoist clutch and brake were of the drum type using a friction band, the hoist clutch was of the internal expanding type and the brake was of the external contracting type. The operation system was direct acting hydraulic control. Under these conditions, experiments were made with the case of boom hoist motion and with the case of compound motions of boom hoist and load lowering.

The locations of the lifted load and the boom point were measured by two methods; one by taking photographs and the other by measuring time for sound waves started from the lifted load (or boom point) to reach a fixed point. The recordings of the boom angle, the hydraulic pressure during operation of clutch and brake, the number of rotations of each drum, etc. were successfully made on an electromagnetic oscillograph.

4.2 Results of experiment

Figure 4 indicates the values of P_B , P_O and ν during boom hoisting. When the clutch is engaged, a vibration with about 0.1 sec. period appears first. This vibration depends on the longitudinal vibration of load hoist rope and disappears within about 1 sec., partly due to viscous damping of ropes and partly due to friction resistance of rotating parts of power lines, drums and sheaves. Then, a vibration with 0.7 ~ 0.8 sec. period appears. This phenomenon is also seen in P_B and ν . This vibration is dependent upon the longitudinal vibration of boom suspension rope and $\dot{\nu}$, P_B and P_O become approximately the same in phase. Generally speaking, before this vibration is damped out, we have finished the boom hoist operation. An approximate expression of direct current P_B' of P_B is

$$P_B' = \frac{\sqrt{a_B^2 + L^2 + 2a_B L \cos(\nu + \Omega_B)}}{a_B \sin(\nu + \Omega_B)} \left[P_O \left\{ \cos \nu \right. \right. \\ \left. \left. - \frac{1}{n} \frac{a_H \sin(\nu + \Omega_H)}{\sqrt{a_H^2 + L^2 + 2a_H L \cos(\nu + \Omega_R)}} \right\} \right. \\ \left. + \frac{1}{2} W_B \cos \nu \right] \dots (41)$$

Fig. 4 P_O , P_B and ν in the case of boom hoist motion.

As ν increases and the length of boom suspension rope decreases, the period of each variable decreases, but, in the ordinary crane operation, the reduction is less than one percent and can be neglected.

Figure 5 shows the values of P_B , P_O and ν during boom lowering. Figures 6 and 7 represent the locus $G_B(x_B, z_B)$ of the

Table 1. Specifications and working conditions of a mobile crane

Type of crane		Small sized mobile crane	
boom	length	L m	15.24
	weight	W_B kg	1014
	modulus of longitudinal elasticity	E_B kg/cm ²	2.1×10^6
	viscous damping coefficient	ρ_B kg·s/m	250
	geometrical moment of inertia	I_{B1} cm ⁴	3.47×10^4
		I_{B2} cm ⁴	1.91×10^4
lifted load	weight	W_O kg	1150
	working radius	x_O m	7.00 ~ 10.00
	(boom angle)	ν deg	66 ~ 53
number of parts of load hoist rope		n	3
dia-meter of rope	load hoist rope	d_H mm	14
	boom suspension rope		
	m1 number of parts in line	d_{B1} mm	12
	m2 number of parts in line	d_{B2} mm	20
modulus of elasticity of rope in tension		E_R kg/cm ²	1.05×10^6
internal viscous damping coefficient of rope		e s	0.006
spring constant	load hoist rope	k_H kg/cm	2400 ~ 2600
	boom suspension rope		
	total	k_B kg/cm	1650 ~ 1700
	m1 number of parts in line	k_{B1} kg/cm	7200 ~ 8200
	m2 number of parts in line	k_{B2} kg/cm	2140

Fig. 5 P_O , P_B and ν in the case of boom lowering motion.

boom point and the locus $G_O(x_O, z_O)$ of the lifted load. In general, the boom lowering line speed is small in comparison with the boom hoist line speed, and in this testing crane, the speed ratio of lowering to hoist is about 0.5. For this reason, in the case of boom hoist motion, the load goes up without moving ahead the boom point, and in the case of boom lowering motion, it goes down with pendulum motion.

Figure 8 indicates the angle of pendulum θ and the velocity v_O of the lifted load in boom lowering motion. Waves with $0.7 \sim 0.8$ sec. period which are remarkably seen in the initial motion take place by the vibration of the boom point due to a change in boom angle ν . Let us investigate the vibration with about 7 sec. period appearing in θ . θ can be obtained as the solution of the following equation containing the variables θ and ν .

$$\ddot{\theta} + \frac{1}{l} \{g + L(\ddot{\nu} \cos \nu - \dot{\nu}^2 \sin \nu)\} \theta - \frac{L}{l} (\ddot{\nu} \sin \nu + \dot{\nu}^2 \cos \nu) = 0 \quad (42)$$

where $\theta \ll 1$. In a mobile crane, since $g \gg L(\ddot{\nu} \cos \nu - \dot{\nu}^2 \sin \nu)$, θ is of the same period $2\pi \sqrt{l/g}$ as a simple pendulum. When θ is zero, v_O takes a maximum in a simple pendulum, while in

Fig. 8 v_O is a minimum, $0.10 \sim 0.15$ (m/s), when it is 7 sec. since the beginning of the motion. At that time, the horizontal component of v_O is approximately zero and v_O consists only of the vertical component corresponding to the boom point vertical displacement.

Let us now discuss compound motions of boom hoist and load lowering. Figure 9 shows G_B and G_O in the case of hoist and load lowering executed at the same time. In the experiment the clutch was operated with on-off control in order to be able to repeat the same performance. With half clutch operation, we could move the load in the horizontal direction.

Figure 10 shows G_B and G_O in the case of boom lowering

Fig. 8 θ and v_O in the case of boom lowering motion.

Fig. 6 Loci of $G_B(x_B, z_B)$ and $G_O(x_O, z_O)$ in the case of boom hoist motion.

Fig. 9 Loci of $G_B(x_B, z_B)$ and $G_O(x_O, z_O)$ in the case of boom hoist and load lowering motion.

Fig. 7 Loci of $G_B(x_B, z_B)$ and $G_O(x_O, z_O)$ in the case of boom lowering motion.

Fig. 10 Loci of $G_B(x_B, z_B)$ and $G_O(x_O, z_O)$ in the case of boom lowering and load hoist motion.

and load hoist executed at the same time. The load hoist speed is large in comparison with the boom point lowering speed, so the load is lifted up.

5. Conclusions

A mobile crane, unlike a pedestal crane or an overhead travelling crane, is a man-to-machine system which depends upon the skill and feeling of the operator. In addition, the working conditions are also complicated. Therefore, the experiment was carefully made by using a real crane. For theoretical analysis, the author obtained the equation of motion, in the case of boom hoist motion, containing $(9+n)$ variables, i.e., $\alpha_0, \alpha_{BI}, \alpha_{HI}, \alpha_{BH}, \theta, \nu, \zeta_B, S, \alpha_{HI}, \alpha_1, \dots, \alpha_n$ [see Eqs. (36) and (38)]. The theoretical computations agreed well with the experimental results, and the theoretical analysis was verified.

In general, the behavior of a load and the tensile force on a load hoist rope, during boom hoisting or during boom hoisting and load lowering executed at the same, depends upon elastic vibrations of the load hoist rope and boom suspension

ropes, inertia force acting at the boom point, and pendulum motion owing to the length of the load hoist rope.

Thus, we have analyzed theoretically and experimentally the behavior of the load lifted by a mobile crane during swinging, load hoisting and boom hoisting (from 1st to 4th reports). On the basis of this analysis, in the next report we shall try to investigate "dynamic load factors" and "tipping stability" which are established as empirical rules in various national codes of mobile cranes in many countries.

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