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Vibration and Dynamic Instability of a Cylindrical shell Conveying a Compressible Fluid.*

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This paper deals with the vibration and the stability of a cylindrical shell conveying a compressible fluid taking into account an effect of the boundary layer of the fluid.

Mizoguchi's shell theory is used and the compressible fluid is treated as an ideal gas.

The frequencies of shell vibrations are obtained for several flow speeds ranging from 10m/sec to Mach 3 under two kinds of the mean static pressures of the fluid.

From the results with the infinitely long pipeline the cases of periodical support may be obtained by means of some simple drawings as in the first author's previous paper.

The instability is discussed based on an energy consideration of the shell and fluid.

1. Introduction

There are many papers related to the vibrations of pipes conveying a fluid, most of which, however are treated as beams conveying an incompressible fluid.^{<1>}

The papers treated upon cylindrical shells are those of Mizoguchi,^{<2>} Paidoussis^{<3>} and Bolotin,^{<4>} the first and second of which are for incompressible fluids and the third gives only a general course of the problems.

In this paper, the vibrational characteristics and the stability of a cylindrical shell conveying a compressible fluid are discussed taking into account an effect of the boundary layer of the fluid.

For a rigorous solution of a thin shell, Mizoguchi's shell theory is used and the compressible fluid is treated as an ideal gas.

To grasp the influence of the flowing fluid precisely the effect of the surrounding media is neglected.

The compressible fluid is treated as an ideal gas flowing in a turbulent state, the velocity distribution of which is assumed such that :

(a) The flow speed is constant except for the inside of the boundary layer of infinitesimal thickness.

(b) The flow speed is constant everywhere neglecting the effect of the boundary layer.

The characteristics of vibration under these two conditions are similar on the whole.

A remarkable difference between them is that in the case (a) no such instabilities as divergence nor flutter appear whereas in the case (b) these will occur under certain conditions.

2. Method of solution

Employing the cylindrical coordinates x, θ, r , the equations of motion of a compressible fluid are given as follows :

$$\left. \begin{aligned} \frac{\partial^2 W_f}{\partial t^2} + c \frac{\partial^2 W_f}{\partial x \partial t} &= - \frac{1}{\rho_0} \frac{\partial q}{\partial r} \\ \frac{\partial^2 U_f}{\partial t^2} + c \frac{\partial^2 U_f}{\partial x \partial t} &= - \frac{1}{\rho_0} \frac{\partial q}{\partial x} \\ \frac{\partial^2 V_f}{\partial t^2} + c \frac{\partial^2 V_f}{\partial x \partial t} &= - \frac{1}{\rho_0} \frac{\partial q}{r \partial \theta} \end{aligned} \right\} (1)$$

where W_f, U_f , and V_f are the infinitesimal perturbed displacements of the fluid in the radial, axial and the circumferential directions respectively, c is the constant flow speed in the positive direction of x , ρ_0 is the mean density of the fluid, and q is the perturbed pressure of the fluid.

Denoting by ρ' the variable density of the fluid, the equation of continuity is expressed as

$$\frac{\partial \rho'}{\partial t} + \frac{1}{r} \frac{\partial (\rho' r \frac{\partial W_f}{\partial t})}{\partial r} + \frac{\partial (\rho' \frac{\partial V_f}{\partial t})}{r \partial \theta} + \frac{\partial \{\rho' (c + \frac{\partial U_f}{\partial t})\}}{\partial x} = 0 \quad (2)$$

Putting $\rho' = \rho_0 (1 + \delta)$ with an infinitesimal quantity δ and substituting this expression into Eq. (2), we obtain the following equation (3) after rejecting the terms of higher infinitesimal

$$\rho_0 \frac{\partial \delta}{\partial t} + \frac{\rho_0}{r} \frac{\partial^2 (r W_f)}{\partial r \partial t} + \frac{\rho_0}{r} \frac{\partial^2 V_f}{\partial \theta \partial t} + \rho_0 \frac{\partial^2 U_f}{\partial x \partial t} + \rho_0 c \frac{\partial \delta}{\partial x} = 0 \quad (3)$$

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Denoting by p_o the mean fluid pressure, the fluid pressure p is expressed by $p_o + q$ and is related with ρ' by an equation

$$p\rho'^{-\gamma} = \text{const} \quad (4)$$

provided the fluid obeys the adiabatic law.

Substituting the equations $p = p_o + q$ and $\rho' = \rho_o(1+\delta)$ into Eq. (4) yields

$$p_o(1+\delta)^\gamma = p_o + q,$$

from which upon expanding the left-hand side and neglecting the terms of higher infinitesimal, we have

$$q = p_o \gamma \delta \quad (5)$$

Eq. (5) may be rewritten with the aid of the acoustic velocity c_o as

$$\delta = \frac{q}{p_o \gamma} = \frac{q}{\rho_o c_o^2} \quad (6)$$

where

$$c_o^2 = \frac{\gamma p_o}{\rho_o} \quad (7)$$

Substituting Eq. (7) into Eq. (3) leads to

$$\frac{\partial}{\partial t} \left[\frac{1}{r} \frac{\partial(rW_f)}{\partial r} + \frac{1}{r} \frac{\partial V_f}{\partial \theta} + \frac{\partial U_f}{\partial x} \right] + \frac{1}{\rho_o c_o^2} \frac{\partial q}{\partial t} + \frac{c}{\rho_o c_o^2} \frac{\partial q}{\partial x} = 0 \quad (8)$$

The equations of motion of a cylindrical shell are the same with those in the case of incompressible fluid^{<2>} and the pressure p_t exerted on the shell surface is expressed by

$$p_t = q + \frac{\bar{p}_o}{a} \left(\frac{\partial^2 W}{\partial \theta^2} + W \right) \quad (9)$$

where $\bar{p}_o$ is the mean gauge fluid pressure. If we put

$$\left. \begin{aligned} q &= f(r) \cos n\theta \sin(\lambda x + \omega t) \\ W_f &= \alpha(r) \cos n\theta \sin(\lambda x + \omega t) \\ V_f &= \beta(r) \sin n\theta \sin(\lambda x + \omega t) \\ U_f &= \gamma(r) \cos n\theta \cos(\lambda x + \omega t) \\ W &= a_{\mu n} \cos n\theta \sin(\lambda x + \omega t) \end{aligned} \right\} \quad (10)$$

as in the author's previous paper^{<2>} and substitute them into Eqs. (1), we have

$$\left. \begin{aligned} \alpha(r) &= \frac{df/dr}{\rho_o \omega [\omega + c\lambda]} \\ \beta(r) &= -\frac{nf/r}{\rho_o \omega [\omega + c\lambda]} \\ \gamma(r) &= \frac{\lambda f}{\rho_o \omega [\omega + c\lambda]} \end{aligned} \right\} \quad (11)$$

Substitute Eqs. (10) into Eq. (8) through Eqs. (11), we have

$$\frac{d^2 f}{dr^2} + \frac{1}{r} \frac{df}{dr} - \left[\lambda^2 - \frac{(\omega + c\lambda)^2}{c_o^2} + \frac{n^2}{r^2} \right] f = 0 \quad (12)$$

$$2.1 \text{ Case of } \lambda^2 - (\omega + c\lambda)^2 / c_o^2 \equiv \xi^2 > 0 \quad (13)$$

$$\text{Put } \xi = r\xi, \quad \bar{\xi} = a\xi \quad (14)$$

then Eq. (12) is reduced to

$$\frac{d^2 f}{d\xi^2} + \frac{1}{\xi} \frac{df}{d\xi} - \left[1 + \frac{n^2}{\xi^2} \right] f = 0 \quad (15)$$

from which we have

$$f = k I_n(\xi) \quad (16)$$

where k is an integration constant and I_n is the modified Bessel function of order n .

Substitution of the last equation of Eqs. (10) in Eq. (9) yields

$$p_t = [k I_n(\bar{\xi}) - a_{\mu n} \frac{\bar{p}_o}{a} (n^2 - 1)] \cos n\theta \times \sin(\lambda x + \omega t) \quad (17)$$

Substitute Eq. (17) and the last equation of Eqs. (10) into the fundamental differential equation of a cylindrical shell acted upon by p_t only,^{<2>} we have

$$a_{\mu n} = \frac{a^4 / D (h/a)^2 I_n(\bar{\xi}) [72(1-\nu)I' - 6K'\Theta^2 + \Theta^4] k}{[72(1-\nu)\{I+P(n^2-1)I'\} - \{72(1-\nu)J + 6P(n^2-1)K'\}\Theta^2 + \{6K+P(n^2-1)\}\Theta^4 - \Theta^6]} \quad (18)$$

where $\Theta, P, I, I', J, K, K'$ are the same quantities with those defined in the author's previous paper.^{<2>}

The condition that the fluid and the shell surface remain always in contact is different between the cases (a) and (b).

2.1.a

The condition may be expressed by

$$\frac{\partial W}{\partial t} = \left(\frac{\partial W_f}{\partial t} \right)_{r=a} \text{ or } W = (W_f)_{r=a} \quad (19)$$

provided the thickness of the boundary layer is very small.

In this case the fluid particle directly under the boundary layer moves at a constant speed c , hence the boundary layer at a new position of the removed fluid particle will get a different thickness, thus the fluid will run more freely than in the case (b) regulating the thickness of the boundary layer. In an actual case such a state will take place more frequently than in the case (b) so long as we restrict the elastic theory within infinitesimal deformations.

From the second and the last equation of Eqs. (10), Eqs. (11), Eq. (16) and Eq. (19), we have

$$a_{\mu n} = \alpha(a) = \frac{k \xi I'_n(\bar{\xi})}{\rho_o \omega [\omega + c\lambda]} = \frac{k \bar{\xi} I'_n(\bar{\xi})}{\rho_o \omega [a\omega + c\mu]} \quad (20)$$

where $I'_n(\bar{\xi}) = dI_n(\xi)/d\xi_{\xi=\bar{\xi}}$ and $\mu = a\lambda$.

From Eq. (18) and Eq. (20), we have the following equation as the frequency equation:

$$\left. \begin{aligned} & (1+\phi)\Theta^6 - \theta \left\{ K + \frac{P}{6}(n^2-1) + K'\phi \right\} \Theta^4 + 72(1-\nu) \\ & \times \left\{ J + \frac{P}{12(1-\nu)}(n^2-1)K' + I'\phi \right\} \Theta^2 - 72(1-\nu) \\ & \times \{ I + P(n^2-1)I' \} + \mu \phi K \Theta \{ \Theta^4 - 6K'\Theta^2 \\ & + 72(1-\nu)I' \} = 0 \end{aligned} \right\} \quad (21)$$

where $\phi = (a/h)(\rho_o/\rho)I_n(\bar{\xi})/\bar{\xi}I'_n(\bar{\xi})$

$$\kappa^2 = \rho h^3 c^2 / D$$

2.1.b

In this case a fluid particle in contact with the shell surface moves with a velocity c maintaining the condition of contact. Hence the fluid cannot move so freely as in the case (a). In practice

this state will occur when the amplitude of vibration grows somewhat significant and gets beyond the ability of the fluid to regulate the thickness of the boundary layer as in the case (a).

For the preservation of contact condition, the velocity component of the fluid particle in the direction perpendicular to the shell surface $(\partial W/\partial t)_{r=a} = c \partial W/\partial x$ should be equal to $\partial W/\partial t$, i.e.

$$\left(\frac{\partial W}{\partial t}\right)_{r=a} = \frac{\partial W}{\partial t} + c \frac{\partial W}{\partial x} \quad (22)$$

By means of the same procedure as in the case (a) through Eq. (22), we have

$$\alpha_{\mu n} = \frac{\omega}{\omega + c\lambda} \alpha(a) = \frac{k\xi I'_n(\bar{\zeta})}{\rho_o[\omega + c\lambda]^2} = \frac{ka\bar{\zeta} I'_n(\bar{\zeta})}{\rho_o[\omega + c\mu]^2} \quad (23)$$

from which we obtain through Eq. (18) the following frequency equation:

$$\begin{aligned} & (1+\phi)\Theta^6 - 6\{K + \frac{P}{6}(n^2-1) + K'\phi - \frac{\kappa^2\mu^2}{6}\phi\}\Theta^4 \\ & + 72(1-\nu)\{J + \frac{P}{12(1-\nu)}(n^2-1)K' \\ & + I'\phi - \frac{\kappa^2\mu^2 K'}{12(1-\nu)}\phi\}\Theta^2 - 72(1-\nu) \\ & \times \{I + P(n^2-1)I' - \kappa^2\mu^2 I'\phi\} + 2\mu\kappa\phi\Theta \\ & \times [\Theta^4 - 6K'\Theta^2 + 72(1-\nu)I'] = 0 \end{aligned} \quad (24)$$

2.2 Case of $\lambda^2 - (\omega + c\lambda)^2/c_o^2 \equiv -\eta^2 < 0$ (25)

$$\text{Put } \zeta = r\eta, \quad \bar{\zeta} = a\eta \quad (26)$$

then Eq. (12) is reduced to

$$\frac{\partial^2 f}{\partial \zeta^2} + \frac{1}{\zeta} \frac{df}{d\zeta} + [1 - \frac{\eta^2}{\zeta^2}]f = 0 \quad (27)$$

from which we have

$$f = kJ_n(\zeta) \quad (28)$$

If we put

$$\psi = (a/h)(\rho_o/\rho)J_n(\bar{\zeta})/\bar{\zeta}J'_n(\bar{\zeta}) \quad (29)$$

it is seen by the same procedure as in Sec. 2.1 that the frequency equations are the same as Eqs. (21) and (24) with ψ instead of ϕ for the cases (a) and (b) respectively.

3. Energy of vibrating system

In order to investigate the characteristics of the foregoing equations, we consider the energy of the vibrating system.

3.1 Energy of shell

The equations of motion of a cylindrical shell are described as follows^{<5>}:

$$\left. \begin{aligned} \rho a h \frac{\partial^2 W}{\partial t^2} &= -(T_\theta + \frac{\partial N_\theta}{\partial \theta} + a \frac{\partial N_x}{\partial x}) + a p_t \\ \rho a h \frac{\partial^2 U}{\partial t^2} &= (a \frac{\partial T_x}{\partial x} + \frac{\partial S}{\partial \theta}) + a f_t \\ \rho a h \frac{\partial^2 V}{\partial t^2} &= (\frac{\partial T_\theta}{\partial \theta} + a \frac{\partial S'}{\partial x} - N_\theta) + a q_t \end{aligned} \right\} \quad (30)$$

Multiplying both sides of Eqs. (30) by $\partial W/\partial t$, $\partial U/\partial t$, and $\partial V/\partial t$ respectively and summing up them, we have

$$\frac{a}{2} \rho h \frac{\partial}{\partial t} [(\frac{\partial W}{\partial t})^2 + (\frac{\partial U}{\partial t})^2 + (\frac{\partial V}{\partial t})^2] = -\frac{\partial}{\partial \theta} (N_\theta \frac{\partial W}{\partial t})$$

$$\begin{aligned} & + N_\theta \frac{\partial^2 W}{\partial \theta \partial t} - a \frac{\partial}{\partial x} (N_x \frac{\partial W}{\partial t}) + a N_x \frac{\partial^2 W}{\partial x \partial t} - T_\theta \frac{\partial W}{\partial t} \\ & + a \frac{\partial}{\partial x} (T_x \frac{\partial U}{\partial t}) - a T_x \frac{\partial^2 U}{\partial x \partial t} + \frac{\partial}{\partial \theta} (S \frac{\partial U}{\partial t}) - S \frac{\partial^2 U}{\partial \theta \partial t} \\ & + \frac{\partial}{\partial \theta} (T_\theta \frac{\partial V}{\partial t}) - T_\theta \frac{\partial^2 V}{\partial \theta \partial t} + a \frac{\partial}{\partial x} (S' \frac{\partial V}{\partial t}) \\ & - a S' \frac{\partial^2 V}{\partial x \partial t} - N_\theta \frac{\partial V}{\partial t} + a p_t \frac{\partial W}{\partial t} + a f_t \frac{\partial U}{\partial t} \\ & + a q_t \frac{\partial V}{\partial t} \end{aligned} \quad (31)$$

Substituting the following equations^{<6>}

$$N_x = \frac{\partial M_x}{\partial x} + \frac{\partial \mathfrak{M}}{\partial \theta}, \quad N_\theta = \frac{\partial \mathfrak{M}}{\partial x} + \frac{\partial M_\theta}{\partial \theta}$$

$$S' = S - \frac{\mathfrak{M}}{a}$$

$$T_x = \frac{12D}{h^2} [\nu \frac{\partial U}{\partial x} + \frac{1}{a} (\frac{\partial V}{\partial \theta} + W)]$$

$$T_\theta = \frac{12D}{h^2} [\nu \frac{\partial U}{\partial x} + \frac{1}{a} (\frac{\partial V}{\partial \theta} + W)]$$

$$S = \frac{6(1-\nu)D}{h^2} [\frac{\partial V}{\partial x} + \frac{1}{a} \frac{\partial U}{\partial \theta}]$$

$$M_x = D [\frac{\partial^2 W}{\partial x^2} + \frac{\nu}{a^2} (\frac{\partial^2 W}{\partial \theta^2} - \frac{\partial V}{\partial \theta})]$$

$$M_\theta = D [\nu \frac{\partial^2 W}{\partial x^2} + \frac{1}{a^2} (\frac{\partial^2 W}{\partial \theta^2} - \frac{\partial V}{\partial \theta})]$$

$$\mathfrak{M} = \frac{(1-\nu)D}{a} [\frac{\partial^2 W}{\partial x \partial \theta} - \frac{\partial V}{\partial x}]$$

into Eq. (31) gives

$$\begin{aligned} & \frac{\partial K_s}{\partial t} + \frac{\partial P_s}{\partial t} = \frac{\partial}{\partial x} (T_x \frac{\partial U}{\partial t}) + \frac{\partial}{\partial \theta} (T_\theta \frac{\partial V}{\partial t}) \\ & + \frac{\partial}{\partial \theta} (S \frac{\partial U}{\partial t}) + \frac{\partial}{\partial x} (S' \frac{\partial V}{\partial t}) - \frac{\partial}{\partial x} (N_x \frac{\partial W}{\partial t}) \\ & - \frac{\partial}{\partial \theta} (N_\theta \frac{\partial W}{\partial t}) + \frac{\partial}{\partial x} (M_x \frac{\partial^2 W}{\partial x \partial t}) \\ & + \frac{\partial}{\partial \theta} [M_\theta \frac{\partial}{\partial t} (\frac{1}{a} \frac{\partial W}{\partial \theta} - \frac{V}{a})] \\ & + \frac{\partial}{\partial x} [\mathfrak{M} \frac{\partial}{\partial t} (\frac{\partial W}{\partial \theta} - \frac{V}{a})] + \frac{\partial}{\partial \theta} [\mathfrak{M} \frac{\partial^2 W}{\partial x \partial t}] \\ & + p_t \frac{\partial W}{\partial t} + f_t \frac{\partial U}{\partial t} + q_t \frac{\partial V}{\partial t} \end{aligned} \quad (33)$$

where

$$K_s \equiv \frac{\rho h}{2} [(\frac{\partial W}{\partial t})^2 + (\frac{\partial U}{\partial t})^2 + (\frac{\partial V}{\partial t})^2]$$

$$\begin{aligned} P_s \equiv & \frac{1}{2Eh} [\nu (T_x - T_\theta)^2 + (1-\nu) T_x^2 + (1-\nu) T_\theta^2 \\ & + 2(1+\nu) S^2] + \frac{1}{2(1-\nu^2)D} [\nu (M_x - M_\theta)^2 \\ & + (1-\nu) M_x^2 + (1-\nu) M_\theta^2 + 2(1+\nu) \mathfrak{M}^2] \end{aligned}$$

3.2 Energy of fluid

Multiplying both sides of Eqs. (1) by $\partial W_f/\partial t$, $\partial U_f/\partial t$, and $\partial V_f/\partial t$ respectively and summing up them gives

$$\begin{aligned} & \frac{\rho_o}{2} \frac{\partial}{\partial t} [(\frac{\partial W_f}{\partial t})^2 + (\frac{\partial U_f}{\partial t})^2 + (\frac{\partial V_f}{\partial t})^2] \\ & + \frac{c\rho_o}{2} \frac{\partial}{\partial x} [(\frac{\partial W_f}{\partial t})^2 + (\frac{\partial U_f}{\partial t})^2 + (\frac{\partial V_f}{\partial t})^2] \end{aligned}$$

that for $n \geq 1$ in order to have a θ tending to zero, ϕ or ψ should tend to infinity.

$I_n(\bar{\zeta})$ or $I'_n(\bar{\zeta})$ takes no zero value except for $\bar{\zeta} = 0$, hence ϕ whose value becomes $1/n(a/h)\rho_o/\rho^{<2>}$ for a small $\bar{\zeta}$ never tends to infinity, whereas ψ may become infinite at the positions for $J'_n(\bar{\zeta}) = 0$.

From this result it is seen that the zero frequencies for $\bar{\zeta} \neq 0$ will occur only in the case of a supersonic flow.

Therefore for a complete incompressible fluid such a phenomenon will not appear.

At the zero frequency the values of $\beta(a)$ and $\gamma(a)$ in Eqs. (11) may become infinite although $a_{\mu n}$ remains finite. This phenomenon is a sort of instability of fluid corresponding to the buckling of shell.

For $n=0$ the zero frequency will occur when $I-PI' = 0$, but the value of p_o in this case is very great and will be out of question in practice.

4.b

In this case the zero frequency may occur in case of

$$\left. \begin{aligned} I+P(n^2-1)I' &= \kappa^2 \mu^2 I' \phi \\ \text{or } I+P(n^2-1)I' &= \kappa^2 \mu^2 I' \psi \end{aligned} \right\} (41)$$

through Eq. (24). First we consider a case of sufficiently small μ . In such a case with the aid of the relation $\phi = \psi = 1/n(a/h)\rho_o/\rho$, Eqs. (41) are reduced to

$$I+P(n^2-1)I' = \kappa^2 \mu^2 I' \frac{1}{n} (a/h) \rho_o / \rho \quad (42)$$

Substituting $\kappa^2 = \rho h^3 c^2 / D$ and $D = Eh^3 / 12(1-\nu^2)$ into Eq. (42), we have

$$c^2 = \frac{I+P(n^2-1)I'}{12(1-\nu^2)\mu^2 I'} \left(\frac{h}{a} \right) \frac{E}{\rho_o} n \quad (43)$$

In the special case of $n=1$, putting $\mu = m\pi a / l$ and neglecting $(h/a)^2$ compared to unity, Eq. (43) is reduced to

$$c^2 = \mu^2 \left(\frac{h}{a} \right) \frac{E}{\rho_o} = \frac{m^2 \pi^2 a h E}{l^2 \rho_o} \quad (43)'$$

This expression agrees with the critical speed given by Housner^{<7>} based on the beam theory.

This result means that the beam theory of vibration with a "follower" flowing fluid is equivalent to the case of neglecting the boundary layer with $n=1$.

The critical speed expressed in Eq. (43)' is proportional to μ and can become sufficiently small, hence it is important in practice if actually exists.

In the case (a) such a phenomenon corresponding to Eq. (43)' does not appear.

The zero frequencies given by Eqs. (41) may occur even for a complete incompressible fluid whereas they do not in the case (a).

The phenomenon of zero frequency is the buckling of a shell, i. e., a sort of instability which is not represented by Eq. (38) because such instabilities are not increasing ones with time.

It should be noted that the zero frequencies are always associated with the fluid instabilities corresponding to shell bucklings.

5. Pipeline regularly supported

The above-mentioned theories are for an infinitely long pipe, but in practice the pipeline is usually regularly supported.

The procedure to obtain the solutions for this case is the same with that of author's previous paper^{<2>}, i. e., for real values of ω

$$\left. \begin{aligned} W &= a_{\mu n} \cos n\theta [\sin(\lambda_1 x + \omega t) \\ &+ \sin(\lambda_2 x - \omega t)] = 2a_{\mu n} \cos n\theta \\ &\times \sin\left(\frac{\lambda_1 + \lambda_2}{2} x\right) \cos\left(\frac{\lambda_1 - \lambda_2}{2} x + \omega t\right) \\ \text{or} \\ W &= b_{\mu n} \cos n\theta [\cos(\lambda_1 x + \omega t) \\ &- \cos(\lambda_2 x - \omega t)] = -2b_{\mu n} \cos n\theta \\ &\times \sin\left(\frac{\lambda_1 + \lambda_2}{2} x\right) \sin\left(\frac{\lambda_1 - \lambda_2}{2} x + \omega t\right) \end{aligned} \right\} (44)$$

where $\frac{\lambda_1 + \lambda_2}{2} = \frac{m\pi}{l}$ (l : distance between the support)

$$m = 1, 2, 3, \dots \dots \dots (45)$$

If we choose λ_1 and λ_2 to satisfy Eq. (45), the conditions of being regularly supported will be satisfied.

This step may be carried out with the aid of $\mu - \theta$ charts ($\mu = a\lambda$) as in the author's previous paper.^{<2>}

$a_{\mu n}$ and $b_{\mu n}$ in Eqs. (44) may be taken arbitrarily, and the general expression of W will be the infinite sum of these expressions corresponding to each m , $a_{\mu n}$ and $b_{\mu n}$ in which may be determined by the initial conditions.

According to the form of $\mu - \theta$ curve, we must take the following expression for W as described in the author's previous paper:^{<2>}

$$\left. \begin{aligned} W &= a_{\mu n} \cos n\theta [\sin(\lambda_1 x + \omega t) - \sin(\lambda_2 x - \omega t)] \\ &= 2a_{\mu n} \cos n\theta \sin\left(\frac{\lambda_1 - \lambda_2}{2} x\right) \\ &\times \cos\left(\frac{\lambda_1 + \lambda_2}{2} x + \omega t\right) \\ \text{with} \\ \frac{\lambda_1 - \lambda_2}{2} &= \frac{m\pi}{l} \quad m = 1, 2, 3, \dots \dots \dots \end{aligned} \right\} (46)$$

6. $\mu - \theta$ Chart

To make the $\mu - \theta$ charts on the basis of above-mentioned theories, the following data are assumed:

Young's modulus of pipe : $2.1 \times 10^6 \text{ kg/cm}^2$

Compressible fluid : Air

Fluid velocities : $10 \text{ m/sec} \sim 993 \text{ m/sec}$

Mean static pressures of fluid : $3, 5 \text{ kg/cm}^2$ (2, 4 kg/cm² gauge)

$a/h = 100$, $n = 0, 1, 2, 3, 4, 5$

In the charts, a pair of θ 's (forward and backward waves in low flow speed range) for the smallest absolute value is selected from the solution of Eq. (21) or (24).

For the convenience to determine the frequency of a shell regularly supported in using Eq. (44) or (46), the curves for negative value of θ are plotted on the pos-

itive side by broken lines.

Figures 1_{a,b} through 9_{a,b} are the μ - θ diagrams for the assigned flow speed in each chart.

All the charts are for $\bar{p}_0 = 4 \text{ kg/cm}^2$ (gauge), because those for 2 kg/cm^2 are not so different except in the range of small μ .

The charts on the left side are for the case (a) and on the right side for the case (b).

Comparing them it is seen that the figures for the case (a) resemble the corresponding ones for the case (b), on the whole.

A remarkable difference is that for $n=1$ and in the small range of μ , the case (a) presents no special instability phenomenon, while the case (b) presents an

instability such as divergence or flutter in the left-side range of vertical segment designated I and a buckling phenomenon of shell at the position designated B adjacent to I which corresponds to that expressed by Eq. (43)'.

Two examples of the magnified curves for the neighborhood of B and I are illustrated in Figs. 10_b and 11_b.

These results confirm the theoretical conclusions stated in 3.3.a and 3.3.b.

In order to generalize the utilization of the μ - θ charts, it is desirable to write the curves of second or third mode, but they are neglected to avoid a confusion in the charts except a few examples illustrated in Figs. 6_{a,b} through 9_{a,b}.

Fig. 1_a Fig. 1_b Fig. 2_a Fig. 2_b Fig. 3_a Fig. 3_b

Fig. 4a

Fig. 4b

Fig. 5a

Fig. 5b

Fig. 6a

Fig. 6b

Fig. 7a

Fig. 7b

Fig. 8a

Fig. 8b

Fig. 9a

Fig. 9b

$\mu - \theta$ charts

Fig. 10a

Fig. 11a

Magnified drawings for the vicinity of I and B.

Conclusions

The above-mentioned results may be summarized as follows :

The $\mu - \theta$ chart for the case (a) which takes into account the effect of boundary layer and for the case (b) which has no such a consideration resemble each other on the whole with the following remarkable differences.

(1) For an infinitely long pipeline (regularly supported case inclusive) in the case (a) no instability such as flutter nor divergence occurs, whereas in the case (b) and $n=1$ such an instability may occur.

(2) In the case (b), at a position adjacent to the onset of instability a shell buckling may occur which agrees with the result of beam theory, whereas this is not the case with (a).

(3) In the case (a) the positions of zero frequencies appear only in supersonic flows associated with fluid instabilities whereas in the case (b) such positions may occur even in the case of a complete incompressible fluid although they are not so different for the present data.

The use of the $\mu - \theta$ charts enables us to obtain easily the frequencies of pipeline regularly supported as in the case of incompressible fluid and it will be seen from the charts that the frequencies become complex if the flow speed gets beyond the acoustic velocity.

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