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A Note on a Lifting Hydrofoil with Finite Cavity in a Solid Wall Tunnel *

By Terukazu OTA** and Takashi MURAKAMI***

A nonlinear theory is developed for an incompressible two-dimensional flow over an inclined flat plate hydrofoil with finite cavity in a solid wall tunnel. The wake model employed is Tulin's double spiral vortex model. The effects of the tunnel wall are evaluated upon the hydrofoil characteristics such as the lift and drag coefficients and the cavity streamline comparing with previous theoretical studies and also existing experimental data.

1. Introduction

The theoretical studies upon the cavitating hydrofoil in the solid wall tunnel have been recognized as being very important in connection with the experimental works and the development of a realistic physical flow model, and therefore there have been many papers published on this flow problem. As far as the asymmetric lifting hydrofoil accompanied with a finite cavity in the solid wall tunnel is concerned, the only nonlinear analyses published until now are those by Wu et al.⁽¹⁾ and Baker⁽²⁾ in which the open wake model was employed and by Ota⁽³⁾ with the single spiral vortex model⁽⁴⁾.

In this note, an inclined flat plate hydrofoil with a finite cavity in the solid wall tunnel is nonlinearly analyzed with the double spiral vortex model⁽⁴⁾. The effects of the tunnel wall proximity upon the hydrofoil characteristics are estimated theoretically and the present numerical results are compared with the previous theoretical and experimental works.

2. Formulation of the problem and analysis

A flat plate of chord length c and angle of attack α is located at any position in the solid wall tunnel of height h and is accompanied with a finite cavity as shown in Fig.1. The velocity and pressure at upstream infinity are U_0 and P_0 respectively. The distance from the leading edge of the plate to the upper tunnel wall is h_a , and h_1 and h_2 denote the distances between the stagnation streamline and the upper or lower walls at upstream infinity. The flow around the plate, cavity, and wake is assumed to be a two-dimensional and irrotational flow of incompressible fluid.

In the double spiral vortex model by Tulin⁽⁴⁾ employed in the present analysis, the following assumptions are made.

- (1) The flow separates smoothly from the leading and trailing edges.
- (2) The velocity and pressure on the cavity streamlines are constant, q_c and P_c .
- (3) The wake downstream of the cavity is replaced by two smooth streamlines of a constant pressure which is equal to that at upstream infinity and they close at downstream infinity.
- (4) Values of the complex potential function $W=\phi+i\psi$ are equal to each other at the upper and lower cavity terminations.

The cavity length l is defined as the arithmetic mean of the distances from the leading edge to the upper and lower terminations in the uniform flow direction.

Fig.1 Flow configuration and coordinate, $z=x+iy$

The complex potential plane constituted from Fig.1 is conformally transformed into the upper half of the auxiliary plane $\zeta=\xi+i\eta$ by

$$W = \frac{1}{\pi} \{ \psi_1 \log(1 - \frac{\zeta}{b}) + \psi_2 \log(1 + \frac{\zeta}{a}) \} \quad \dots \quad (1)$$

Fig.2 Auxiliary plane, $\zeta=\xi+i\eta$

* Received 28th July, 1977.

** Assistant Professor, Department of Mechanical Engineering, Akita University, Akita.

*** Engineer, Fuji Electric Co. Ltd.

where $\psi_1 = U_0 h_1$, $\psi_2 = U_0 h_2$, $h_1/h_2 = b/a$ based on the Schwarz-Christoffel formula as shown in Fig. 2. Following relation is obtained from the assumption (4)

$$\frac{b-k_c}{b+k_d} = \frac{(a-k_d)/b}{a+k_c} \quad \dots\dots\dots(2)$$

Now the complex function ω is introduced as

$$\omega = \log\left(\frac{1}{U_0} \frac{dw}{dz}\right) = \log\left(\frac{q}{U_0}\right) - i\theta \quad \dots\dots\dots(3)$$

where q and θ are the velocity and direction of the flow respectively. Then the present problem to determine the flow field around the plate, cavity, and wake is transformed into one of finding an analytic function ω satisfying the physical boundary conditions which are prescribed on the real axis of the ζ -plane as follows:

(i) On the tunnel wall, $\theta=0$,

$$\text{Im}(\omega)=0; \quad -\infty < \xi < -a, \quad b < \xi < \infty. \quad \dots\dots(4)$$

(ii) On the wake streamlines downstream of the cavity termination, the velocity is constant, U_0 ,

$$\text{Re}(\omega)=0; \quad -a < \xi < -k_d, \quad k_c < \xi < b. \quad \dots\dots(5)$$

(iii) On the cavity streamlines, $q=q_c$,

$$\text{Re}(\omega)=\log(q_c/U_0); \quad -k_d < \xi < -1, \quad k_a < \xi < k_c \quad \dots\dots(6)$$

where q_c is expressed with the cavitation number

$$\sigma = (P_0 - P_c) / \frac{1}{2} \rho U_0^2 \quad \dots\dots\dots(7)$$

from the Bernoulli formula, that is, $q_c = U_0 \sqrt{1+\sigma}$.

(iv) On the wetted side of the plate surface, the fluid flows along the plate,

$$\begin{aligned} \text{Im}(\omega) &= \alpha; & -1 < \xi < 0 & \text{ and} \\ \text{Im}(\omega) &= \alpha - \pi; & 0 < \xi < k_a. \end{aligned} \quad \dots\dots(8)$$

This is one of the well known Riemann-Hilbert problems on $\omega^{(5)}$, and the solution is easily obtained as follows:

$$\begin{aligned} \omega(\zeta) &= \frac{1}{\pi} \sqrt{H(\zeta)} \left(\int_{-kd}^{-1} \frac{\log(q_c/U_0)}{(\xi'-\zeta)\sqrt{H(\xi')}} d\xi' - \int_{-1}^0 \frac{\alpha}{(\xi'-\zeta)\sqrt{H(\xi')}} d\xi' + \int_0^{ka} \frac{\pi-\alpha}{(\xi'-\zeta)\sqrt{H(\xi')}} d\xi' - \int_{ka}^{kc} \frac{\log(q_c/U_0)}{(\xi'-\zeta)\sqrt{H(\xi')}} d\xi' \right) \quad \dots\dots\dots(9) \end{aligned}$$

where $H(\zeta) = (\zeta+a)(\zeta+1)(\zeta-k_a)(\zeta-b)$.

The above solution contains five unknowns a , b , k_a , k_c , and kd and they are determined by the following four physical conditions and Eq.(2). The physical plane $z = x+iy$ is determined from Eq.(3) as

$$z = (1/U_0) \int \exp\{-\omega(\zeta)\} (dw/d\zeta) d\zeta. \quad \dots\dots(10)$$

(a) The disturbances due to the hydrofoil and the cavity vanish at the upstream and downstream infinities,

$$(P_1 - P_4) \log(q_c/U_0) - \alpha(P_2 + P_3) + \pi P_3 = 0, \quad \dots\dots(11)$$

$$(Q_1 - Q_4) \log(q_c/U_0) - \alpha(Q_2 + Q_3) + \pi Q_3 = 0 \quad \dots\dots(12)$$

where

$$\begin{aligned} P_1 &= \int_{-kd}^{-1} \frac{d\xi}{\sqrt{H(\xi)}}, \quad P_2 = \int_{-1}^0 \frac{d\xi}{\sqrt{H(\xi)}}, \quad P_3 = \int_0^{ka} \frac{d\xi}{\sqrt{H(\xi)}}, \\ P_4 &= \int_{ka}^{kc} \frac{d\xi}{\sqrt{H(\xi)}}, \quad Q_1 = \int_{-kd}^{-1} \frac{\xi d\xi}{\sqrt{H(\xi)}}, \quad Q_2 = \int_{-1}^0 \frac{\xi d\xi}{\sqrt{H(\xi)}}, \\ Q_3 &= \int_0^{ka} \frac{\xi d\xi}{\sqrt{H(\xi)}}, \quad Q_4 = \int_{ka}^{kc} \frac{\xi d\xi}{\sqrt{H(\xi)}}. \end{aligned}$$

(b) The ratio of the chord length to the tunnel height is given in the physical plane,

$$\begin{aligned} \frac{c}{h} &= -\frac{1}{\pi} \left(\int_0^{ka} \exp\left\{-\frac{\sqrt{H(\xi)}}{\pi} \tilde{\phi}(\xi)\right\} \frac{\xi d\xi}{(\xi+a)(\xi-b)} - \int_{-1}^0 \exp\left\{-\frac{\sqrt{H(\xi)}}{\pi} \tilde{\phi}(\xi)\right\} \frac{\xi d\xi}{(\xi+a)(\xi-b)} \right) \quad (13) \end{aligned}$$

where

$$\begin{aligned} \tilde{\phi}(\xi) &= \int_{-kd}^{-1} \frac{\log(q_c/U_0)}{(\xi'-\xi)\sqrt{H(\xi')}} d\xi' - \int_{-1}^0 \frac{a}{(\xi'-\xi)\sqrt{H(\xi')}} d\xi' \\ &+ \int_0^{ka} \frac{\pi-\alpha}{(\xi'-\xi)\sqrt{H(\xi')}} d\xi' - \int_{ka}^{kc} \frac{\log(q_c/U_0)}{(\xi'-\xi)\sqrt{H(\xi')}} d\xi'. \end{aligned}$$

(c) The position of the leading edge in the tunnel is given in the z -plane,

$$\begin{aligned} \frac{ha}{h} &= \frac{1}{\pi} \left(\frac{U_0}{q_c} \int_{ka}^{kc} \sin\left\{\frac{\sqrt{H(\xi)}}{\pi} \tilde{\phi}(\xi)\right\} \frac{\xi d\xi}{(\xi+a)(\xi-b)} + \int_{-1}^0 \sin\left\{\frac{\sqrt{H(\xi)}}{\pi} \tilde{\phi}(\xi)\right\} \frac{\xi d\xi}{(\xi+a)(\xi-b)} \right) \quad \dots\dots(14) \end{aligned}$$

Five unknowns included in the solution are thus expressed with the angle of attack, the cavitation number, the chord length-tunnel height ratio, and the plate position in the tunnel.

3. Hydrofoil characteristics

The lift and drag, L and D , can be determined from the integral of the pressure difference between the wetted and the cavity sides of the plate, and then the lift and drag coefficients are expressed as

$$C_L = F_c \cos(\alpha), \quad C_D = F_c \sin(\alpha) \quad \dots\dots\dots(15)$$

where $C_L = L / \frac{1}{2} \rho U_0^2 c$, $C_D = D / \frac{1}{2} \rho U_0^2 c$, and

$$\begin{aligned} F_c &= 1 + \sigma + \frac{1}{\pi} \left(\int_{-1}^0 \exp\{-\theta(\xi)\} \frac{\xi d\xi}{(\xi+a)(\xi-b)} - \int_0^{ka} \exp\{-\theta(\xi)\} \frac{\xi d\xi}{(\xi+a)(\xi-b)} \right) \end{aligned}$$

$$\Theta(\xi) = \frac{\sqrt{H(\xi)}}{\pi} \tilde{\varphi}(\xi), \quad I_0 = \int_0^{ka} \exp\{\Theta(\xi)\} \times$$

$$\frac{\xi d\xi}{(\xi+a)(\xi-b)} - \int_{-1}^0 \exp\{\Theta(\xi)\} \frac{\xi d\xi}{(\xi+a)(\xi-b)}.$$

The shapes of the cavity streamlines can be calculated from Eqs. (1), (9), and (10), and the cavity length defined above is given by

$$\frac{2l}{c} = \cos(\alpha) + \frac{U_0}{I_0 q c} \left(\int_{ka}^{kc} \cos\{\Theta(\xi)\} \frac{\xi d\xi}{(\xi+a)(\xi-b)} - \int_{-kd}^{-1} \cos\{-\Theta(\xi)\} \frac{\xi d\xi}{(\xi+a)(\xi-b)} \right). \quad \dots (16)$$

These results express the characteristics of an inclined flat plate hydrofoil which is located at any place in the solid wall tunnel and is accompanied with a finite cavity. Those of the unbounded and the choking flows are easily derived from these results as special cases.

4. Numerical results and discussion

Numerical calculations were made for a flat plate accompanied with a finite cavity in which the stagnation streamline at the upstream infinity was situated at the center of the tunnel. The relation, $a=b$, is derived under such flow condition. Therefore in the following numerical results, the plate center, in general, differs from that of the tunnel, but it may not be supposed that the present numerical results lose the generality of estimating the wall effects upon the hydrofoil characteristics.

Unknown parameters found in the solution were determined by the following procedure.

1. Under a given value of the angle of attack, an appropriate value is designated for a , afterwards a pair of values kc and ka are determined in order to satisfy the above conditions by the iteration method.
2. Values of a , kc , and ka determined in the procedure 1 are introduced into Eqs. (11) through (17), and then σ , C_L , C_D , and c/h are calculated.
3. For a given value of c/h , the values of σ , C_L , and C_D are obtained from above results by the interpolation method.

Under the aim to verify the appropriateness of the present analysis and the numerical procedure, numerical evaluations were made for a flat plate with a finite cavity in an unbounded flow and for a choking flow condition. However their results are not shown in this note because of the space limitation. Present theoretical results for an unbounded flow employing the double spiral vortex model are a little larger than the previous results by the single spiral vortex model⁽⁶⁾ and also the open wake model⁽²⁾, but their agreement is pretty good. Comparisons were also made

for the choking flow condition between the present numerical results and Ai's results⁽⁷⁾ and the agreement between them is very good.

Representative numerical results are shown in the following four figures. Present numerical results are compared with previous nonlinear studies employing different flow model, that is, the open wake model⁽²⁾ and the single spiral vortex model⁽³⁾ with regard to the lift coefficient in Fig. 3. The present double spiral vortex model estimates the lift coefficient larger than other flow models, however the difference among them is not found to be large from the figure.

Fig. 3 Comparison of lift coefficient with other nonlinear theories

The wall effects on the lift coefficient are shown in Fig. 4 for various angles of attack and various tunnel heights. It is clearly shown in the figure that at the same cavitation number, the proximity of the tunnel wall reduces the lift coefficient, but its reduction is relatively small.

Fig. 4 Wall effects on lift coefficient

On the other hand, the tunnel wall has extremely large effects upon the shape of cavity streamline and therefore the cavity length. Typical examples are distinctly shown in Fig. 5 where the proximity of the tunnel wall increases markedly the cavity length at the same cavitation number. Therefore the wall effects on the

cavity length are quantitatively estimated and shown in Fig.6 comparing with the existing experimental data⁽⁸⁾ and the linearized theory employing the double spiral vortex model⁽⁹⁾. Present results agree well with the experimental data and it is clear that the cavity length estimated by the nonlinear theory is smaller than that by the linearized theory.

Fig.5 Wall effects on shape of cavity streamline

Fig.6 Comparison of cavity length with linearized theory and experimental data

5. Concluding remarks

A nonlinear analysis is developed for a lifting flat plate hydrofoil with a finite cavity in a solid wall tunnel employing Tulin's double spiral vortex model.

Numerical results reveal that the proximity of the tunnel wall reduces the lift and drag coefficients at the same cavitation number, however its reduction is quite small. On the other hand, the shape of the cavity streamline and then the cavity length are extremely affected by the proximity of the tunnel wall and the cavity becomes remarkably long with the decrease of the tunnel height.

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References

- (1) Wu et al., CIT Rept. E-111A.5 (1969).
- (2) Baker, E.S., Naval Ship Research and Development Center Rept. 3688 (1972).
- (3) Ota, T., ZAMM, 55 (1975), 227.
- (4) Tulin, M. P., J. Ship Res., 7 (1964), 16.
- (5) Muskhelishvili, N. I., Singular Integral Equations, Noordhoff, (1953).
- (6) Larock, B. E., and Street, R. L., J. Ship Res., 9 (1965), 170.
- (7) Ai, D. K., Trans. ASME, Ser. D, 88-1 (1966), 132.
- (8) Wade, R. B., CIT Rept. E-133.3 (1965).
- (9) Ota, T., Zeit. Flugwiss., 22-6 (1975), 201.