Vibration of Helical Gears

Part 2 Experimental Investigation

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Dynamic strain and accelerations of three-directional vibrations of helical gears are measured and their natural frequencies, modes and exciting force are discussed in comparison with those of spur gears. Some interesting facts are found: As in spur gears, in helical gears also the most prominent natural frequency is the one which corresponds to torsional vibration of the geared system. Some natural frequencies and modes which are not found in spur gears appear in vibrations of helical gears. As the result of resonance in these frequencies the axial vibration level and dynamic load of helical gears both increase to some extent. The exciting force due to periodic variation of tooth stiffness decreases with an increase of helix angle. Any remarkable difference is not found between levels of exciting forces of two pairs of helical gears having same helix angle but different face widths.

1. Introduction

Noise of helical gears has been investigated experimentally by Niemann and Hösel and by Attia.4) Kuhro and others investigated dynamic load on helical gears through measurements of dynamic strains on teeth.5) Some interesting dynamic behaviours of helical gears have been pointed out in these reports. But theoretical analysis of vibrations is not dealt with in these reports. The reason for distinctive dynamic behaviours of helical gears was, therefore, found merely in exciting force or was not explained clearly.

The authors have constructed and analyzed a vibration system of helical gears and shafts.6) It has been demonstrated by the analysis that the distinctive characteristics of vibrations of helical gears have been found also in their natural frequencies and modes. In the present paper the dynamic behaviours of helical gears will be investigated through measurements of acceleration levels which will show more sensitive behaviours than noise or dynamic strains. The experimental results will be compared with the theoretical ones and some distinctive behaviours of helical gears will be explained. Also some interesting behaviours which may be reduced to characteristics of exciting forces of gear vibrations will be shown.

<table>
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<th>Item</th>
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<tr>
<td>Circulating torque</td>
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<td>Tangential force</td>
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<td>100 Ps</td>
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<td>Power of motor</td>
<td>10 Ps</td>
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</table>

Table 1 Outline of the apparatus

Fig. 1 Outline of the apparatus of a power-circulating type.

* Received 12th April 1976.

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Fig. 1 shows an outline of the experimental apparatus. The apparatus consists of two pairs of gears. One is a pair of gears tested and the other is a pair through which the driving power is supplied. The pair of gears tested are supported by four slide-bearings and bearing-supports. This pair is vibrationally isolated from the other pair by torsion-rods and fly-wheels. The power-supplying pair has helical teeth and is driven by a direct-current motor through a plain belt and a pulley. The rotational speed may be changed continuously in a wide speed range of the experiment. The driven gear of this power-supplying pair is mounted with a straight spline on the gear-carrying shaft. The gear can therefore be moved in the axial direction. In order to change the transmitted load, this driven gear is squeezed by a system of an oil pressure pump and a piston. The oil pressure pump is operated manually. Thus the desired transmitted load on a pair of gears tested is susceptible of a continuous change during the motion.

Table 2 Dimensions of tested pairs of gears.

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>b</th>
<th>x</th>
<th>( \beta_b )</th>
<th>( \beta_y )</th>
<th>( \varepsilon_s )</th>
<th>( \varepsilon_f )</th>
<th>( \varepsilon_h )</th>
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<tr>
<td>J10</td>
<td>4</td>
<td>49</td>
<td>20</td>
<td>0.125</td>
<td>0°</td>
<td>0°</td>
<td>1.486</td>
<td>1.486</td>
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<tr>
<td>H1</td>
<td>3</td>
<td>65</td>
<td>20</td>
<td>0</td>
<td>10°</td>
<td>9°23'</td>
<td>1.756</td>
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</tr>
<tr>
<td>H2</td>
<td>3</td>
<td>62</td>
<td>20</td>
<td>0</td>
<td>20°03'</td>
<td>18°48'</td>
<td>1.628</td>
<td>0.725</td>
</tr>
<tr>
<td>H3</td>
<td>3</td>
<td>57</td>
<td>20</td>
<td>0</td>
<td>30°16'</td>
<td>28°16'</td>
<td>1.427</td>
<td>1.079</td>
</tr>
<tr>
<td>H4</td>
<td>3</td>
<td>62</td>
<td>20</td>
<td>0</td>
<td>20°03'</td>
<td>18°48'</td>
<td>1.628</td>
<td>1.088</td>
</tr>
</tbody>
</table>

Note: m:module, z:tooth number, b:face width (mm), x:addendum modification coefficient, \( \beta_b, \beta_y \):helix angle on pitch and base cylinder, \( \varepsilon_s, \varepsilon_f, \varepsilon_h \):profile, face and total contact ratio.

Table 3 Makers and types of measuring instruments

<table>
<thead>
<tr>
<th>instrument</th>
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<th>type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerometer Pick up</td>
<td>Bruel &amp; Kjaer</td>
<td>4335</td>
</tr>
<tr>
<td>Pre-amplifier</td>
<td>Bruel &amp; Kjaer</td>
<td>1606</td>
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<tr>
<td>Frequency Analyzer</td>
<td>Bruel &amp; Kjaer</td>
<td>2107</td>
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<tr>
<td>Microphone Amp.</td>
<td>Bruel &amp; Kjaer</td>
<td>2603</td>
</tr>
<tr>
<td>Level Recorder</td>
<td>Bruel &amp; Kjaer</td>
<td>2305</td>
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<tr>
<td>Strain Gauge</td>
<td>Kyowa Elec.</td>
<td>KFC-1C2-11</td>
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<tr>
<td>Dynamic Strain Ampl.</td>
<td>Shinko Tsushin</td>
<td>D9/PX</td>
</tr>
<tr>
<td>Dynamic Strain Meter.</td>
<td>Shinko Tsushin</td>
<td>Pw7-P</td>
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<tr>
<td>F-V Converter</td>
<td>Yokogawa Elec.</td>
<td>Sr/13S 93660</td>
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<tr>
<td>Slip-ring</td>
<td>Shinko Tsushin</td>
<td>R 500 S</td>
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<tr>
<td>Data Recorder</td>
<td>TEAC</td>
<td>3051</td>
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<tr>
<td>Log Convereter</td>
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<tr>
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<td>Yokogawa Elec.</td>
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<tr>
<td>Dc Amplifier</td>
<td>Sanei Sokki</td>
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</table>

Fig. 2 Dimensions of gear-carrying shafts of the experimental apparatus.

3. Results of measurements

3-1 Acceleration levels of three-directional vibrations and dynamic factors.

In Fig. 3 acceleration levels of three-directional vibrations of spur gears Ho are plotted against the tooth-mesh frequency. A kind of dynamic factor is also plotted in Fig. 3. This dynamic factor means the ratio of the maximum value of a dynamic strain to that of the static one which is measured at a low gear speed, that is, at \( f_g=200 \text{ Hz} \).

Many peaks occur on the curve of torsional vibration \( \alpha_r \). But most peaks can be classified into three kinds of resonance. One set of these peaks is due to the first- the second- and the
higher-order resonances with a natural frequency of about 2600 Hz. Another set is due to the resonances with a natural frequency of about 3200 Hz. The third set seems to be caused by resonances with a natural frequency of about 4300 Hz. These three kinds of resonance and the order of resonance of each peak are distinguished by \( n \) and \( m \) (\( n = 1, 2, 3, \ldots \)) which are noted at each peak of the curve of \( a_z \). Comparing these natural frequencies with the theoretical results, it becomes apparent that these are natural frequencies which have been named \( f_{22} \), \( f_{33} \) and \( f_{44} \), respectively.

When the curve of the axial vibration \( a_z \) is observed, corresponding peaks are seen to occur at the frequencies where the curve of \( a_z \) forms peaks. But other peaks do on the curve of \( a_z \) at a frequency region of 1800~2300 Hz where the curve of \( a_z \) has no peak. The former peaks on the curve of \( a_z \) are chiefly caused by an increase of the dynamic load on gears which is transmitted in the axial direction through gear-carrying shafts and bearing supports. The latter peaks are due to some resonances of the longitudinal, the rotational, and the plate-mode vibrations which are included in the axial vibration. Thus it is seen that the axial vibration in the apparatus is not coupled noticeably through teeth of the spur gears.

On the other hand each peak on the curve of \( a_z \) corresponds to that on the curve of \( a_z \). This fact means that the radial vibration is coupled tightly with the torsional one. It should be noted that the directions of the pick-ups are not fixed in space. Therefore the acceleration level of a vibration in one fixed lateral direction is to be measured 3 dB less than the practical value. It has been also observed that the wave forms of vibrations at the first- and the second- order resonances in Fig. 3 are almost sinusoidal. Taking these facts into consideration, the ratio of the displacement of a lateral vibration to that of the torsional one, that is, \( x_r/x_t \) can be estimated approximately by \( f_2/f_3 \). Then the values of \( x_r/x_t \) at the resonances with the natural frequencies of \( f_{22} \) and \( f_{33} \) are estimated at 0.2 and 0.1 respectively. The theoretical values corresponding to then are 0.5 and 0.34 respectively.

The radial component in the coupled mode of frequency \( f_{22} \) is obviously larger than that of frequency \( f_{33} \). This tendency coincides with the theoretical result. But the ratios \( x_r/x_t \) which have been experimentally obtained are less than those of theoretical result. The reason for this discrepancy may be found in an under-estimation of equivalent mass of gear-carrying shaft in the theoretical analysis.

The dynamic load factor builds up noticeably at the first- and the second-order resonances with natural frequencies \( f_{22} \) and \( f_{33} \). The dynamic factors at these resonances with \( f_{22} \) are larger than those at the corresponding resonances with \( f_{33} \). This tendency seems to be contradictory to the tendency of the levels of \( a_z \) at two kinds of resonances. But this opposite tendency can be explained by the theoretical results that the natural mode \( f_{22} \) tends to cause tooth deflection more than \( f_{33} \) does.

Thus it can be concluded that the dynamic behaviours of spur gears H0 are almost identical to those theoretically predicted.

In Fig. 4 similar results of measurement in the helical gears H2 are shown. At the top of Fig. 4, theoretical values of resonant frequencies for the higher-order coupled natural frequencies \( f_{21} \) and \( f_{12} \) are marked.

In this pair of helical gears also, the dynamic factor builds up noticeably by the resonance with the natural frequency \( f_{21} \) and \( f_{12} \). This tendency coincides with that of the

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**Fig. 3** The three-directional vibrations and the dynamic factor of the spur gears H0.

**Fig. 4** The three-directional vibrations and the dynamic factor of the helical gears H2.
theoretical result. But the values of $f_{22}$ and $f_{32}$ are not similar to those in Table 2 in the preceding report. They are rather similar to those in Fig. 11 in the preceding report.}

Comparing the curves of the three-directional vibrations, it is seen that most peaks on the curve of $a_2$ and $a_3$ correspond to those on the curve of $a_1$. Some peaks are found in the frequency region of 1800~2300 Hz where the curve of $a_2$ of the gears H0 also presents some peaks. In the helical gears H2, however, these peaks appear not only on the curve of $a_2$ but also on the curves of $a_1$ and $a_3$. Thus it is clear that the axial vibration in the helical gears is tightly coupled with the vibrations in other directions. The ratio of the level of $a_2$ to that of $a_3$ is larger in the gears H2 than in the gears H0. This tendency coincides with the theoretical results shown in the preceding report.

Generally in the three-directional vibrations at a resonant frequency, components of vibrations of several natural modes are included. It is sometimes difficult, therefore, to distinguish the ratios of the three-directional components of one natural mode under consideration from those of the others. This difficulty increases particularly when the resonance under consideration is not strong. What is more, the components of the rotational vibrations and the plate-mode vibrations as well as of the longitudinal vibration are included in the axial vibration $a_2$. It is seen, however, that the ratios of the level $a_2$ to the level $a_3$ are larger in frequency regions of 1800~2300 Hz and 3400~4000 Hz than at the resonances with the natural frequency $f_{22}$ and $f_{32}$.

When the gear H2 is reduced to a circular plate of radius 99 mm and width 20 mm which is fixed by an inner circle of radius 25 mm, natural frequencies of plate-mode vibration $f_{p2}=3740$ and $f_{p3}=6670$ Hz are obtained. The modes corresponding to $f_{p2}$ and $f_{p3}$ have two and three diameters of node respectively.

One of the peaks in the frequency region of 1800~2300 Hz seems to be caused by the first-order resonance with the natural frequency $f_{22}$. One of the peaks in the frequency region of 3400~4000 Hz seems to be caused by resonances with the natural frequency of the plate-mode vibration named $f_{p2}$. The higher-order resonances with the natural frequency $f_{32}$ ($1 > 3$) also may play some role in the building-up of the acceleration levels in these frequency regions.

It is apparent therefore that the natural modes whose predominant component is the longitudinal or the plate-mode vibration play some role in the increase of the dynamic load. This result also coincides with that which has been theoretically predicted.

It should also be mentioned that the levels of the three-directional vibrations and dynamic factor in H2 are lower than those in H1. This tendency becomes remarkable in the frequency region lower than 2000 Hz. This may be due to the difference between exciting forces of both pairs of gears.

In Fig. 5 a similar result of experiments for the gears H3 is shown. It is seen that the acceleration level $a_3$ is higher than those of $a_3$ and $a_4$. It is seen also that the resonances in the frequency regions of about 2000 Hz and higher than 3200 Hz are remarkable than those in the frequency region of 2500~3200 Hz. Thus the distinctive behaviours of helical gears can be seen more clearly in Fig. 5 than in Fig. 4.

Fig. 6 shows similar results for the gears H4 whose face width is one and half times that of H2. The value of the coupled natural frequency $f_{p2}$ is found larger and that of $f_{p3}$ less than those of the case of H2. The reason for this tendency may be found in the strength of coupling of vibrations, since the coupling of the vibrations through teeth tends to increase with the face width.

There is not any peak at about 3600 Hz where a peak appeared in Fig. 4. It is also seen that the forms of peaks at frequency about 2300 Hz are different from those in Fig. 4.
These differences are caused by the difference between the plate mode vibrations of both gear pairs. The values of the natural frequencies \( f_{p2} \) and \( f_{p3} \) of the gears H4 can be estimated at 5600 Hz and 10000 Hz respectively from the results given by Fukuma.\(^7\)

3-2 Frequency spectra of the vibrations.

Fig. 7 shows frequency spectra of the accelerations of three-dimensional vibrations of the gears H0 at some typical rotational speeds. It is seen that most peaks on the spectra of \( a_r \) correspond to the tooth-mesh frequency and its higher order frequencies. When the \( m \)-th order tooth-mesh frequency \( f_{mz} \) approaches 2600 Hz or 3200 Hz, its frequency component of \( a_q \) builds up remarkably. These frequencies correspond to the natural frequencies \( f_{q2} \) and \( f_{q3} \) respectively.

The curves of \( a_q \) and \( a_r \) are found to form peaks at frequencies corresponding to those of \( a_q \). But the ratios of the levels of these three vibrations vary with the frequency. The level of \( a_q \) builds up noticeably in the frequency region of 3500~4000 Hz. It is noted also that the ratio of \( a_q \) to \( a_q \) in the frequency region of 1800~2300 Hz is larger than that in 2500~3200 Hz. Thus it may be concluded that there are at least two natural modes in which the axial component of the vibration is predominant. It is found in Fig. 3 that some peaks have appeared in the frequency region of 1800~2300 Hz only on the curve of \( a_q \). Some of these peaks may be reduced to the first-order resonance with a natural frequency of about 2000 Hz and to the second-order resonance with a natural frequency of about 3600 Hz.

It is also seen that the level of \( a_q \) is larger than that of \( a_q \) in the frequency region less than 1000 Hz and higher than 8000 Hz. In particular, a peak at the frequency region of about 8000~9000 Hz builds up visibly. This peak may be caused by the resonance with the second-order natural frequency of the lateral vibration.

Fig. 8 shows similar results of the gears H2. It is found in this figure that the ratios of \( a_q \) to \( a_q \) larger than those in the gears H0. It has been demonstrated theoretically that the ratio \( a_q/a_q \) increases in some natural modes of helical gears. There are natural frequencies of about 3600, 4000 and 6000 Hz in addition to those of 2600 and 3200 Hz. It is also distinctive in helical gears that the spectra of vibrations have not visible peaks in the high frequency region. This characteristic is due to the exciting force of the vibrations of helical gears. It has been shown by Umezawa that the periodic variation of stiffness of helical gears is not so intense as that of spur gears.\(^8\) Therefore it can be expected that the exciting force of the vibration of helical gears has not Fourier's components of the high-order which excite the frequency component of the corresponding order.

In the spectra of the vibrations of the gears H3 which are shown in Fig. 9, is recognized a further typical tendency of the vibrations of helical gears. The level of the axial vibration is larger than that of \( a_q \) or \( a_q \) in most frequency regions. The resonances with the natural frequencies other than \( f_{q2} \) and \( f_{q3} \) stand out visibly.

It should be also marked that the second-order frequency component is larger than the first-order one in each curve of spectrum. The reason for this fact may be also found in Fourier's components of the exciting force. In the helical gears of large helix angle the shapes of the function of an exciting force due to manufacturing errors and the variation of the tooth stiffness should be completely different from those of spur gears. It is not
likely therefore that the first-order Fourier's component is always larger than the second-order one in the exciting force of the vibration of helical gears. For example, in the spectra of \( a_x \), \( a_y \) or \( a_z \), the second-order frequency component at \( f_x = 1220 \) Hz is larger than the first-order frequency component at \( f_x = 2328 \) Hz. Since the exciting frequencies for the frequency components under consideration are almost similar in both cases, the difference between the amplitudes should be reduced to the difference between those of the exciting forces.

Fig. 10 shows a similar result for the gears \( H_4 \). Natural frequencies are in the frequency regions of 4000 to 4500 Hz and of about 8000 Hz. On the other hand, there are no natural frequencies of about 3600 Hz and 6000 Hz which have been found in the other gears. Since these natural frequencies vary noticeably with the face width, they seem to be those of the plate-mode vibrations. From observations of wave forms of axial vibrations over one rotation of a gear they have been identified to be the natural frequencies \( f_2 \) and \( f_3 \).

3-3 The influence of transmitted load
Figs. 11, 12, 13 and 14 show the relationship of the acceleration level and the transmitted load per unit face width of gears \( H_0 \), \( H_1 \), \( H_4 \) and \( H_5 \) respectively. As seen in Fig. 11, the acceleration level \( a_y \) of the gears \( H_0 \) at a fixed tooth-mesh frequency is not proportional to the transmitted load. Furthermore, the tendency to show a change in the level depends on the tooth-mesh frequency. These characteristics have been explained previously.

In the helical gears \( H_1 \), however, the level \( a_y \) is almost proportional to the transmitted load \( W \), and the tendency to show a change in the level is almost similar in each tooth-mesh frequency. In helical gears \( H_6 \) each level \( a_z \) increases with the load \( W \) but the rate of the increase is slower than that of \( H_1 \). In helical gears \( H_3 \) which have the largest helix angle in the gears tested, the levels of \( a_y \) do not depend on the transmitted load at every tooth-mesh frequency. These tendency are similar to those which Niemann and Høsel have shown for the noise of helical gears.

One of the reasons for this slow increase of \( a_y \) may be found in the fact that the exciting force due to the periodic variation of the tooth stiffness tends to be smaller as the helix angle becomes larger. That is, helical
gears with large helix angle are excited merely by the errors of the gears, which do not depend on the transmitted load. But when it is taken into consideration that the exciting force due to profile errors of gears tends also to be smaller with an increase of the helix angle, the other reason may be found in the non-linearity of the stiffness of the helical teeth.

Fig. 15 shows the dynamic factors of the five pairs of gears plotted against the tooth-mesh frequency by taking the transmitted load as a parameter. The dynamic factor of the helical gears H2 decreases generally with an increase of the transmitted load. This tendency is also observed in the case of the gears H4 which have the same helix angle as H2. The gears H3 which have the largest helix angle among the five pairs of gears also show this tendency more clearly. The increase of the dynamic factor of H3 under the smaller transmitted load seems to be caused by errors of gears. On the other hand, the dynamic factor of the spur gears H0 has no tendency of this kind. At some resonant frequencies the dynamic factor rather increases with the transmitted load. The dynamic factor of the helical gears H1 also shows a tendency similar to that of the spur gears.

These tendencies are almost similar to those which have been shown by Kubo and others, and correspond to the relationship between $a_t$ and $W$ which has been mentioned above. But it should be marked that the dynamic factor of helical gears may build up not only by the resonance with $f_{22}$ and $f_{23}$ but also by the resonance with the vibration modes whose predominant components are axial vibrations.

It is seen also that the relationship between the dynamic factor and the transmitted load in the helical gears of H4 is almost similar to that in H2. Thus the gears H4 have shown a tendency similar to that of the gears H2 in most dynamic behaviours which are related to the characteristics of the exciting force. Comparing contact ratios of five pairs of gears shown in Table 2, it is apparent that the pair H4 has the largest face contact ratio and the largest total contact ratio. It seems therefore that the distinctive characteristics of the exciting force of the helical gears may depend rather on the helix angle than on the contact ratio.

One of the important characteristics of engagement of a pair of helical teeth is that a tooth begins and ends contact at an edge point of the tooth. The authors have shown the effects of this point-contact on the reduction of the vibration of the spur gears whose face width is made narrow at some region of the engagement. In addition to this the mean value of stiffness of mating teeth increases
with an increase of the total contact ratio. In such cases, the ratio of the variation of the stiffness of teeth to the mean value decreases in some extent. But this amount of decrease is not expected to become comparable to the amount of decrease of an exciting force because of a constant value of the equivalent stiffness of shafts which should be added to the mean value of the stiffness of teeth. Consequently, the increase of the total contact ratio through the increase of the face width is not so effective as the increase of the helix angle. This may be a reason for the fact that the gears H3 show the distinctive characteristics of helical gears more clearly than the gears H4.

4. Conclusions

The dynamic behaviors of helical gears whose face width is not large have been investigated through observation of the acceleration levels of three-directional vibrations and dynamic factors. The results of the observations may be summarized as follows.

(1) In the vibration of helical gears the torsional vibration is coupled tightly with the axial vibration as well as the radial vibration through teeth of gears. Therefore attention should be paid to many kinds of natural frequencies in discussing dynamic behaviors of helical gears.

(2) In general the ratio of the level of the axial vibration to that of the torsional one increases with the helix angle. In some vibration modes the ratio of the level of the radial vibration to that of the torsional one increases with the helix angle.

(3) In helical gears as well as in spur gears, the natural mode which presents the largest dynamic factor is one in which the torsional component of the vibration is predominant. But in some natural modes in which the axial component of the vibration plays an important role the dynamic load may increase in some extent.

(4) In general the amplitude of the exciting force decreases with an increase of the helix angle. Especially when the helix angle amounts to 30°, the exciting force due to the periodic variation of stiffness of teeth becomes negligibly small. It is also found that Fourier's components of the exciting force of helical gears vary with the helix angle. Even in the case of a small helix angle of 10°, the resonances due to the high-order Fourier's components of the exciting force are reduced markedly in comparison with the spur gears.

(5) The increase of the acceleration level of the vibration with the transmitted load tends to be slower in helical gears. Especially in the case of the helix angle of 30° the acceleration level is almost independent of the transmitted load.

(6) At least in helical gears of small face width as that used in the present experiment, a more marked effect of the reduction of the vibration can be obtained by increasing the helix angle than by increasing the face width.

References

(1) Niemann, G. und Hüseler, Th., Konstruktion, Bd. 18, Heft 4 (1966-4), s. 129.