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Natural Convective Oscillatory Flow in an Enclosed Space*

(Part 2, The Case of a Line Heat Source Placed along
the Axis of a Horizontal Rectangular Chamber)

By Tamotsu IGARASHI**

Experiments were carried out to investigate the natural convective oscillatory flow of air caused by a line heat source placed concentrically along the axis of a horizontal rectangular chamber. For various values of the aspect ratio of the cross section of the chamber Z , the frequency of this oscillatory flow motion and the initial conditions under which the oscillation of the flow commences are obtained. The flow patterns are found to be specific to the oscillation frequency and can be divided into three distinct regimes depending on the range of aspect ratios Z . For a given regime, the frequency depends only on the horizontal distance between the heat source and the side wall.

1. Introduction

In the previous paper⁽¹⁾, the author investigated experimentally the characteristics of natural convective oscillatory flow from a heated wire placed along the axis of a horizontal cylinder. The frequency of this oscillatory flow and the initial conditions under which the oscillation of the flow commenced were obtained. The system of the experiments was two-dimensional and the simplest one whose heat source was negligible. The shape and dimensions of the system were represented by one characteristic length, that is, the outer cylinder diameter D . The practical system, however, is three-dimensional. Even if the system can be considered two-dimensional, the geometrical variables of the system consisting of circular cylinder or rectangular chamber with a heat source are two or three, respectively. As the second step, the purpose of the present study is to clarify the effects of the dimensions of gravitational direction and transverse direction on natural convective oscillatory flow under the influence of gravitational force. Experimental studies have been performed on natural convection from a line heat source placed along the axis of a horizontal rectangular chamber.

Concerning the previous theoretical studies on natural convection from a horizontal line heat source in an enclosed space, Honda, et al.⁽²⁾, for example, solved steady laminar natural convection in square chamber for low Grashof number; $Gr = 500, 1000$ and 2500 , and $Pr = 0.70$. But there is no report for a considerably high Grashof number accompanied with oscillatory phenomena. This oscillatory flow is the object of the present study. Experimental study

for high Grashof number was done by Itoh, et al.⁽³⁾ As the result of measurements of the frequency of the oscillatory flow, they found the change of the flow patterns of oscillation with the value of the aspect ratio of the chamber. These phenomena, however, have been left without quantitative as well as qualitative clarification.

The test model was 300 mm long in the direction of the axis to keep the two-dimensionality of the flow field. The aspect ratio of the rectangular section Z , was varied from 0.4 to 2.5. The experiments were carried out in the modified Grashof number region of $6 \times 10^5 \leq Gr^* Pr \leq 3 \times 10^8$, based on the characteristic length B , which was the horizontal distance between the line heat source and the side wall, and heat input Q . The range of expansion numbers which represents the degree of heating was within $0.093 \leq \beta(\theta_w - \theta_\infty) \leq 1.78$. In the present experiment, the frequency and the initiation conditions of this oscillatory flow motion were obtained in the same way as in the previous report. And, the temperature distribution and vertical velocity component in the plume above a line heat source were measured. The results were related to the frequency of the oscillatory flow and were compared with Fujii's theory⁽⁴⁾ for steady laminar natural convection above a line heat source.

Nomenclature

- A : distance from line heat source to upper wall
- B : horizontal distance between line heat source and side wall
- C : constant
- D : outer circular cylinder diameter
- d : line heat source diameter
- f : frequency of oscillatory flow = $1/\tau$
- f_s : dimensionless stream function
- Gr : Grashof number, $Gr_B = B^3 g \beta \Delta \theta / \nu^2$, $Gr_D = D^3 g \beta \Delta \theta / \nu^2$
- Gr^* : modified Grashof number, $Gr_B^* = B^3 g \beta \theta / \nu^2$, $Gr_D^* = D^3 g \beta \theta / \nu^2$, $Gr_x^* = x^3 g \beta \theta / \nu^2$
- h : dimensionless temperature

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** Associate Professor, National Defense Academy, Yokosuka.

L : length of chamber, length of line heat source
 Pr : Prandtl number
 Q : heat rate per unit length due to conduction and convection
 Ra^* : modified Rayleigh number = $Gr \cdot Pr$
 Re : Reynolds number = ux/ν
 Re_f : dimensionless frequency, $Re_{fB} = fB^2/\nu$, $Re_{fD} = fD^2/\nu$
 T : absolute temperature
 u : vertical velocity component
 x : elevation above line heat source
 y : horizontal distance from mid plane of the plume
 Z : aspect ratio = A/B
 β : volumetric thermal expansion coefficient = $1/T$
 Θ : number with dimension of temperature defined by Eq. (3)
 θ : temperature
 θ_w : line heat source temperature
 θ_∞ : wall temperature, ambient fluid temperature
 θ_f : film temperature = $(\theta_w + \theta_\infty)/2$
 $\Delta\theta$: temperature difference = $\theta - \theta_\infty$, $\theta_w - \theta_\infty$
 λ : thermal conductivity
 ν : kinematic viscosity
 ξ : independent variable of the similar solution
 τ : period of oscillatory flow

where the reference temperatures of fluid properties with subscript f and without it are θ_f and θ_∞ respectively.

2. Experimental apparatus and procedure

A schematic diagram of the apparatus is shown in Fig. 1. The test model consisted of a horizontal rectangular chamber made of acrylic resin and a line heat source of nichrome wire. Air was used as a working medium. In the previous paper⁽¹⁾, five nichrome wires of different diameters from 0.29 mm to 1.0 mm were used as a line heat source. The same results were obtained regardless of the diameter of the heated wire. Therefore, in the present experiment,

Fig. 1 Schematic diagram of apparatus

the diameter of the line heat source was 1.0 mm. The length of the chamber with rectangular cross section L , was 300 mm and the dimensions of the horizontal and vertical direction of the cross section, $2B$ and $2A$ respectively, were varied from 70 mm to 200 mm. The line heat source was set up along the axis of the chamber under proper tension all the time. Thus, the distances between the line heat source and the side wall, and upper wall were A and B respectively. The dimensions and the aspect ratio $Z (=A/B)$, of the cross section of the chamber are shown in Table 1. Experiments of the combinations of blank in the table were not performed. The aspect ratio Z , was varied from 0.4 to 2.5.

Experiments have been done in the same manner as in the previous paper. The frequency and the initiation conditions of this oscillatory motion were measured. Flow visualizations were achieved by optical means where incense stick smoke was introduced in the rectangular chamber. And the temperature distribution and vertical velocity component above a line heat source were obtained. The velocity of the plume was estimated by using the traces of the temperature variation of two or three thermocouples of 0.05 mm diameter, that had no effect on velocity and temperature field, at different points above the line heat source. When the lower probe was slightly displaced instantaneously, a small disturbance induced by the lower probe propagated to the upper probe. Thus the mean velocity over the two points could be calculated by the time delay of small temperature fluctuations. The mean velocity in the plume of oscillatory flow for high heat input was obtained by the phase difference of the two sinusoidal traces of the temperature variation at two different points. The results of these two different methods agreed with each other.

3. Experimental results and discussion for a square chamber

3.1 Oscillation frequency

Typical flow patterns of natural convection above a line heat source mounted along the axis of a square chamber are shown in Fig. 2. Figure 2(a) is a representative photograph of steady and stable symmetric flow occurring at small heat input. By increasing heat input to the line heat source, an oscillatory flow of the swaying plume is observed as shown in Figs. 2(b) and (c). In arranging the experimental data of these

Table 1 Dimensions and aspect ratio

$Z = A/B$		2B mm						
		70	80	100	105	120	150	200
2A mm	70	1.0		0.70				
	80		1.0	0.80		0.67	0.53	0.40
	100	1.43	1.25	1.0		0.83	0.67	0.50
	105						0.70	
	120		1.50	1.20		1.0	0.80	0.60
	150		1.88	1.50	1.43	1.25	1.0	0.75
	200		2.50	2.0		1.67	1.33	1.0

oscillatory phenomena, the result of previous paper provides a good guide. In case of a line heat source placed concentrically along the axis of a horizontal cylinder, the oscillation frequency and the initiation conditions of the oscillatory flow can be given by the following Eqs. (1) and (2) : for oscillation frequency :

$$1.5 \times 10^8 \leq Gr_D^* Pr \leq 1.5 \times 10^9 ;$$

$$Re_{fD} = 0.12 (Gr_D^* Pr)^{0.40} \dots\dots\dots(1)$$

and for initiation conditions :

$$\beta_f \Delta \theta \geq 400 / (D^3 g / \nu_f^2)^{0.40} \dots\dots\dots(2)$$

where $Re_{fD} = fD^2/\nu$, $Gr_D^* = D^3 g \beta \theta / \nu^2$, $\theta = Q/\lambda Pr$ and the reference temperatures of the fluid properties with subscript f and without it are the film temperature θ_f and the wall temperature of the chamber θ_∞ respectively. Horizontal distance between the line heat source and the side wall B, which corresponds to the diameter D for circular section, was chosen as the characteristic length of the dimensionless frequency Re_f and modified Grashof number Gr^* for square section. These dimensionless numbers are defined by the following equations.

$$Re_{fB} = fB^2/\nu, \quad Gr_B^* = B^3 g \beta \theta / \nu^2,$$

$$\theta = Q/\lambda Pr \dots\dots\dots(3)$$

The results are shown in Fig. 3. It has the same tendency as that obtained for circular section. The dimensionless frequency can be given as follows :

$$Z = 1, \quad 6 \times 10^6 \leq Gr_B^* Pr \leq 2.5 \times 10^8 ;$$

$$Re_{fB} = 0.087 (Gr_B^* Pr)^{0.40} \dots\dots\dots(4)$$

Assuming a similarity between the oscillation frequency of square section and that of circular section, for instance, the equivalent diameter is defined by the use of perimeter of the circulating flow region. In this region, the fluid above the line heat source rises up to the ceiling of the chamber, then goes down along the side wall and returns to the heat source. The following simple relation has been assumed :

$$(1 + \pi/4)D = 4B (= 4A) \dots\dots\dots(5)$$

By substituting Eq. (5) into Eq. (1), the oscillation frequency for square chamber can be estimated as follows :

$$Z = 1, \quad 9.1 \times 10^6 \leq Gr_B^* Pr \leq 9.1 \times 10^7 ;$$

$$Re_{fB} = 0.063 (Gr_B^* Pr)^{0.40} \dots\dots\dots(6)$$

The experimental results expressed by Eq. (4) are about 40 % larger than the estimated values obtained with Eq. (6) and the range of the modified Rayleigh numbers at which the oscillation flow commences is wider than the calculated one. This means that the natural convection in a square chamber is unstable and it sways easily, its oscillation frequency being larger than in a circular chamber.

3.2 Initiation conditions

In the same manner as in the previous experimental results, the film temperature θ_f is taken as the reference temperature of the fluid properties. The initial conditions under which the flow oscillation commences are plotted with two dimensionless groups, $\beta_f \Delta \theta$ against $(8A^3 g / \nu_f^2) Pr$, on a log-log graph. The results are shown in

Fig. 2 Flow patterns

Fig. 3 Dimensionless frequency correlations for square chamber

Fig. 4 Initiation conditions for square chamber

Fig. 4. For square section, the value of the aspect ratio Z is one, namely, $A=B$, but the characteristic length of dimensionless frequency and initial conditions are given by B and A respectively. This is for the purpose of correspondence to the results of rectangular section described in chapter 4. The periodic flow oscillation commences when the expansion number $\beta_f \Delta \theta$, satisfies the following equation.

$$\beta_f \Delta \theta \geq 1100 \{ (8A^3 g / \nu_f^2) Pr \}^{-0.50} \dots (7)$$

By substituting Eq. (5) into Eq. (2), the initiation conditions for square section are predicted as follows;

$$\beta_f \Delta \theta \geq 2900 \{ (8A^3 g / \nu_f^2) Pr \}^{-0.40} \dots (8)$$

Comparing the values of the expansion number $\beta_f \Delta \theta$, for the initiation of the oscillation, the experimental values are found to be smaller than the predicted ones. This means that the natural convection in a square chamber sways easily for a small temperature difference. Although those oscillation frequencies of both cases show a similar tendency, the values of the exponents in Eqs. (7) and (8) are different.

By substituting the property values of air and rearranging Eq. (7) for a given wall temperature θ_w , the relations between the height of the chamber $2A$, and the temperature difference $\Delta \theta (= \theta_w - \theta_\infty)$, are obtained as shown in Fig. 5. Just as with natural convection in a circular chamber, there exists a critical (minimum) length of the side of the chamber $2A_{\min}$, for a given wall temperature θ_w at which a periodical oscillatory flow commences. If the height of the chamber $2A$ is smaller than $2A_{\min}$, however high the temperature of the line heat source θ_w may be, an oscillation cannot take place. The minimum length $2A_{\min}$ for

Fig. 5 Transition from steady flow to oscillatory flow for air

Fig. 6 Critical dimensions of chamber

present experiment and the minimum diameter for previous experiment $D_{\min}$ against wall temperature θ_∞ are shown in Fig. 6. From this figure, it is seen that the natural convection in a square chamber is unstable and becomes an oscillatory flow easily.

4. Experimental results and discussion for a rectangular chamber

4.1 Oscillation frequency

Typical experimental results are shown in Figs. 7, 8 and 9. In order to investigate the effect of the width of the chamber on the oscillation frequency, the results are shown for various widths of the chamber at constant height of the chamber. Figure 7 shows the relations between the oscillation frequency f and the number with dimension of temperature $\Theta = Q / \lambda Pr$, transformed heat input per unit length Q , at the height of the chamber 120 mm. The relation of f vs. Θ shows the same tendency as that of square section as follows:

$$f \propto \Theta^{0.40} \dots (9)$$

The oscillation frequency f is affected considerably and decreases monotonously with an increase in width of chamber $2B$. But in the case of $2B=200$ mm ($Z=0.60$), the oscillation frequency is higher than the other cases. The same tendency was obtained in the experiment of Itoh, et al.⁽³⁾, for $2B=182$ mm and $2A=100$ mm ($Z=0.55$). Figure 8 shows the effect of width of chamber on oscillation frequency at $2A=100$ mm. Similar results are obtained to those in the previous figure. In the case of $2B=150$ mm ($Z=0.67$) and $2B=200$ mm ($Z=0.50$), the oscillation frequencies shift to higher frequencies. When the aspect ratio Z is smaller than a certain value, the frequency

Fig. 7 Effect of width of chamber on oscillation frequency

Fig. 8 Effect of width of chamber on oscillation frequency

increases discontinuously. Figure 9 shows the effect of the height of the chamber $2A$ on the oscillation frequency at $2B=120$ mm. All data fall into three straight lines. The same results were obtained in the other cases with different widths of the chamber. The oscillation frequency can be classified into three patterns depending on the value of the aspect ratio Z .

The following characteristics become clear. The oscillation frequency f is independent of the height of the chamber in the direction of the gravity and is affected significantly by the width of the chamber in the horizontal direction. The frequency decreases with an increase of the length of width $2B$ and increases proportionally to $\Theta^{0.40}$, and varies stepwise at a certain aspect ratio, $Z(=A/B)$, of the chamber.

It is obvious that the width of the chamber is taken as the reference length of the dimensionless frequency. These experi-

mental data are rearranged by dimensionless groups based on the horizontal distance between the line heat source and the side wall B , in the same way as in the case of square chamber. The results are shown in Fig. 10. They are distributed on three straight lines. The dimensionless frequency, Re_{fB} , is related to the modified Rayleigh number, Ra^* , in the same form as Eq.(7) and can be expressed as follows:

$$6 \times 10^6 \leq Ra_B^* \leq 2.5 \times 10^8, 0.4 \leq Z \leq 2.0;$$

$$Re_{fB} = C (Ra_B^*)^{0.40} \dots\dots\dots(10)$$

By examining in detail which lines each datum belongs to, it is seen that the frequency varies discontinuously with the range of the aspect ratio Z . Assuming that constant C in Eq. (10) is dependent only on Z , $C(Z)$ can be expressed as follows:

$$C(Z) = Re_{fB} / (Ra_B^*)^{0.40} \dots\dots\dots(11)$$

The value of $C(Z)$ are calculated for all experimental data. The relations between aspect ratio Z and $C(Z)$ are shown in Fig. 11. The value $C(Z)$ is held constant over a range of the aspect ratios Z and decreases stepwise as Z increase. The value of constant $C(Z)$ against Z is given by the following equations.

$$\left. \begin{aligned} C(Z) &= 0.140 \quad (0.40 \leq Z \leq 0.70) \\ C(Z) &= 0.087 \quad (0.75 \leq Z \leq 1.50) \\ C(Z) &= 0.070 \quad (1.67 \leq Z \leq 2.0) \end{aligned} \right\} \dots\dots\dots(12)$$

4.2 Initiation conditions

The flow patterns of natural convection in an enclosed space vary with the shape and dimensions of the system and heat input. The flow patterns in rectangular chamber are very complicated compared with those in circular section and square section. Thus, it is difficult to summarize over a wide range of the aspect ratios. Then, the height of the chamber in the direction of gravity is taken as the reference length and the effect of the horizontal distance is considered by introducing the aspect ratio Z . Considering the results of square section, it has been assumed that the initial conditions under which the flow oscillation commences can be expressed by use of the following three dimensionless groups as follows:

$$\beta_f \Delta \theta = F \{ (A^3 g / \nu_f^2) Pr, Z \} \dots\dots\dots(13)$$

The relation of $\beta_f \Delta \theta$ vs. Z is plotted in Fig. 12. The dimensions of rectangular section for all points are expressed by $(2B, 2A)$ and are written on the graph. For $Z > 1$, it is seen from this figure that expansion number $\beta_f \Delta \theta$ is primarily dependent on aspect ratio Z . For $Z < 1$, the value of $\beta_f \Delta \theta$ has a strong dependence on the height of the chamber $2A$ and decreases in inverse proportion to $2A$. This same tendency was obtained for all cases in the range of the experiments. But the effect of the width of the chamber on

Fig. 9 Effect of height of chamber on oscillation frequency

Fig. 10 Dimensionless frequency correlations for rectangular chamber

Fig. 11 Relationship between constant C and aspect ratio Z

expansion number does not show clearly, then it can be considered that there is little effect of the width of the chamber. The relationships of $(8A^3g/v_f^2)Pr$ vs. $\beta_f\Delta\theta$ are shown in Fig. 13 in the same manner as the results of the square section. The results of $Z=1$ are shown in this figure. All of the data for $Z<1$ and $Z=1$, are distributed near the two straight lines respectively. In conclusion, the initiation conditions are given as follows:

$$\left. \begin{aligned} \beta_f\Delta\theta &= 88\{(8A^3g/v_f^2)Pr\}^{-0.33} \quad (0.4 \leq Z < 1) \\ \beta_f\Delta\theta &= 1100\{(8A^3g/v_f^2)Pr\}^{-0.50} \quad (Z=1) \\ \beta_f\Delta\theta &= 0.19 Z^{-1.70} \quad (1 < Z \leq 2) \end{aligned} \right\} \dots\dots\dots(14)$$

These equations correlate all of the data with maximum errors of $\pm 10\%$.

The results above are summarized as follows: the flow in the chamber with aspect ratio $Z > 1$ is very unstable and the expansion number at the initiation conditions is small, on the other hand, the flow of $Z \leq 1$ is fairly stable and the value of $\beta_f\Delta\theta$ is considerably large. This can be considered as follows. For $Z > 1$, the up-flow from the heat source is interfered with the down-flow along side walls because these flows are close and become unstable. For $Z \leq 1$, these flows are free from interference and remain stable.

5. Flow patterns

The flow patterns of oscillatory flow as shown in Fig. 2 are difficult to understand. The stable flow patterns were observed and photographed. Figure 14 shows representative photographs of flow patterns for various aspect ratios Z . Figure 15 shows the flow patterns for an increasing heat input, that is, the stable flows from (a) to (c) become oscillatory (d). The following characteristics became clear through the observations of the flow patterns. (i) The fluid in the upper region of the heat source moves at a relatively high speed and circulates, while the fluid in the lower region is nearly stagnant but moves at a very low speed. (ii) The circulating flow in the upper region forms a "scroll" flow moving towards the central part of the eddy. (iii) The fluids in the upper and lower regions move slowly in longitudinal direction of the chamber. Namely, the fluid in a plume forming a scroll, flows toward the end wall, then falls along the wall, and comes back to the central part of the longitudinal direction. Then the slowly moving fluid rises up in the nearly stagnant region of the lower side, moves up to the line heat source and joins again the circulating flow. (iv) This circulating flow

Fig. 12 Expansion number at initiation point

Fig. 13 Initiation condition for rectangular chamber

Fig. 14 Flow patterns for varying Z at $2A = 120$ mm

keeps its pattern unchanged regardless of the heat input.

As shown in Fig. 14, the scrolls of the circulating flow in upper region of the heat source can roughly be classified into three flow patterns depending on the aspect ratio of the chamber, Z , as follows: (a) egg-shaped eddy, (b) kidney-shaped eddy and (c) Γ -shaped eddy. The values of the aspect ratio for the three patterns, (a), (b) and (c) range $1 < Z$, $0.67 \leq Z \leq 1$ and $Z < 0.60$, respectively. These patterns depend not only on the aspect ratio Z , but also on the dimension of the chamber. For example, the third pattern (c), " Γ -shaped eddy", occurs only at low value of Z and at large dimension of the chamber.

There is a remarkable difference between the stable flow and the oscillatory flow as shown in Figs. 2 and 15. The fluid near the line heat source C flows up to and along the upper wall and goes down along the side wall, then separates from the side wall at the point A, where the force of inertia and the buoyancy force balance each other. When the buoyancy force surpasses the force of inertia, the fluid flows upward and goes over the peak B and returns to the line heat source. The primary differences between these two flows are in the shapes of the near peak B. For a stable flow, the peak descends as a gradual slope; for an oscillatory flow, the peak with steep slant falls down as a breaking of a wave crest. Then the plume above the heat source has a tilt to the broken side. This is considered to be the cause for "oscillation". As the heat input increases, the separation point A descends and the peak B rises. Then the sharp peak falls and an oscillatory flow occurs. Finally, the initiation conditions are considered to be given by the instability of the circulating region depending on the dimension, and the shape of lower side of the circulating region and the locus of the motion from separating point A to heat

source C, which depends on the heat input.

Examining the results of the oscillation frequencies and those of flow patterns, it is found that three types in regard to oscillation frequencies as classified depending on the aspect ratio Z , correspond respectively to three types of flows as shown in Fig. 14. As the aspect ratio Z increases, the size of the circulating region increases and the length of the locus of the circulating fluid increases. As the result, the period becomes longer and the oscillation frequency decreases corresponding to the three types as shown in Fig. 11.

6. Temperature profile and vertical velocity component in the plume

From the results of the present experiments for rectangular chamber and of previous report on circular cylinder, the frequencies of these oscillation phenomena are given by the following equation.

$$Re_f = C (Gr \cdot Pr)^{0.40} \dots \dots \dots (15)$$

In order to clarify the oscillation phenomena, it is important to establish a certain relation among the frequency and the temperature profiles and the velocity profiles in the chamber. However it is difficult to measure the unsteady flow with small fluid velocity in an enclosed chamber. Therefore, the measurements of the time averaged temperature profiles and the maximum vertical velocity component in the plume above a heat line source were performed.

In order to compare the experimental results with the theoretical solution, a plume above an infinitely long horizontal line heat source is considered as shown in Fig. 16. According to the results of Fujii's ⁽⁴⁾ study on steady laminar natural convection, the dominant nondimensional parameters in these phenomena are:

$$Gr_x^* = x^3 g \beta \theta / \nu^2 \text{ and } \theta = Q / \lambda Pr \dots \dots \dots (16)$$

The temperature profiles and the vertical velocity components are written as follows:

$$\theta - \theta_\infty = \theta Gr_x^{*1/5} h(\xi) \dots \dots \dots (17)$$

Fig. 15 Flow patterns for varying $\Delta\theta$ at $Z = 0.67$ ($2A = 80$ mm, $2B = 120$ mm)

Fig. 16 Geometry

$$ux/v = Gr_x^{*1/5} f_s'(\xi) \quad \text{.....(18)}$$

where $\xi = Gr_x^{*1/5} (y/x)$, h : nondimensional temperature, f_s : dimensionless stream function and f_s' : nondimensional vertical velocity component. The maximum vertical velocity component u_{max} and the maximum temperature $(\theta - \theta_\infty)_{max}$ are given at just above the line heat source $\xi = 0$ ($y = 0$). For $Pr = 0.70$ (air), the values of $h(0)$ and $f_s'(0)$ are 0.373 and 0.808 respectively. This theoretical solution is confirmed experimentally by Brodoricz, et al.⁽⁵⁾ and Fujii, et al.⁽⁶⁾

The measurements have been performed in the case of square chamber of $2A = 2B = 120$ mm. As is evident from Eq. (17), the vertical velocity component u does not change with the location x drastically. Then, two probes were set up at the elevation of $x = 10$ mm and 20 mm just above the line heat source ($y = 0$ mm). The vertical velocity component u_m at $x = 15$ mm is estimated by the average velocity between these probes. Some typical results are shown in Figs. 17, 18 and 19. Figure 17 (a) and (b) are the temperature profiles for steady flow and oscillatory flow, respectively. For steady flow, its experimental conditions are almost the same as those of Brodowicz,

(a) Steady flow

(b) Oscillatory flow

Fig. 17 Temperature distributions above a line heat source

et al.⁽⁵⁾, then, these same temperature profiles have been obtained. Figure 17 is rewritten by the universal functions ξ and h , and are shown in Fig. 18. For steady flow, the values near the heat source where x and y are small and agree fairly well with Fujii's theory. This indicates that the plume is laminar. When either x or y increases, the temperature of the ambient fluid of the plume rises due to the wall effects. Therefore, the experimental values are higher than those of the theoretical ones. The above tendency intensifies remarkably for the oscillatory flow regime.

Figure 19 shows the mean velocities u_m at $x = 15$ mm and $y = 0$ mm. There is no difference between steady flow and oscillatory flow. The results agree well with Fujii's theoretical solution: Eq. (18).

$$u_m x/v = 0.808 Gr_x^{*2/5} \quad \text{.....(19)}$$

Therefore, the vertical velocity component, u , is related to heat input, Θ , as follows:

$$u \propto \Theta^{2/5} \quad \text{.....(20)}$$

From Eq. (9) or Eq. (15), the oscillation frequency f is found to be in proportion to the vertical velocity component u . There exists a general relationship as shown in the following form.

$$Re_f \propto Re \quad \text{.....(21)}$$

This suggests that the period of the flow oscillation, $\tau (=1/f)$, corresponds to the period of the flow circulation.

Fig. 18 Dimensionless temperature distribution above a line heat source

Fig. 19 Dimensionless vertical velocity above a line heat source

7. Conclusions

Experimental investigations on the natural convective oscillatory flow of air caused by a line heat source placed concentrically along the axis of a horizontal rectangular chamber were carried out. The following conclusions were obtained.

(1) A periodic oscillation phenomenon can be observed in the modified Rayleigh number region of $6 \times 10^5 \leq Gr_B * Pr \leq 3 \times 10^8$.

(2) The frequency of the oscillatory flow can be classified into three types depending on the range of the values of the aspect ratio of the chamber Z . For each range of aspect ratios, the frequency can be expressed as a function of both the heat input and the width of the chamber regardless of the height of the chamber. The oscillation frequencies are given as follows:

$$Re_{fB} = C(Z) (Gr_B * Pr)^{0.40}$$

where

$$C(Z) = 0.140 \quad (0.40 \leq Z \leq 0.70)$$

$$C(Z) = 0.087 \quad (0.75 \leq Z \leq 1.50)$$

$$C(Z) = 0.070 \quad (1.67 \leq Z \leq 2.0)$$

(3) The larger the aspect ratio Z , the more the oscillatory flow is apt to occur because of the instability of the system. The initial conditions under which the oscillatory motion commences can be expressed in terms of the expansion number $\beta_f \Delta \theta$, as follows;

$$\beta_f \Delta \theta = 88 \{ (8A^3 g / \nu_f^2) Pr \}^{-0.33} \quad (0.4 < Z < 1)$$

$$\beta_f \Delta \theta = 1100 \{ (8A^3 g / \nu_f^2) Pr \}^{-0.50} \quad (Z = 1)$$

$$\beta_f \Delta \theta = 0.19 Z^{-1.70} \quad (1 < Z \leq 2)$$

(4) The plume above a line heat source becomes a circulating flow and forms a scroll moving towards the center. Three stable types of scrolls are observed depending on the value of the aspect ratio as

follows: (a) egg-shaped eddy, (b) kidney-shaped eddy and (c) Γ -shaped eddy.

(5) The temperature distributions in the plume above a horizontal line heat source are considerably higher than those of theoretical solution of the steady laminar natural convection in the infinite space, but the maximum vertical velocity components in the plume agree well with those of theory.

(6) The oscillation frequency f is closely related with the vertical velocity component in the plume above a line heat source u by the following equation.

$$f B^2 / \nu \propto u x / \nu ; (Re_f \propto Re).$$

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