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New Measuring Method of Impact Force*

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The impact force is measured usually by the method of Hopkinson Pressure Bar. However, the measurable time range of this method is restricted by the length of the bar. The author suggests a usefull measuring method based on the theory of propagation of longitudinal elastic stress wave.

In this method, the external force, ranging from quasi-static to impact, at the end of a cylindrical bar can be estimated by measuring stresses at two suitable points on it. Also the length of the measuring bar can be shorter than in another methods.

1. Introduction

In the high speed blanking and compression test, the impact force is measured usually by the method of Hopkinson bar using a long cylindrical bar with a uniform cross sectional area.

However, because the measurable time range is restricted by the length of the bar, this method can not be used when the measuring time is too long.

In this paper, the author shows that, by measuring the stresses at two suitable points, stress waves traveling in two opposite directions can be separated by use of the theory of propagation of longitudinal elastic stress wave, and the impact force working at the end of the bar can be estimated without the restriction of its time range.

In this method, the measuring bar of a considerably short length can be used.

2. Separation of the stress wave

In the cylindrical bar with uniform cross sectional area, we take points *a* and *b* with distance l_1 as shown in Fig.1-(a).

In the case that the stress wave travels from the L-direction, the stress passing over the point *a* at the time *T* is $\sigma_L(T)$. This stress wave which travels in the R-direction with its velocity $C = \sqrt{gE/\gamma}$ will pass over the point *b* after time distance $t_1 = l_1/C$ as shown in Fig.1-(b), where *g* is the acceleration of the gravity, *E* is the modulus of elasticity of material and γ is weight per unit volume.

The relation is formulated as follows:

$$\sigma_L(T) = \sigma_a(T) = \sigma_b(T + t_1) \quad (1)$$

In the case of the stress wave traveling from the R-direction as shown in Fig.1-(c),

we have

$$\sigma_R(T) = \sigma_a(T) = \sigma_b(T - t_1) \quad (2)$$

If two waves traveling in two opposite directions come together as shown in Fig.1-(d), two momentary stresses $\sigma_a(T)$ and $\sigma_b(T)$ at the time *T* are formulated as follows:

$$\sigma_a(T) = \sigma_L(T) + \sigma_R(T) \quad (3)$$

$$\sigma_b(T) = \sigma_L(T - t_1) + \sigma_R(T + t_1) \quad (4)$$

From Eqs.(3) and (4), we have

$$\sigma_L(T) = \sigma_a(T) - \sigma_R(T) \quad (5)$$

$$\sigma_R(T) = \sigma_b(T - t_1) - \sigma_L(T - 2t_1) \quad (6)$$

$$\therefore \sigma_L(T) = \sigma_a(T) - \sigma_b(T - t_1) + \sigma_L(T - 2t_1) \quad (7)$$

The last term of the right hand side of Eq.(7) is expressed as follows:

$$\left. \begin{aligned} \sigma_L(T - 2t_1) &= \sigma_a(T - 2t_1) - \sigma_b(T - 3t_1) + \sigma_L(T - 4t_1) \\ \sigma_L(T - 4t_1) &= \sigma_a(T - 4t_1) - \sigma_b(T - 5t_1) + \sigma_L(T - 6t_1) \\ &\dots\dots\dots \\ \sigma_L(T - 2kt_1) &= \sigma_a(T - 2kt_1) \\ &\quad - \sigma_b(T - t_1 - 2kt_1) + \sigma_L(T - 2(k+1)t_1) \end{aligned} \right\} \quad (7)'$$

$$\therefore \sigma_L(T) = \sum_{n=0}^k \sigma_a(T - 2nt_1) - \sum_{n=0}^k \sigma_b(T - t_1 - 2nt_1) + \sigma_L(T - 2(k+1)t_1) \quad (8)$$

Fig.1 Stress due to the superposition of elastic waves

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Substituting Eq.(8) into Eq.(6), we find

$$\begin{aligned}\sigma_R(T) &= \sigma_b(T-t_1) - \sum_{n=1}^k \sigma_a(T-2nt_1) \\ &+ \sum_{n=1}^k \sigma_b(T-t_1-2nt_1) - \sigma_L(T-2(k+1)t_1) \\ &= \sum_{n=0}^k \sigma_b(T-t_1-2nt_1) - \sum_{n=1}^k \sigma_a(T-2nt_1) \\ &- \sigma_L(T-2(k+1)t_1) \dots\dots\dots (9)\end{aligned}$$

If $\sigma_L(T-2(k+1)t_1)$ is known, two momentary stresses $\sigma_L(T)$ and $\sigma_R(T)$ will be separated by Eqs.(8) and (9) using the measured stresses $\sigma_a(T)$ and $\sigma_b(T)$. If static stress σ_s is applied to the cross sectional area of the bar at time $T-2(k+1)t_1$, the stress $\sigma_L(T-2(k+1)t_1)$ is equal to $\sigma_s/2$. In the case that the stress is not applied, the stress $\sigma_L(T-2(k+1)t_1)$ is zero.

3. Stress at an arbitrary point c

The stress at a point c with arbitrary distance l_2 from the point a can be obtained from the separated stress waves traveling in two opposite directions (Fig.2). The stress wave traveling from L-direction passes over the point c at the time distance $t_2=l_2/c$ before it passes over the point a. The stress wave traveling from R-direction passes over the point c at the time interval t_2 after it passes over the point a. Therefore, the momentary stress at the point c at the time T can be represented in the form,

$$\sigma_c(T) = \sigma_L(T+t_2) + \sigma_R(T-t_2) \dots\dots\dots (10)$$

Substituting Eqs.(8) and (9) as staggered by the time t_2 into Eq.(10), we find

$$\begin{aligned}\sigma_c(T) &= \sum_{n=0}^k \sigma_a(T+t_2-2nt_1) \\ &- \sum_{n=0}^k \sigma_b(T-t_1+t_2-2nt_1) + \sigma_L(T+t_2-2(k+1)t_1) \\ &+ \sum_{n=0}^k \sigma_b(T-t_1-t_2-2nt_1) \\ &- \sum_{n=1}^k \sigma_b(T-t_2-2nt_1) - \sigma_L(T-t_2-2(k+1)t_1) \dots\dots\dots (11)\end{aligned}$$

In this case, as stresses $\sigma_L(T+t_2-2(k+1)t_1)$ and $\sigma_L(T-t_2-2(k+1)t_1)$ are known, the stress $\sigma_c(T)$ can be obtained from the stresses measured at points a and b. In order to simplify the above equation, we take the same distance between points a and c, and between a and b. Then we obtain, $l_2=l_1$ and $t_2=t_1=l_1/c$. Substituting these into Eq.(11), we find

$$\begin{aligned}\sigma_c(T) &= \sum_{n=0}^k \sigma_a(T+t_1-2nt_1) \\ &- \sum_{n=0}^k \sigma_b(T-2nt_1) + \sigma_L(T+t_1-2(k+1)t_1) \\ &+ \sum_{n=0}^k \sigma_b(T-2(n+1)t_1) \\ &- \sum_{n=1}^k \sigma_a(T-t_1-2nt_1) - \sigma_L(T-t_1-2(k+1)t_1) \dots\dots\dots (12)\end{aligned}$$

$$\begin{aligned}\sigma_c(T) &= \sigma_a(T+t_1) + \sigma_a(T-t_1) - \sigma_a(T-t_1-2kt_1) \\ &- \sigma_b(T) + \sigma_b(T-2(k+1)t_1) \\ &+ \sigma_L(T+t_1-2(k+1)t_1) - \sigma_L(T-t_1-2(k+1)t_1) \dots\dots\dots (13)\end{aligned}$$

Rearranging this equation by using Eq.(7), we have

$$\begin{aligned}\sigma_L(T+t_1-2(k+1)t_1) &= \sigma_a(T+t_1-2(k+1)t_1) \\ &- \sigma_b(T-2(k+1)t_1) + \sigma_L(T-t_1-2(k+1)t_1) \dots\dots\dots (14)\end{aligned}$$

Substituting this formula into Eq.(13), we find

$$\sigma_c(T) = \sigma_a(T+t_1) + \sigma_a(T-t_1) - \sigma_b(T) \dots\dots\dots (15)$$

Therefore, the stress at the point c is obtained from the stresses measured at points a and b.

4. Impact force applied to the end of the bar

Consider now the case of the left end of the bar being free as shown in Fig.3-(a). The stress wave from the R-direction passes over the point b at first time. This stress wave passes over the point a after time interval $t_1=l_1/c$, and arrives at the end of the bar at time interval $t_2=l_2/c$ after passing over the point a. This stress wave is reflected at the free end of the bar as a similar opposite wave and travels in the R-direction. After a time interval t_2 , the reflected wave arrives at the point a. This time, the stress $\sigma_L(T)$ from the L-direction at the point a is

$$\sigma_L(T) = -\sigma_R(T-t_2) \dots\dots\dots (16)$$

Fig.2 Location of measuring points a, b, and calculated point c

Fig.3 Stresses due to impulsive load and traveling wave

If the external force is applied to the left end of the bar at the time T , the stress $\sigma_F(T)$ which is generated at this point arrives at the point a after the time interval t_2 . In this case, the magnitude of the stress wave traveling from the L -direction at time T is

$$\sigma_L(T) = \sigma_F(T - t_2) - \sigma_R(T - 2t_2) \quad \dots\dots\dots(17)$$

$\sigma_F(T)$ is given by

$$\sigma_F(T) = \sigma_L(T + t_2) + \sigma_R(T - t_2) \quad \dots\dots\dots(18)$$

The right side of this equation is the same as that of Eq.(10). Therefore, the stress $\sigma_F(T)$ which is generated by impact force F applied to the end of the bar can be obtained by the use of the same equations as Eqs.(11) and (15). In the case of $l_1 \neq l_2$, Eq.(18) becomes

$$\begin{aligned} \sigma_F(T) = & \sum_{n=0}^k \sigma_a(T + t_2 - 2nt_1) - \sum_{n=0}^k \sigma_b(T - t_1 + t_2 - 2nt_1) \\ & + \sigma_L(T + t_2 - 2(k+1)t_1) + \sum_{n=0}^k \sigma_b(T - t_1 - t_2 - 2nt_1) \\ & - \sum_{n=1}^k \sigma_b(T - t_2 - 2nt_1) - \sigma_L(T - t_2 - 2(k+1)t_1) \quad \dots\dots\dots(19) \end{aligned}$$

In this case, if the stresses $\sigma_L(T + t_2 - 2(k+1)t_1)$ and $\sigma_L(T - t_1 - 2(k+1)t_1)$ are known, $\sigma_F(T)$ is obtained. In the case of $l_1 = l_2$, Eq.(19) becomes

$$\sigma_F(T) = \sigma_a(T + t_1) + \sigma_a(T - t_1) - \sigma_b(T) \quad \dots\dots\dots(20)$$

The stress $\sigma_F(T)$ which is generated by the impact force F applied to the end of the bar can be obtained from the stresses measured at points a and b . Assuming that the stress $\sigma_F(T)$ is uniformly distributed over the cross section at the measuring point of an arbitrary distance from the end of the bar, the impact force F applied to the end of the bar with cross sectional area A is given by

$$\begin{aligned} F(T) &= \sigma_F(T)A \\ &= (\sigma_a(T + t_1) + \sigma_a(T - t_1) - \sigma_b(T))A \quad \dots\dots\dots(21) \end{aligned}$$

5. Experiment and result

In order to verify this measuring method, an experiment was made as follows. A block diagram of the experimental arrangement is shown in Fig.4. The impact bar which is accelerated to a velocity of about 5 meters per second by the accelerator or using the compressed air collides with the tail of the measuring bar. The output of the semiconductor strain gauges installed at two points of the measuring bar is conveyed to the oscilloscope and the data are recorded by a 35mm camera.

The measuring bar is shown in Fig.5.

The measuring points a and b have distances 25mm and 50mm from the end of the bar.

The measured results of the stresses at two points a and b are shown in Fig.6-(a,b). The stress of the end of the measuring bar calculated from the stresses at these two points is shown in Fig.6-(c).

Fig.4 Block diagram of the experimental arrangement

Fig.5 Measuring bar

Fig.6 Measured and calculated stresses

In this experiment, as the top of the measuring bar is not applied with the external force, the stress at the top of the bar must be zero. The calculated result is in good agreement with the above prediction.

6. Conclusions

From the result obtained by use of the theory of propagation of longitudinal elastic stress wave, the conclusion about the measuring method of the impact force applied to the end of the bar is obtained as follows

- (1) Making use of the measured stresses at two points with distances l_1 and l_2 respectively from the end of the bar, without the restriction of time, the external force applied to the end of the bar can be obtained.
- (2) In this method, the part with a uniform cross section of measuring bar can be as short as the length which is required for the uniform stress distribution at the measuring points.

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