Response Analysis of 500KV Circuit Breaker with Nonlinear Damping Devices under Seismic Excitation

By Shigeru FUJIMOTO**, Taro SHIMOGO*** and Mitsuru ARII****

This paper deals with a fundamental research for the aseismic design of a 500KV air circuit breaker. In particular, effects of nonlinearity of the damping device, which is connected to stays, on the seismic response of the circuit breaker are theoretically investigated.

The model for the analysis is constructed of a simplified dynamic model of the circuit breaker structure, and it is assumed that the seismic wave is a nonstationary Gaussian white noise in the horizontal direction.

The Fokker-Planck equation is first established with the aid of equations of motion which are derived without consideration of geometrical non-linearities. Further, moment equations are introduced with the assumption of Gaussian random responses, and the time history of the mean squared response is obtained by numerical solutions of the moment equations. From this result, effects of various parameters of the structure were made clear.

The transient response to the impulse acceleration is also investigated through simulation by a digital computer.

1. Introduction

The aseismic design of electrical distribution equipment has been developed on the basis of a static analysis or a dynamic analysis of the sinusoidal wave input.

Since the size of such equipment has recently become larger, the dynamic response analysis of the equipment under nonstationary random excitation has attracted attention in the field of aseismic design).

A large circuit breaker, such as a 300KV air circuit breaker, is installed on the top of a slender support column, 6 - 7 m in height, and is supported by three stays connected with friction dampers, as roughly illustrated in Fig. 1. For such a structure, the nonlinear characteristics of the friction damper play an important role in the aseismic design.

In this paper, the dynamic model of the 500KV air circuit breaker is simplified as illustrated in Fig. 2, and the response of the circuit breaker subjected to seismic wave is investigated by theoretical calculation, particularly in order to study the influence of the initial tension of the friction dampers and the stiffness of the bottom of the support column.

In this analysis, it is assumed that the seismic wave is a nonstationary random wave, and the time history of the mean squared value of response caused by this random input is calculated. And further, the impulse response of this model is examined through a digital computer simulation.

In the analytical treatment, the equations of motion are first established in consideration of the hysteresis characteristics of friction dampers, and then the Fokker-Planck equation governing the probability density function of the response is derived. From this equation, nonlinear simultaneous differential equations with respect to the moments of response are obtained with the assumption that the probability density of response has a Gaussian distribution. These moment equations are numerically solved under the assumption that the seismic wave input has the power spectral density of nonstationary white noise.

Nomenclature

qi, bj : x, y coordinates of the lower end of stay (i = 1,2,3).

Fig. 1 The scheme of 500KV AEM-type air circuit breaker (1/2 phase)
\( A_x, A_y, B_x, B_y, A_5, B_5 \): coefficients in equations of motion.
\( c_1, c_2 \): parameters of exponential function included in \( D(t) \).
\( c_1', c_2' \): dimensionless expression of \( c_1 \) and \( c_2 \).

\( D(t) \): Function expressing the nonstationarity of \( U(t) \).
\( D_{eq} \): coefficient in \( D(t) \).
\( D_0 \): dimensionless expression of \( D_{eq} \).
\( q \): gravitational acceleration.
\( K \): stiffness of the bottom of support column (supporting porcelain).
\( K_4 \): dimensionless expression of \( K \).
\( k, k' \): spring constants of ring spring.
\( k_d, k_d' \): dimensionless expressions of \( k \), \( k' \).
\( k_0 \): mean value of \( k_d \) and \( k_d' \).
\( L \): height of support column (supporting porcelain).
\( L_0 \): length of stay.
\( M_1 = m_1 + m_p/3 + m_3 \).
\( M_2 = m_1 + m_p/2 \).
\( m_1 \): mass of interrupting chamber.
\( m_2 \): mass of support column.
\( m_3 \): mass of each stay.
\( M_{ij} \): 2nd order moment of \( n_i, n_j \) (1,1, 2, 3, 4).
\( m_{ij} \): 2nd order moment of \( y_i, y_j \).
\( p \): joint probability density function of \( y_1, y_2, y_3, y_4 \).
\( P_0 \): frictional force of ring spring at equilibrium point.
\( P^* \): dimensionless expression of \( P_0 \).
\( r \): radius of circle on which the lower ends of stays are located.
\( r_1 = L/L \).
\( S(q(t)) \): power spectral density function of \( U(t) \).
\( T_0 \): initial tension of each stay.
\( T_d \): dimensionless expression of \( T_0 \).
\( T_1 \): tension of stay \((i = 1,2,3)\).
\( t \): time.
\( U(t) \): acceleration of seismic excitation.
\( U(t) = U(t)/g \).
\( W(q(t)) \): dimensionless expression of \( S(q(t)) \).
\( x, y, z \): relative displacements of head of circuit breaker.
\( y_1 = x, y_2 = y, y_3 = z, y_4 = y \).
\( \alpha_i \): parameter of the frictional force of ring spring \((i = 1,2,3)\).
\( \gamma \): angle between the direction of \( U(t) \) and \( x \)-axis.
\( \delta \): variation of the deflection of ring spring \((i = 1,2,3)\).
\( c(t) \): function expressing the irregularity of \( U(t) \).
\( n_i = y_i/L \) \((i = 1,2,3,4)\).
\( \kappa_i \): parameter of the spring constant of ring spring \((i = 1,2,3)\).
\( \mu = M_2/M_1 \).
\( \nu \): dimensionless expression of \( t \).
\( \omega_0 \): reference angular frequency.

2. Dynamic model of circuit breaker

The structure of the 500KV air circuit breaker is schematically shown in Fig. 12. The circuit breaker is installed on the top of a support column (supporting porcelain), the bottom of which is connected with a foundation rack. Three stays are fixed to the top of the support column, and the lower end of each stay is connected with the foundation rack through a ring spring. Initially, tension is given to each stay for stabilizing the support of the circuit breaker.

The auxiliary circuit breaker is also supported on the same foundation rack. Both of the circuit breakers are connected with each other by a joint conductor and together make up a 1/2 phase circuit breaker. In this paper, however, the coupled effect of an auxiliary circuit breaker is not taken into account.

In order to examine the effects of the frictional force of the ring spring and the stiffness of the support column on the seismic response of the circuit breaker, the dynamic model of structure is simplified under the following assumptions:

1. Interrupting chamber (the head of the circuit breaker)

According to the vibration test results of the circuit breaker subjected to 3 cycles of 0 3g sinusoidal wave of resonance frequency, the deformation of the interrupting chamber itself is negligibly small and the chamber is considered to be rigidly connected with the support column. Since the distance between the center of gravity of the interrupting chamber and the upper end of the stay is small compared with the height of the support column (about 1/11), the interrupting chamber is assumed to be represented by a single mass point on the upper end of the support column.

2. Support column

According to the above mentioned vibration test results, the maximum deflection of the support column is small in comparison with the height of the support column (about 0.01 - 0.015). In this case, the effect of geometric nonlinearity caused by the deflection of the column can be neglected. Further, since the vibration of the higher modes of the support column is considered to be small compared with that of the first mode, it can be assumed that the support column is a uniform rigid bar, and its bottom is replaced by a flexible joint. Under this assumption, the restoring moment produced in the column is proportional to the angle of inclination.

Fig. 2 The dynamic model of circuit breaker structure
of the column, and its stiffness is assumed to be constant regardless of the direction of inclination. The equivalent mass of the column is added to the interrupting chamber.

(3) Stays
According to the vibration test results, a transverse vibration of the first mode of the stay itself is observed. However, since the mass of the stay is small compared with the mass of the interrupting chamber (about 1/15 - 1/10) and the amplitude of the vibration is much smaller than the length of the stay, the influence of the vibration of the stay on the response of the circuit breaker can be neglected. Then, it is assumed that the three stays are rigid, both ends of each stay are supported by turning pairs and the equivalent masses of the stays are added to the interrupting chamber.

(4) Ring springs
The ring springs act as friction dampers whose characteristics are given as nonlinear hysteresis described in section 4. According to the vibration test, the hysteresis curve on the load-deflection diagram of the ring spring can be fairly well approximated by the characteristics as shown in Fig. 3.

(5) Foundation rack
Since the foundation rack is constructed so rigidly as not to resonate with the circuit breaker, the deformation of the foundation rack is neglected.

(6) Seismic input
In this paper, the seismic input is given as a horizontal vibration. However, it is difficult to represent the nonstationary characteristics and the power spectrum of the seismic wave for the general case. Since the purpose of this paper is to investigate the effects of the initial tension of stays and the stiffness of the support column, the power spectrum of the seismic wave is assumed to be uniform in the range of frequencies which are significant for the response of the circuit breaker, and in the analysis of this paper, the seismic input is approximated by a nonstationary Gaussian white noise as described in section 3.

The simplified dynamic model of the circuit breaker structure, which is constructed under the above mentioned assumptions, is illustrated in Fig. 2.

3. Input seismic wave
In general, the seismic input is regarded as a random vibration having remarkably nonstationary characteristics. The input acceleration $U(t)$ of the foundation is expressed here as the product of a function $D(t)$, which varies slowly, and a stationary random function $Z(t)$ (the mean value $E(Z) = 0$). That is,

$$Z(t) = D(t)Z(t)$$  \hspace{1cm} (1)

and the mean value $E(U(t)) = D(t)E(Z(t)) = 0$. If the function $D(t)$ is expressed by Eq. (2), it has greater possibilities for the description of envelopes of real seismic accelerations.

$$D(t) = D_0(e^{-\frac{t}{C_1}} - e^{-\frac{t}{C_2}})$$

where $C_1$ and $C_2$ are respectively determined according to the time when the envelope of the seismic wave is maximum. ($0 < C_1 < C_2$)

Suppose $Z(t)$ is a Gaussian white noise in order to simplify the theoretical treatment, although $Z(t)$ has generally a dominant frequency. Then the auto-correlation function of $Z(t)$ is represented by

$$\rho(Z(t), Z(t+\Delta t)) = \delta(\Delta t)$$  \hspace{1cm} (2)

where $\delta(t)$ is Dirac's delta function, and the auto-correlation function of $U(t)$ is expressed as follows:

$$U(t) = \rho(D(t)Z(t)) = D(t)D(t+\Delta t)\delta(\Delta t) = D(t)^2$$

The power spectral density function of $U(t)$ is therefore reduced to

$$S(U) = 2D(t)^2 = 2D_0^2(e^{-\frac{t}{C_1}} - e^{-\frac{t}{C_2}})^2$$

($-\infty < t < \infty$)  \hspace{1cm} (3)

It is now assumed that the direction of input acceleration is horizontal and makes an angle $\gamma$ with the x-axis (Fig. 2).

4. Characteristics of friction damper
A load-deflection diagram of the ring spring, which is provided for each stay, is shown in Fig. 3. The tension acting on the stay behaves as a compressive load on the ring spring. When the tension $T$ is increasing, the stiffness of the ring spring is denoted by $k$, and when $T$ is decreasing, the stiffness is $k' (< k)$. When the direction of spring deflection $w$ changes, namely $\dot{w} = 0$, it is assumed that the value of $T$ jumps on the broken line which lies in the middle of bilinear characteristics. When the amount of the spring deflection is $w_0$, the frictional force $F_0$ at this equilibrium point is estimated by the expression:

$$F_0 = (k-k')\overline{W} = T_0(k-k')$$

That is, $F_0$ is proportional to $T_0$. If the variation of deflection from the equilibrium point is expressed by $\delta$, then the variation of tension acting on the stay is reduced to

$$T(\delta) = T_0 + k\delta$$

$$(\delta > 0)$$

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$$(\delta < 0)$$

$$F_0, k, k' > 0$$

where $k$ and $k'$ represent the spring constants of the ring spring.

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where $k$ and $k'$ represent the spring constants of the ring spring.

5. Equation of motion
On account of the balance of the moments about the bottom of the support column (origin 0 in Fig. 2), the following equation is introduced:

$$\tau'(x) + [H + K_T(T_1 + T_2 + T_3 + G_1 + G_2)/2] = 0$$  \hspace{1cm} (4)

where $\tau$: position vector of the center of gravity of the interrupting chamber; $H$: inter-
tia force vector acting on \( P \) due to the equivalent mass \( m_1 \) of the interrupting chamber, the equivalent mass \( m_2/3 \) of the support column and the equivalent mass \( 3(m_3/3) \) of the three stays; \( K \): restoring force vector acting on \( P \) by the restoring moment at the bottom of the support column; \( T_1, T_2, T_3 \): tension vector acting on each stay, \( G_1 \): gravity vector acting on \( P \); \( G_2 \): gravity vector acting on the center of gravity of the support column.

When the relative position vector of \( P \) is represented by \( r(x, y, z) \) and the input acceleration vector at the origin is given by \( \Omega(u \cos \psi, v \sin \psi, 0) \), the absolute acceleration vector at \( P \) becomes \( (x + u \cos \psi, y + u \sin \psi, z) \) and the inertia force vector is reduced to

\[
H = (m_1 + m_2/3 + m_3) \cdot (x + u \cos \psi, y + u \sin \psi, z)
\]

(10)

If \( x \) and \( y \) are very small compared with the height of the support column \( L \), then \( z \approx L \) and \( \psi \approx 0 \), and the inclination angle \( \theta \) of the support column for the z-axis becomes \( \alpha = \sqrt{x^2 + y^2}/L \). Denoting the stiffness of the bottom of the support column by \( K \), the restoring moment is \( K \theta \) and the magnitude of the restoring force vector acting on \( P \) becomes \( K \theta = K \sqrt{x^2 + y^2}/L \). Thus

\[
K = (K/L^2)(x, y, 0)
\]

(11)

If the lower end position of each stay and the magnitude of tension are expressed by \( T_i \) \((x_i, y_i, 0)\) and \( T_1 \) respectively, then

\[
T_i = \left( T_1/\sqrt{(x_i - x)^2 + (y_i - y)^2 + z^2} \right) \cdot (a_i - x, b_i - y, -z) \quad (i = 1, 2, 3)
\]

(12)

where \( z \approx L \). Arranging the lower ends of each stay at equal intervals on a circle with radius \( R \) as shown in Fig. 2, then

\[
a_1 = -a_2 = (\sqrt{3}/2) R, b_1 = b_2 = (1/2) R, a_3 = 0, b_3 = -R.
\]

The gravity vectors acting on the head of the circuit breaker and on the support column are

\[
G_1 = (0, 0, -m_2 g)
\]

\[
G_2 = (0, 0, -m_3 g)
\]

respectively. Substituting Eqs. (10) - (14) into Eq.(9) and separating the equation into \( x \) and \( y \) components, the equations of motion are represented as follows:

\[
x \cdot (K/M_1 L^2 - M_2 g/M_1 L) x = \frac{1}{2} M_1 \frac{d^2}{dt^2} \left( (x - x_i)^2 + (y - y_i)^2 + z^2 \right) = -u \cos \psi
\]

\[
y \cdot (K/M_1 L^2 - M_2 g/M_1 L) y = \frac{1}{2} M_1 \frac{d^2}{dt^2} \left( (x - x_i)^2 + (y - y_i)^2 + z^2 \right) = -u \sin \psi
\]

where \( M_1 = m_1 + m_2/3 + m_3, M_2 = m_1 + m_2/2 \).

From Eq. (7) the variation of tension \( F_i \) can be expressed as follows:

\[
F_i = a_i x_i + b_i y_i + d_i \quad (i = 1, 2, 3)
\]

(16)

where

\[
a_1 = m_0, \quad c_i = k \quad \text{for} \quad \delta < 0
\]

\[
a_1 = 0, \quad c_i = (k + k')/2 \quad \text{for} \quad \delta = 0
\]

\[
a_1 = -m_0, \quad c_i = k' \quad \text{for} \quad \delta > 0
\]

The deflection \( \delta \) and its velocity \( \dot{\delta} \) of the ring spring are

\[
\delta = \sqrt{(a_i - x)^2 + (b_i - y)^2 + L^2 - L_0}
\]

(18)

respectively, where

\[
L_0 = \left( \sqrt{(a_i - x)^2 + (b_i - y)^2 + L^2} \right)^2
\]

(19)

Hence the tension acting on each stay becomes

\[
T_i = T_0 + a_i x_i + b_i y_i + d_i
\]

(20)

Substituting Eq. (20) into Eqs. (12) and (15), and expanding the third term of the left side of each equation in power series of \( x \) and \( y \), and further neglecting the small terms higher than the second order, then

\[
x \cdot A_2 x + b_0 y + d_0 + \cdots = \frac{1}{2} \cdot u \cos \psi
\]

\[
y \cdot B_2 x + b_0 y + d_0 + \cdots = \frac{1}{2} \cdot u \sin \psi
\]

(21)

where

\[
A_2 = a_2 x + b_2 y + c_2 \quad \text{for} \quad \delta_0
\]

\[
B_2 = a_2 x + b_2 y + c_2 \quad \text{for} \quad \delta_0
\]

\[
A_0 = a_0 x + b_0 y + c_0 \quad \text{for} \quad \delta_0
\]

\[
B_0 = a_0 x + b_0 y + c_0 \quad \text{for} \quad \delta_0
\]

\[
c_2 = (1/M_1) \cdot \frac{a_2}{a_0} \quad \text{for} \quad \delta_0
\]

\[
c_0 = (1/M_1) \cdot \frac{a_0}{a_0} \quad \text{for} \quad \delta_0
\]

\[
c_1 = (1/M_1) \cdot \frac{a_1}{a_0} \quad \text{for} \quad \delta_0
\]

\[
c_0 = (1/M_1) \cdot \frac{a_0}{a_0} \quad \text{for} \quad \delta_0
\]

\[
b_0 = a_0 x + b_0 y + c_0 \quad \text{for} \quad \delta_0
\]

\[
b_1 = a_1 x + b_1 y + c_1 \quad \text{for} \quad \delta_0
\]

\[
b_2 = a_2 x + b_2 y + c_2 \quad \text{for} \quad \delta_0
\]

\[
b_0 = a_0 x + b_0 y + c_0 \quad \text{for} \quad \delta_0
\]

\[
b_1 = a_1 x + b_1 y + c_1 \quad \text{for} \quad \delta_0
\]

\[
b_2 = a_2 x + b_2 y + c_2 \quad \text{for} \quad \delta_0
\]

(23)

where \( a_x, a_y, a_0, a_1, a_2 \) and \( b_x, b_y, b_0, b_1, b_2 \) are equal to the expressions of \( a_0, a_1, a_2 \) and \( b_0, b_1, b_2 \) where \( m_0 \) is replaced by \( m_1 \).

The parameters shown in Eq.(22) are the functions of \( \delta_0 \) and \( \delta_1 \), which vary discretely according to the sign of \( \delta_0 \) as shown in Eq. (17). The sign of \( \delta_0 \) is determined in accordance with the sign of \( (x - a_0)^2 + (y - b_0)^2 \). As shown in Eq. (18), that is,

\[
a_i = P_0, \quad c_i = k \quad \text{for} \quad a_i x + b_i y < 0
\]

\[
a_i = 0, \quad c_i = (k + k')/2 \quad \text{for} \quad a_i x + b_i y = 0
\]

\[
a_i = P_0, \quad c_i = k' \quad \text{for} \quad a_i x + b_i y > 0
\]

(24)

6. Fokker-Planck equation

In the response analysis of a nonlinear system with a nonstationary random input, the Fokker-Planck equation governing the probability density function of the response is derived for the general case.

Putting \( x = y_1, y = y_2, z = y_3, \dot{x} = y_4 \) and rewriting the equations of motion (21) by the first order differential equations with respect to four variables, then
\[ \begin{align*}
    \dot{y}_1 &= y_2 \\
    \dot{y}_2 &= \frac{1}{2} \left( 3v y_1 - 2y_2 \right) \\
    \dot{y}_3 &= -A y_1 y_3 - A y_2 y_4 - A - \cos y_3 \\
    \dot{y}_4 &= -B y_1 - B y_2 - B_0 - \sin y_4
\end{align*} \] (25)

When the input \( U(t) \) is a nonstationary Gaussian white noise as described section 3, with the aid of Eq. (25), the Fokker-Planck equation relating to the joint probability density function \( p(y_1, y_2, y_3, y_4, t) \) is

\[ \begin{align*}
    \frac{\partial}{\partial t} p &= \frac{\cos^2 y_1}{2} \left( \frac{\partial^2 p}{\partial y_1^2} + \frac{\sin^2 y_1}{2} \frac{\partial^2 p}{\partial y_3^2} - \frac{\partial p}{\partial y_1} \right) \\
    &+ \frac{3}{2} \frac{\partial p}{\partial y_2} + \frac{3}{2} \frac{\partial p}{\partial y_3} \\
    &- \frac{3}{2} \frac{\partial p}{\partial y_4} + \frac{3}{2} \left( A y_1 y_2 + A y_3 y_4 + A y_4 y_1 \right) \\
    &+ \frac{3}{2} \left( B y_1 y_2 + B y_3 y_4 + B y_4 y_1 \right)
\end{align*} \] (26)

If the initial conditions are \( y_1(0) = y_2(0) = y_3(0) = y_4(0) = 0 \) at \( t = 0 \), then \( p(0,0,0,0,t) = \delta_4(t) \). (\( \delta_4(t) \): Dirac's delta function)

7. Moment equation

Since it is difficult to solve the Fokker-Planck equation derived in the preceding section, the authors derive the equations relating to the moments of the response from the Fokker-Planck equation.

Multiplying both sides of Eq. (26) by \( y_1 y_2 y_3 y_4 \) and integrating them over the whole range of \( y_1, y_2, y_3, \) and \( y_4 \), the moment equation is expressed as follows:

\[ \begin{align*}
    \frac{3}{2} \frac{\partial}{\partial t} [M] &= \frac{\cos^2 y_1}{2} \left( \frac{\partial^2 M}{\partial y_1^2} + \frac{\sin^2 y_1}{2} \frac{\partial^2 M}{\partial y_3^2} \right) \\
    &+ \frac{3}{2} \frac{\partial M}{\partial y_2} + \frac{3}{2} \frac{\partial M}{\partial y_3} \\
    &- \frac{3}{2} \frac{\partial M}{\partial y_4} + \frac{3}{2} \left( A y_1 y_2 + A y_3 y_4 + A y_4 y_1 \right) \\
    &+ \frac{3}{2} \left( B y_1 y_2 + B y_3 y_4 + B y_4 y_1 \right)
\end{align*} \]

where \( M = y_1 y_2 y_3 y_4 \), and \( E[\cdot] \) means the ensemble average of the function. The conditions

\[ p = \dot{p} = \dot{\dot{p}} = \dot{p}_y = \dot{p}_y = 0 \]

are used to derive the above moment equation. Then, the moment equation is reduced to the following simultaneous differential equations relating to the second order moments \( E[y_1 y_3] \):

\[ \begin{align*}
    \dot{m}_{12} &= 2m_{12} \\
    \dot{m}_{13} &= m_{12} + 3E[y_1 y_2 y_3] - E[y_1 y_3] \\
    \dot{m}_{14} &= 3E[y_1 y_2 y_4] - E[y_1 y_4] \\
    \dot{m}_{23} &= m_{23} + 3E[y_2 y_3] - E[y_2 y_3] \\
    \dot{m}_{24} &= 3E[y_2 y_4] - E[y_2 y_4] \\
    \dot{m}_{34} &= m_{34} - 2E[y_3 y_4] - 2E[y_3 y_4] + E[y_3 y_4] \\
    \dot{\dot{m}}_{12} &= \frac{\partial^2 M}{\partial y_1^2} + \frac{\partial^2 M}{\partial y_3^2} \\
    \dot{\dot{m}}_{13} &= \frac{\partial^2 M}{\partial y_1^2} + \frac{\partial^2 M}{\partial y_3^2} \\
    \dot{\dot{m}}_{14} &= \frac{\partial^2 M}{\partial y_1^2} + \frac{\partial^2 M}{\partial y_3^2} \\
    \dot{\dot{m}}_{23} &= \frac{\partial^2 M}{\partial y_2^2} + \frac{\partial^2 M}{\partial y_3^2} \\
    \dot{\dot{m}}_{24} &= \frac{\partial^2 M}{\partial y_2^2} + \frac{\partial^2 M}{\partial y_3^2} \\
    \dot{\dot{m}}_{34} &= \frac{\partial^2 M}{\partial y_3^2} + \frac{\partial^2 M}{\partial y_4^2} \\
    \dot{\dot{m}}_{44} &= \frac{\partial^2 M}{\partial y_4^2} + \frac{\partial^2 M}{\partial y_4^2}
\end{align*} \]

where \( m_{ij} = E[y_i y_j] \) and \( M_{ij} = E[y_i y_j y_k y_l] \).
8. Numerical example

In numerical examples, the values of dimensionless parameters $T_d$ and $K_d$ corresponding respectively to the initial tension of the stays and the bottom stiffness of the support column, are taken as follows:

$$T_d = 0.0904, \quad 0.1356, \quad 0.1810$$

$$K_d = 1.152, \quad 5.756$$

The values of dimensionless stiffness of the friction damper are $k_d = 83.25$ and $k_d = 70.14$, the dimensionless parameters for the locations of lower ends of the stays are $r_1 = r_2 = 0.1428$, $r_3 = -0.2857$, $r_4 = 1.04$, and the mass ratio is $u = 0.959$. Values of dimensionless parameters of the power spectral density of input acceleration are taken as

$$D_g^2 = 0.0471, \quad c_f = 0.676, \quad c_f^a = 1.352$$

When the direction of input coincides with the $y$-axis, the moment equations (30) are numerically solved.

The time history of the power spectral density function (dimensionless) $W(\tau)$ of the input acceleration is illustrated in Fig. 4. In Figs. 5 and 6, the time histories of the mean squared value $M_0$ of the relative displacement $\eta_0$ of the interrupting chamber are plotted. The initial tension $T_d$ and the bottom stiffness $K_d$ are taken as parameters in Figs. 5 and 6, respectively. From these figures, it is seen that the mean squared displacement decreases, when the value of $T_d$ or $K_d$ increases. In Figs. 7 and 8, the time histories of the mean squared value $M_{\eta_0}$ of the relative velocity $\eta_0$ of the interrupting chamber are plotted. From these Figures, it is seen that the response velocity has the same tendency with respect to the effect of the initial tension $T_d$ of the stays as the response displacement, and the stiffness $K_d$ of the column bottom has only a little effect on the mean squared response velocity.

In order to examine the transient response of the circuit breaker, the responses to the impulse input acceleration (that is, a velocity step) are plotted in Figs. 9, 10, 11 and 12 for the previously described numerical example of the parameters of structure. In this example, the value of the dimensionless

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Fig. 4 The time history of the input acceleration power spectral density function

Fig. 5 The time histories of the mean squared response displacement of the circuit breaker (The effects of the initial tension of stays)

Fig. 6 The time histories of the mean squared response displacement of the circuit breaker (The effects of the bottom stiffness of the support column)

Fig. 7 The time histories of the mean squared response velocity of the circuit breaker (The effects of the initial tension of stays)

Fig. 8 The time histories of the mean squared response velocity of the circuit breaker (The effects of the bottom stiffness of support column)
representation \( \dot{U} = U/g \) of input acceleration is chosen as
\[
\dot{U}(\tau) = \begin{cases} 
0.91 & (0 \leq \tau \leq 0.0443) \\
0 & (\tau > 0.0443)
\end{cases}
\]
and the equations of motion are numerically solved. Figures 9 and 11 show that the maximum values of the transient responses decrease and the responses diminish rapidly, as the value of the initial tension \( T_0 \) of the stays becomes large. Figures 10 and 12 show that the maximum value of the velocity varies little in spite of the decrease of the maximum value of the displacement and the wave length of the response becomes short, as the value of the stiffness \( K_d \) of the column bottom increases.

9. Conclusions

As already described in the previous section, it is possible to reduce the mean squared response of the circuit breaker by the adoption of a large initial tension of the stays. This is due to the fact that the frictional force increases in proportion to the initial tension (see Eq.(6)) and accordingly the absorbed energy becomes larger. This fact can be also seen from transient response to the acceleration impulse.

While the stress at the bottom of the support column decreases as the response displacement of the interrupting chamber becomes smaller, the stresses at the stays and friction dampers increase as the initial tension of the stays increases. Hence the most suitable value of initial tension for the aseismic design can be determined.

The reason why the response velocity does not decrease in spite of the stiffened column bottom is due to the fact that the response frequency shifts to the higher frequency range. This fact is also seen from the wave length of the impulse response. In this numerical example, the wave length decreases to about 0.86 times, when the stiffness increases five-fold.

The approximate value of dominant period of the circuit breaker can be estimated from the ratio of \((f_0)_{\text{max}}^2 \) to \((f_0)_{\text{max}}^2 \) given in Table 1. In the case of (1) and (4) in Table 1, these values are about 1.65 and 1.45 respectively, and the ratio of these values is 1.45/1.65 = 0.878. As the result, the dominant period decreases to about 0.878 times when the stiffness of the support column increases to about 5 times, and this value agrees with the ratio of the wave lengths of the response to the above mentioned impulse input.

In the case where the column stiffness increases, the variation of response due to the initial tension is considered to be smaller. In other words, the effect of the friction damper should be expected to appear...
only in the case where the stiffness of the support column is relatively small.

The maximum values of the mean squared responses and the time $t_m$ when the responses attain the maximum values are shown in Table 1. The values of $t_m$ are larger than the time ($t = 1.05$) when the maximum value of the input power spectrum is attained. Although the analytical model treated in this paper is extremely simplified, the effects of the initial tension acting on the stays and of the bottom stiffness of the support column on the seismic response of the circuit breaker have been ascertained by theoretical calculation and these results can be applied to the seismic design of the circuit breaker. However, a number of fundamental problems should be further investigated in future.

<table>
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<th>$(g^2/I_p)^{\text{max}}$</th>
<th>$t_m$</th>
<th>$(g^2/\text{g})^{\text{max}}$</th>
<th>$t_m$</th>
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<td>7.20x10^{-4}</td>
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<td>(4) 3.81</td>
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<td>7.16</td>
<td>2.04</td>
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Table 1 The maximum value of mean squared response and the time $t_m$ when the maximum value is attained

Acknowledgment

The invaluable assistance of Mr. K. Nagata, research worker of Heavy Apparatus Engineering Laboratory, Tokyo Shibaura Electric Co., Ltd., with the numerical calculations, is gratefully acknowledged.

Appendix "Computer simulation with an actual seismic input"

In order to examine the applicability of the method described in this paper to an actual seismic wave, a simulation was carried out by a digital computer. In this simulation, the solutions of Eq. (21) are calculated by Runge-Kutta method for the El Centro seismic wave (N-S component, the maximum acceleration $U_{\text{max}} = 0.5g$) as shown in Fig. A1. Some examples of the results are indicated in Figs. A2 and A3. The mean squared values of these responses for one second intervals are evaluated, and the maximum values of these mean squared responses are indicated for various parameters of the circuit breaker in Table A1.

From Table A1, it is seen that the maximum mean squared responses of the relative displacement and the velocity decrease as the value of the initial tension $T_0$ of the stay increases, and these results are similar to those in Table 1. As the column stiffness $K$ increases, the maximum mean squared value of the relative displacement slightly decreases and that of the relative velocity increases. These tendencies are a little different from those in Table 1. These results can be explained from the fact that when the bottom stiffness of the column increases, then the resonance frequency of the circuit breaker increases and consequently the frequency components of the input having dominant influences upon the response change. However, since the level of the power spectral density of the El Centro seismic wave has a positive inclination in the vicinity of the resonance frequency ($f_0 = \frac{f_0}{\omega_0} = 0.58$ for $K_0 = 1.152$) of the circuit breaker as shown in Fig. A5, the results are different from those of the case where the input earthquake is assumed to be a white noise. In this case, the input must be approximated by a white noise in order to calculate the time history of the r.m.s. value of the response by using the moment equation (30). In this study, the input is approximated by an equivalent white noise as follows:

The envelope of the El Centro seismic wave, shown in Fig. A4, is obtained by a lag window and is normalized so that the maximum value is equal to 1.0. Further, the power spectral density of the stationary wave, which is obtained from the seismic wave (Fig. A4), is represented in Fig. A5 (this power spectral density is defined for both sides of plus and minus frequency range). Now, consider the case where the values of the parameters of the circuit breaker are $K_0 = 1.152$ and $T_0 = 0.1356$. In this case, the resonance frequency $f_0 (= \frac{f_0}{\omega_0})$ is about 0.58 and the equivalent damping ratio is about 0.26. Then, it may be assumed that the response of the circuit breaker mainly depend-
s on the frequency components of the input over the range of about ± 60% of the resonance frequency \( f_0 \) (the shadowed portion in Fig. A5). Therefore, if the input seismic wave is approximated by a white noise whose power spectrum has the same value as the average level of the input power spectrum over the above-mentioned frequency range, the resultant error of the r.m.s. value of response may be fairly small. For the input power spectrum shown in Fig. A5, the dimensionless power spectrum of the equivalent white noise is \( W_0 = 1.20 \times 10^{-3} \), and this value is nearly equal to that of the average power spectrum calculated from the frequency range of ± 100% of the resonance frequency. Since this level of the power spectral density corresponds to the maximum value of the power spectral density of the nonstationary input, the time history \( W(t) \) of the power spectral density of the nonstationary white noise is obtained as the product of \( W_0 \) and the squared value of the envelope function (see in Fig. A6). Solving the moment equation (30) by using this input power spectrum \( W(t) \), the r.m.s. values of the responses are obtained as indicated in Figs. A7 and A8 (solid lines). On the other hand, the envelopes of the squared values of the responses are obtained by the computer simulation (Figs. A2 and A3), and the square roots of these envelopes are indicated in Figs. A7 and A8 (broken lines). Though the peak values of two curves at \( t = 4 \) sec in Fig. A7 do not agree with each other, these curves show similar tendencies on the whole. The r.m.s. value of the responses obtained by the simulation and the input power spectral density used in the moment equation depend on the number of repetitions for smoothing which is required to obtain the envelope functions. Further, since the dominant frequencies of the actual seismic wave somewhat vary with time, it may not always be expected that the analytical results agree with the results of simulation using the actual seismic wave.

In general, the seismic wave cannot be treated as an ergodic process as far as its nonstationary behavior is taken into account. Since the r.m.s. values of the responses obtained by the analytical method in this paper mean the ensemble averages, the r.m.s. values should be obtained from many sample functions in simulation.

In this paper, however, the simulation results for Eq. (21) obtained from only

![Fig. A3](image)

**Fig. A3** The time histories of the response relative velocity of the circuit breaker to El Centro seismic wave (The effects of the initial tension of stays)

![Fig. A4](image)

**Fig. A4** The normalized envelope function of El Centro seismic wave

![Fig. A5](image)

**Fig. A5** The acceleration power spectral density function on El Centro seismic wave

![Fig. A6](image)

**Fig. A6** The time history of the equivalent acceleration power spectral density function (the white noise) of El Centro seismic wave

<table>
<thead>
<tr>
<th>( (\gamma^2 / L_j)^{\text{max}} )</th>
<th>( (\delta^2 / g L_j)^{\text{max}} )</th>
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<td>2.236 \times 10^{-6}</td>
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<td>(3) 3.357</td>
<td>1.85</td>
</tr>
<tr>
<td>(4) 5.332</td>
<td>2.590</td>
</tr>
</tbody>
</table>

\( J\gamma \approx 1.152, \quad J\gamma \approx 0.109 \)

\( J\gamma \approx 1.152, \quad J\gamma \approx 0.109 \)

\( J\gamma \approx 1.152, \quad J\gamma \approx 0.109 \)
one example of the actual seismic waves (i.e. one sample function) were compared with the analytical results. Hence the above-mentioned comparison is not sufficient for evaluating the analytical method. However, the analytical method described in this paper is available to estimate the r.m.s. response to the actual seismic wave, if the time history of the acceleration power spectral density of the input in the vicinity of the resonant frequency of the system can be estimated as the ensemble average of the actual seismic waves.

Fig. A7 The time histories of the root mean squared response relative displacement obtained from the moment equation and the simulation

Fig. A8 The time histories of the root mean squared response relative velocity obtained from the moment equation and the simulation

References