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A Numerical Study on the Flow Field
in a Wall Attachment Amplifier *

1st Report, In the Case of No Vent, One Output Port Closed.

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At the first stage of this study, on a simplified model with no vent and one output port closed, some numerical computations and visualized experiments have been performed at Reynolds numbers up to 2000 in various flow rate ratios of control flow to main jet. Numerical solutions by finite difference method include an up-wind difference scheme for the convection term of a basic equation and indicate good agreement with the experimental results by hydrogen bubble method and tuft method. These investigations have made clear stream lines, equivorticity lines, velocity distribution, separation point, reattachment point and thickness of attached vortex in the present model.

1. Introduction

The flow field in a wall attachment amplifier, on which many experimental and/or theoretical researches have been carried out, can not be said to have been made clear thoroughly. This is due to the fact that it is so complicated with diverse relevant factors that its theoretical examination is very difficult.

Bourque and Newman⁽¹⁾, who were the first to be engaged in this research field, analysed the two-dimensional attachment jet of an incompressible fluid by use of a model and the momentum theory. Then the scope of such methods was enlarged by Levin and Manion⁽²⁾, Sher⁽³⁾, Brown⁽⁴⁾, Perry⁽⁵⁾ and Kimura et al.⁽⁶⁾, who were concerned with the reattachment distance and the shape of side walls, based on some assumptions at the stage of modeling.

On the other hand, with the development of computer, there has arisen another method to solve the Navier-Stokes equation of motion under various boundary conditions. It is, however, not necessarily easy to obtain stable solutions for a complicated flow field. So far some partial problems such as those on the effect of control flow without splitter⁽⁷⁾⁻⁽⁹⁾ and of splitter without control flow⁽¹⁰⁾ were analysed at Reynolds numbers less than 100.

As for higher Reynolds numbers, a laminar wall attachment flow and a reattachment flow in a rapid expansion conduit⁽¹¹⁾ (Re = 5000) were dealt with.

There can be hardly found any reports

on a flow field similar to the actual amplifier, which has splitter, control flow and higher Reynolds numbers. This study purports to analyse the flow field in the same model as the conventional amplifier at practical Reynolds numbers.

At the first stage on a model with no vent and one output port closed, which is relatively easy for analysis, at Reynolds numbers as high as 2000 some numerical computations and visualized experiments such as hydrogen bubble method and tuft method have been made and their results have been compared with each other.

2. Main Symbols

As: aspect ratio
bc: control nozzle width
bs: main nozzle width
d: offset
L₁: separation point distance
L₂: reattachment point distance
L₃: vortex thickness
p: pressure
Q_c: control flow rate
Q_o: output flow rate
Q_s: supply flow rate
Re: Reynolds number
u: x velocity component
u_o: average velocity at main nozzle inlet
v: y velocity component
ζ: vorticity
ν: kinematic viscosity coefficient
ρ: density
ψ: stream function

3. Numerical Computations

3.1 Basic Equations

Assuming that the flow in an amplifier is a two-dimensional, laminar and steady flow of an incompressible and viscous fluid without body forces, the Navier-Stokes equation of motion and the continuity equation in the Cartesian coordinates are

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$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (1)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

The definitions of stream function and vorticity are

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (4)$$

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad (5)$$

Eliminating the pressure term in (1), (2) and using Eqs.(3),(4) and (5) gives

$$\frac{\partial \psi}{\partial y} \frac{\partial \zeta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \zeta}{\partial y} = \nu \left(\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2} \right) \quad (6)$$

Substituting Eq.(4) into Eq.(5) yields

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\zeta \quad (7)$$

Taking main nozzle width and average velocity at main jet inlet as references, the following dimensionless quantities

$$\left. \begin{aligned} x^* &= \frac{x}{b_s}, & y^* &= \frac{y}{b_s}, & \zeta^* &= \frac{\zeta b_s}{u_0} \\ \psi^* &= \frac{\psi}{u_0 b_s}, & R_s &= \frac{u_0 b_s}{\nu} \end{aligned} \right\} \quad (8)$$

are substituted into Eqs.(6) and (7) and moreover asterisk is omitted.

Thus the basic equations

$$\frac{\partial \psi}{\partial y} \frac{\partial \zeta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \zeta}{\partial y} = \frac{1}{R_s} \left(\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2} \right) \quad (9)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\zeta \quad (10)$$

are obtained.

3.2 Derivation of Finite Difference Equations

As shown in Fig.1, the field of interest, which has splitter, control nozzle and

Fig.1 Analysed Domain

no offset, is covered with a grid network in such a way that the nodal points just fall on the oblique walls and grid distances in x direction of control nozzle are as fine as those in y direction of main nozzle. According to Gosman et al.⁽¹³⁾, finite difference approximation of Eqs.(9) and (10) is derived by the integration over a small domain element (Fig.2) around one node enclosed by four points nw,sw,se and ne which refer not to nodes but to relevant corners of integration domain element, whereas e,n,w and s lie midway between adjacent nodes respectively. Integrating both sides of Eq.(9) yields

$$\int_s^n \int_w^e \left(\frac{\partial \psi}{\partial y} \frac{\partial \zeta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \zeta}{\partial y} \right) dx dy = \frac{1}{R_s} \int_s^n \int_w^e \left(\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2} \right) dx dy \quad (11)$$

The left-hand side is divided as follows

$$\int_s^n \zeta_e \left(\frac{\partial \psi}{\partial y} \right)_e dy - \int_s^n \zeta_w \left(\frac{\partial \psi}{\partial y} \right)_w dy - \int_w^e \zeta_n \left(\frac{\partial \psi}{\partial x} \right)_n dx + \int_w^e \zeta_s \left(\frac{\partial \psi}{\partial x} \right)_s dx \quad (12)$$

The respective terms are denoted by I_{a1}, I_{a2}, I_{a3} and I_{a4} . Then

$$I_{a1} = \int_s^n \zeta_e \left(\frac{\partial \psi}{\partial y} \right)_e dy \quad (13)$$

Since ζ is a continuous function along se-ne face, there exists an average value denoted by $\bar{\zeta}_e$.

$$\bar{\zeta}_e = \frac{\int_s^n \zeta_e \left(\frac{\partial \psi}{\partial y} \right)_e dy}{\int_s^n \left(\frac{\partial \psi}{\partial y} \right)_e dy} \approx \frac{I_{a1}}{\psi_n - \psi_s} \quad (14)$$

Thus

$$I_{a1} = \bar{\zeta}_e (\psi_n - \psi_s) \quad (15)$$

The left-hand side of Eq.(11) - convection term - becomes predominant at higher Reynolds numbers, to which the up-wind difference method promising good convergence is applied. In this method, taking the flow direction into consideration, $\bar{\zeta}_e$ takes the value possessed by the node in the up-stream to ne-se face of the domain, that is, when the flow direction is P → E ($\psi_n - \psi_s > 0$), $\bar{\zeta}_e = \zeta_P$ and when E → P ($\psi_n - \psi_s < 0$), $\bar{\zeta}_e = \zeta_E$.

To satisfy these two cases, Eq.(15) is rewritten as follows.

Fig.2 Integration Domain Element

$$I_{a1} = \zeta_P \left\{ \frac{(\phi_{ne} - \phi_{se}) + |\phi_{ne} - \phi_{se}|}{2} \right\} + \zeta_E \left\{ \frac{(\phi_{ne} - \phi_{se}) - |\phi_{ne} - \phi_{se}|}{2} \right\} \dots\dots\dots (16)$$

, where stream functions at the edge points of integration domain such as ϕ_{ne} and ϕ_{se} are assumed as an arithmetic mean of four stream functions of surrounding nodes, for example;

$$\phi_{ne} = \frac{\phi_{NE} + \phi_N + \phi_P + \phi_E}{4} \dots\dots\dots (17)$$

Applying the same procedure to the other terms I_{a2} , I_{a3} , and I_{a4} , and adding together the results gives Eq.(12) as transformed;

$$I_a = \sum_{i=1}^4 I_{ai} = A_E(\zeta_P - \zeta_E) + A_W(\zeta_P - \zeta_W) + A_N(\zeta_P - \zeta_N) + A_S(\zeta_P - \zeta_S) \dots\dots\dots (18)$$

, where

$$\left. \begin{aligned} A_E &= \frac{1}{8} \{ (\phi_{SE} + \phi_S - \phi_{NE} - \phi_N) + |\phi_{SE} + \phi_S - \phi_{NE} - \phi_N| \} \\ A_W &= \frac{1}{8} \{ (\phi_{NW} + \phi_N - \phi_{SW} - \phi_S) + |\phi_{NW} + \phi_N - \phi_{SW} - \phi_S| \} \\ A_N &= \frac{1}{8} \{ (\phi_{NE} + \phi_E - \phi_{NW} - \phi_W) + |\phi_{NE} + \phi_E - \phi_{NW} - \phi_W| \} \\ A_S &= \frac{1}{8} \{ (\phi_{SW} + \phi_W - \phi_{SE} - \phi_E) + |\phi_{SW} + \phi_W - \phi_{SE} - \phi_E| \} \end{aligned} \right\} \dots\dots\dots (19)$$

Similarly the right-hand side of Eq.(11) is divided into four terms, of which the first term is denoted by I_{b1} .

$$I_{b1} = \int_s^n \left(\frac{\partial \zeta}{\partial x} \right)_e dy \dots\dots\dots (20)$$

Then

From an approximation

$$\left(\frac{\partial \zeta}{\partial x} \right)_e \approx \frac{\zeta_E - \zeta_P}{x_E - x_P} \dots\dots\dots (21)$$

$$I_{b1} = \int_s^n \frac{\zeta_E - \zeta_P}{x_E - x_P} dy = \frac{\zeta_E - \zeta_P}{x_E - x_P} \frac{y_N - y_S}{2} \dots\dots\dots (22)$$

is obtained.

Repeating the same procedure to the other terms I_{b2} , I_{b3} , and I_{b4} and adding together the results yields

$$I_b = \sum_{i=1}^4 I_{bi} = -\frac{1}{R_e} \{ B_E(\zeta_P - \zeta_E) + B_W(\zeta_P - \zeta_W) + B_N(\zeta_P - \zeta_N) + B_S(\zeta_P - \zeta_S) \} \dots\dots\dots (23)$$

, where

$$B_E = \frac{y_N - y_S}{2(x_E - x_P)}, \quad B_W = \frac{y_N - y_S}{2(x_P - x_W)}, \quad B_N = \frac{x_E - x_W}{2(y_N - y_P)}, \quad B_S = \frac{x_E - x_W}{2(y_P - y_S)} \dots\dots (24)$$

Equating Eq.(18) and Eq.(23) and solving it in reference to ζ_P gives

$$\zeta_P = \frac{\left(A_E + \frac{B_E}{R_e} \right) \zeta_E + \left(A_W + \frac{B_W}{R_e} \right) \zeta_W + \left(A_N + \frac{B_N}{R_e} \right) \zeta_N + \left(A_S + \frac{B_S}{R_e} \right) \zeta_S}{A_E + A_W + A_N + A_S + \frac{1}{R_e} (B_E + B_W + B_N + B_S)} \dots\dots\dots (25)$$

In the same way, the integration of Eq.(10) yields

$$\phi_P = \frac{B_E \phi_E + B_W \phi_W + B_N \phi_N + B_S \phi_S + C_P \zeta_P}{B_E + B_W + B_N + B_S} \quad (26)$$

, where

$$C_P = (x_E - x_W)(y_N - y_S)/4 \dots\dots (27)$$

Eqs.(24) and (27) show that calculation is also possible in the case of unequal grid distances. Inspection of Eqs.(25) and (26) reveals that the absolute values of the sum of the coefficients of the variables on the nodes, all positive, will not exceed unity, which satisfies the convergence condition of iterative solution of equations.

3.3 Boundary Conditions

3.3.1 At a wall

Since a wall is impermeable, the stream function at a wall is constant.

The vorticity is derived under the following assumptions;

- (1) the fluid does not slip on a wall surface;
- (2) the variation of vorticity along a wall is negligibly small compared with that perpendicular to a wall; and
- (3) the vorticity varies linearly with the distance from a wall.

From the Taylor expansion of stream function around the node NP and the above as-

sumptions, the vorticity at the wall parallel to x coordinate or the oblique walls is obtained.

$$\zeta_P = - \left\{ \frac{3(\phi_{NP} - \phi_P)}{(y_{NP} - y_P)^2} + \frac{1}{2} \zeta_{NP} \right\} \dots\dots (28)$$

, where the suffix NP means an interior node adjacent to a wall. As the vorticity at the edge H of the splitter is taken an average value on its upper and lower nodes calculated from Eq.(28) and as the vorticities at the other edge points (N,B,K and E) are taken the average values of the adjacent nodes in x and y directions.

3.3.2 At the inlet

Assuming equi-velocity distribution at the inlet, the vorticity there is zero.

3.3.3 At the outlet

The second degree extrapolation of the values of upstream nodes gives

$$\left. \begin{aligned} \phi_{i,j} &= 3\phi_{i-1,j} - 3\phi_{i-2,j} + \phi_{i-3,j} \\ \zeta_{i,j} &= 3\zeta_{i-1,j} - 3\zeta_{i-2,j} + \zeta_{i-3,j} \end{aligned} \right\} \dots\dots (29)$$

3.4 Calculation Procedure

Here is applied the successive over-relaxation method. The procedure is summarized below:

- (1) The initial values of vorticities and stream functions are assigned;
- (2) vorticities and stream functions of inner nodes are found from Eqs.(25) and (26) respectively;
- (3) vorticities and stream functions on the boundary are found from Eqs.(28) and (29), using the values obtained from (2).
- (4) the procedure is repeated until the convergence condition as specified in Eq.(30) is attained;

$$\sum_P |\zeta_P^{(n)} - \zeta_P^{(n-1)}| \leq 0.15 \dots\dots\dots (30)$$

, where n is the iterated cycle number; and

- (5) after (4) is satisfied, velocity distribution is found.

4. Experimental Method

To verify the results of numerical computations, some visualized experiments have been carried out. The experiment system is illustrated in Fig.3. Working fluid water is pumped up into a tank, from which it is fed to the supply port and the control port through flow-nozzle and valve. Supply and control flow rates are measured

Fig.3 Experiment System

by flow-nozzle. The amplifier model which is adopted has the shape as shown in Fig.4 and is made of acryl resin.

Fig.4 Dimension of Amplifier Model used for Experiment

Hydrogen bubble method and tuft method have been used for visualization. In hydrogen bubble method, stream lines were observed by setting a kinked platinum wire at the outlet of main nozzle. In tuft method, with short-cut silk threads suspended from the whole upper surface of the model, the movement of the threads was observed. Varying control flow rates at a constant supply flow rate in the range of $Q_c/Q_s = 0, 0.05, 0.1$ and 0.2 , the experiments were done at Reynolds numbers $50-2000$ defined as $Re = u_0 b_s / \nu$.

5. Results and Consideration

By numerical computations vorticities, stream functions and velocity distribution of the flow field were found with the cases of $Q_c/Q_s = 0, 0.05, 0.1$ and 0.2 in the range of Reynolds numbers $20-2000$. The flow patterns obtained by visualized methods in the same conditions have good agreement with those by numerical method.

As some examples, Fig.5 shows a flow

pattern without control flow at Reynolds number 300, Fig.6, one without control flow at Reynolds number 2000, and Fig.7, one with control flow $Q_c/Q_s = 0.2$ at Reynolds number 2000. Fig.5 (b) shows a visualized flow pattern by hydrogen bubble method, Fig.6 (d) and Fig.7 (d), ones by tuft method. These patterns seem to confirm the results of numerical computations. As seen in these figures, there are three vortices in the model. The first is in the neighbourhood of outlet of control nozzle, the second is in the neighbourhood of inlet of non-at-

tachment output port and the third is an attached vortex. Of these vortices the second is the most clearly seen in visualized experiments. The velocity distribution shows that it is non-symmetric at main nozzle outlet and the flow is pushed to the attached side with an increase of control flow rate. Fig.8 shows the scale of attached vortex at Reynolds numbers 200, 300, 500, 1000 and 2000, determined from numerical computations and visualized experiments. Attached vortex generates at Reynolds numbers 200-300 and it does at

(a) Equi-Vorticity Lines

(b) Stream Lines

(c) Velocity Distribution

(d) Flow Pattern visualized by Tuft Method

Fig.6 Results of Numerical Computation
and Visualized Experiment
 $Re = 2000$, $Q_c/Q_s = 0$

(a) Equi-Vorticity Lines

(b) Stream Lines

(c) Velocity Distribution

(d) Flow Pattern visualized by Tuft Method

Fig.7 Results of Numerical Computation
and Visualized Experiment
 $Re = 2000$, $Q_c/Q_s = 0.2$

Fig.8 Thickness of Attached Vortex
(the numbers in the figures
are Reynolds numbers)

higher Reynolds numbers with an increase of control flow rate. From these figures the separation point, the reattachment point and the thickness of attached vortex are measured and the results are summarized in Fig.9, where L_1 is the separation point distance, L_2 the reattachment point distance and L_3 is the thickness of attached vortex. L_1 is shifted to the downstream side with an increase of control flow rate and to the upstream side at higher Reynolds numbers. L_2 is shifted to the down-

Fig.9 Separation Point Distance , Re-
attachment Point Distance and
Thickness of Attached Vortex
in relation to Reynolds numbers

stream side as Reynolds number increases but it is nearly independent of control flow rate, while L_3 becomes larger at higher Reynolds numbers and it is hardly dependent on control flow rate too, which needs further examination, however.

6. Conclusions

On a wall attachment amplifier model with non-attachment output port closed, some numerical computations and visualized experiments have been performed at Reynolds numbers up to 2000, whose results are enumerated below:

- (1) The application of the up-wind difference method to the convection term of the Navier-Stokes equation, which becomes predominant at higher Reynolds numbers, has proved to be effective with superior convergence for the numerical analysis of the flow field of a wall attachment amplifier under various conditions.
- (2) The results of numerical computations have good agreement with those of experiments, proving the availability of the present numerical method.
- (3) Attached vortex begins to occur at Reynolds number about 200 and it generates at higher Reynolds numbers as control flow rate increases.
- (4) Separation point of attached vortex is shifted to the upstream side at higher Reynolds numbers and to the downstream side with an increase of control flow rate, but reattachment point and thickness of vortex are hardly dependent on control flow rate.

The computer used for the present numerical computations is UNIVAC 1108 at the computer centre of Tokai University.

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