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A Study of the Packaging of a Small Engine*
(1st Report Relationship between Vibration and Noise)

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Noise sources of an engine can be roughly classified into exhaust noise, suction noise and engine surface noise. The exhaust and suction noises can be effectively reduced by attaching mufflers, but it is not always easy to reduce the noise emitting from the engine surface.

The authors come to the conclusion that there is no way except enclosing the engine in a package to drastically reduce the engine surface noise.

This paper shows the relation between the noise and vibration of a package enclosing a small engine.

1. Foreword

The demands of society for the development of a low-noise engine have been increasingly strong.

Noises from an engine can be roughly classified into exhaust noise, suction noise, and engine noise. Exhaust noise as well as suction noise can be reduced by using a silencer, but noises produced by the engine-body can hardly be reduced at all. In view of the results obtained through years of study by the authors, it seems most difficult to reduce them by more than 10 dB, notwithstanding any improvements in engines or any noise protection.

Judging from this fact, it has been concluded that the most effective way to substantially reduce engine noise is to completely enclose the engine itself. The purpose of this study is to meet social demands by conducting systematic research on packaging.

2. Experimental equipment and experimental method

Fig. 1 shows the experimental equipment. The engine used for the experiments is a single-cylinder, small size, two-cycle, air-cooled gasoline engine. It was fixed on a basis plate with a thickness of 9 mm and then covered with package fixed to the plate so that it could be completely enclosed.

Partly to prevent overheating in the package due to the enclosure and partly to supply air, a blower (air blasting capacity of 1.2 m³/min) was installed. The length of the intake pipe of engine is 4,000 mm. The exhaust pipe, equipped with a silencer, is led under the surface plate to a sound-proof room. The length of the exhaust pipe is 3,050 mm.

The acceleration level of vibration was measured in the vertical direction at the center point of the upper plate of the package. Both the sound pressure level and the sound level were also measured at the same time by means of a microphone, which was installed along the vertical line from the center of the upper plate.

The following are the instruments used for the experiments: B & K 2203 sound level meter, B & K 4145 condenser microphone, B & K 3226 frequency analyzer and B & K 2603 vibration meter. Also, a 1/3 octave band filter was used for frequency analysis.

3. Experimental approach by the use of a primitive cubic package for enclosing purpose⁽¹⁾

3.1 Experimental method

Is it really possible, as is commonly thought, to reduce noises just by enclosing an engine in a package? In order to ascertain this question and, additionally,

Fig. 1 Experimental equipment

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to clarify the relationship between vibrations and noises of a package, experiments were carried out by using three types of packages alternately.

That is to say, the packages used for the experiments were cubic in shape and made of rolled steel plate with a thickness of 1.6 mm and the length of the side of each cube was 300 mm, 400 mm, and 500 mm. Noises were measured at a point 300 mm over the center of the upper plate.

3.2 Experimental results

Table 1 shows SPL (sound pressure level), SL (sound level), and VAL re Gal (vibration acceleration level) when no package was provided or when three types of the packages were used alternately. Fig. 2 shows the frequency spectrum for

Table 1 The effect of the size of packages

	SPL dB	SL dB(A)	VAL re Gal, (160Hz) dB
Without any package	93	89	
With package □ -300	93	88	48
With package □ -400	95	92	56
With package □ -500	96	93	57

Fig. 2 Changes in vibration acceleration and noise with a change in the size of packages

each of the items mentioned above. In each of Fig. 2, a solid line stands for sound pressure level, and a broken line for vibration acceleration level. f_1 and f_2 are forced frequency and its second harmonic, respectively, based on the number of engine combustions per second. In this case, the engine speed N is 5,000 rpm and, accordingly, $f_1=83$ Hz and $f_2=167$ Hz.

As shown in Table 1, sound level increased with an increase in the size of the package. The sound level measured when a package was provided was higher than that measured when no package was used. According to Fig. 2, it can be seen that, with an increase in the size of packages, both the vibration acceleration level and the sound pressure level increased in the low-frequency range, causing the overall value (OA) to increase. Consequently, it can be said that increasing the size of a package may be not effective in reducing noise but noise reduction should be paid to the vibration of the package. Fig. 2 shows that the vibration acceleration characteristics corresponds closely with the sound pressure characteristics.

4. Relationship between vibration and noise of a cylindrical package

4.1 Experimental method

In each of the primitive cubic packages mentioned above, the upper plate is equal in thickness to the side plates and, accordingly, the noise produced in the package was easily transmitted outward through the side plates. In order to prevent this, the following experiments were carried out by building a cylindrical package having a thick side plate so that all the noises produced by the engine could be radiated from the upper plate.

The package used for the experiments, being made of rolled steel plate, is cylindrical in shape with an internal diameter 300 mm, a height 300 mm, and a side wall thickness 9 mm. A thin upper plate was provided on the upper part of the package to enclose it.

Noises were measured at a point 500 mm over the upper plate, vertically above its center.

4.2 Vibration mode of the package upper plate

In order to basically clarify the relationship between vibration and noise of the cylindrical package, the vibration mode of the package upper plate was examined in the first place.

Fig. 3 shows the vibration mode obtained by measuring the vibration of the package upper plate with the use of an acceleration-type vibration pickup. The numbers in the figures indicate the values of vibration acceleration levels.

As shown in the Figure, the vibration mode of the package is nearly symmetrical with its center. So, in the experiments

mentioned below, the acceleration-type vibration pickup was fixed at the center of the disc shown in Fig. 3 for measuring the vibration of the package.

Fig. 3 Vibration mode of the package

4.3 Frequency of vibration of normal mode of the package upper plate

A basic formula for the vibration of a plane plate is expressed as follows:⁽²⁾

$$\gamma \frac{\partial^2 w}{\partial T^2} + \frac{Et^2}{12(1-\sigma^2)} \nabla^2 w = 0 \quad (1)$$

where

- w : Displacement
- E : Young's modulus of the plate
- γ : Density of the plate
- t : Thickness of the plate
- σ : Poisson's ratio
- T : Time

If the frequency of vibration of the normal mode of the disc (i.e., the upper plate of the package) is deduced from this equation, the frequency of vibration of normal mode f_{nm} can be expressed by the following formula, provided that the disc is fixed on the periphery.

$$f_{nm} = \frac{\lambda^2 t}{4\pi a^2} \sqrt{\frac{E}{3\gamma(1-\sigma^2)}} \quad (2)$$

where, a is the effective radius of the vibrating surface of the disc, while λ is the value shown in Table 2 in relation to the number of nodal diameter n and the number of nodal circle m . So, by substituting $\lambda=3.2$ into Eq. (2), the frequency of vibration of normal mode f_0 ($n=m=0$) is computed for some conditions as shown in Table 3. The measured values are also

Table 2 λ of a disc

$m \backslash n$	0	1	2
0	3.20	4.61	5.91
1	6.31	7.80	9.20
2	9.44	11.0	12.4

Table 3 Frequency of vibration of normal mode (Hz) of package

Thickness of plate mm	1.0	1.6	2.3	3.2	4.5	6.0
Calculated value	77	122	176	245	344	459
Measured value	71	117	164	241	344	404

$a=18\text{cm}$

shown in Table 3. That is, the upper plate of the package was hit with a hammer and the vibration was detected by a pickup fixed on the center of the plate.

According to Table 3, the calculated values are nearly equal to the measured values, but the former values are a little less than the latter values.

4.4 General relationship between noise and vibration

Now, let us consider the general relationship between noises and vibrations of the disc.

As shown in Fig. 4, suppose the radius of a disc is a and vibration velocity is

$$u = \sum_n u_{n\max} e^{j\omega_n t} \quad (3)$$

and the disc vibrates uniformly as a piston. When $r \gg a$, the sound pressure p at a distance r away from the center of the disc in the infinite baffle, inclining at an angle θ to the z axis can be expressed by the following equation.⁽³⁾ The density of air is taken to be ρ kg/m³ and the sound velocity in the air is taken to be c m/s

$$p = j\rho c \frac{a}{r} \sum_n u_{n\max} \exp[j(\omega_n t - k_n r)] \times \left[\frac{J_1(k_n a \sin \theta)}{\sin \theta} \right] \quad (4)$$

On the z axis (i.e., $\theta=0$), the value of the term in the brackets in Eq. (4) will be as follows:

$$\lim_{\theta \rightarrow 0} \frac{J_1(k_n a \sin \theta)}{\sin \theta} = \frac{k_n a}{2} \quad (5)$$

Substituting this value into Eq. (4) yields,

$$p = j\rho c \frac{a^2}{2r} \sum_n k_n u_{n\max} \exp[j(\omega_n t - k_n r)] \quad (6)$$

and rewriting Eq. (6), the following equation is obtained.

$$p = \rho \frac{a^2}{2r} \sum_n (j\omega_n u_{n\max} e^{j\omega_n t}) \exp(-jk_n r) \quad (7)$$

In Eq. (7), ω_n , $u_{n\max}$ are the values proportional to the frequency. Consequently, the value in the parentheses in Eq. (7) is in proportion to the vibration acceleration. According to the theoretical analyses mentioned above, it can be deduced that the sound pressure at a distant point on the center axis of a disc is in proportion to the vibration acceleration, provided that the disc vibrates uniformly as a piston.

4.5 Experiments on the response of sound pressure level and that of vibration acceleration level⁽⁴⁾

In order to experimentally ascertain the above-mentioned relationship between

Fig. 4 Radiated sound from the disc

sound and vibration, the frequency characteristics of the sound pressure level (solid line) and the vibration acceleration level (broken line) were analyzed with two conditions. That is, the engine vibration was isolated in one condition and not isolated in the other condition, so that the effect of the vibrating isolation suspension could be examined. Fig. 5 shows the results obtained by the experiments. In these experiments, three pieces of vibration isolating rubber with a spring constant of $K=67 \text{ kgf/cm}$ were used for the purpose of isolating the vibration.

It can be seen in Fig. 5 that the frequency characteristic of the sound pressure level coincides nearly with that of the vibration acceleration level. However, there are some frequency bands showing that the two levels do not coincide with each other. As the reasons for the discrepancy, the following might be mentioned, that is, the flexural vibration was superposed in practice, though Eq. (7) was deduced with an assumption that the vibration was uniform as a piston, and there was no baffle in practice, though Eq. (4) was established with an assumption that a baffle was provided.

Fig. 6 shows the results obtained by measuring the sound pressure level (OA), the sound level (OA), and the vibration acceleration level when the engine speed is changed.

As shown in Fig. 6 with an increase in the engine speed, both the vibration acceleration level and the sound pressure level increase in proportion to the logarithm of the engine speed. The increasing slope of the acceleration level is slightly greater than that of the sound pressure level.

If the effective value of the sound pressure is taken to be p, Pa and the effective value of the vibration acceleration is to be $\dot{a}, \text{m/s}^2$, the sound pressure level SPL and the vibration acceleration

level VAL are expressed by $20\log_{10} p/p_0$ ($p_0 = 2 \times 10^{-5} \text{ Pa}$) and $20\log_{10} \dot{a}/\dot{a}_0$ ($\dot{a}_0 = 1 \times 10^{-2} \text{ m/s}^2$), respectively. Hence, the following formula is obtained, according to Fig. 6.

$$p \propto N^{2.7} \dots \dots \dots (8)$$

$$\dot{a} \propto N^{4.0} \dots \dots \dots (9)$$

According to Fig. 6, too, the relationship between the vibration acceleration level and the sound pressure level can be expressed as in Fig. 7. In general, sound pressure and vibration acceleration will be

$$p \propto \dot{a}^{0.94 \pm 0.06} \dots \dots \dots (10)$$

That is, sound pressure is in fair proportion to vibration acceleration.

Fig. 6 The effect of engine speed on the sound pressure level and the vibration acceleration level

Fig. 5 Comparison between the frequency characteristics of sound pressure level and vibration acceleration level

Fig. 7 Relationship of vibration acceleration level to sound pressure level and sound level (without engine vibration isolating suspension)

4.6 Theory and experiments with regard to wall surface vibrations and radiated sound⁽⁴⁾

4.6.1 SPL and VAL

A description was made in 4.4 above about the vibration of plate and radiated sound on the assumption that the plate vibrates uniformly as a piston. In real structures, however, many types of vibrations are produced, depending on the vibration excitation conditions. Of all these conditions, flexural waves are more related to the radiation of sound than to anything else. As a correction to the flexural vibration of plate, therefore, a coefficient of correction will be introduced here. First, transforming Eq. (7) into

$$p = \rho \frac{a^2}{2r} \sum_n \{ \alpha_{n\max} e^{i(\omega_n t - k_n r)} \} \quad (11)$$

According to Eq. (11), the effective value of the sound pressure will be

$$p_e = \frac{\rho a^2}{2r} \sqrt{\sum_n \left(\frac{1}{\sqrt{2}} \alpha_{n\max} \right)^2} \quad (12)$$

Consequently, SPL or the sound pressure level is expressed as the following:

$$\text{SPL} = 20 \log_{10} (p_e / p_0) = 20 \log_{10} \frac{\rho a^2}{2r} \frac{\sum_n \left(\frac{1}{\sqrt{2}} \alpha_{n\max} \right)^2}{p_0} \quad (13)$$

Next, the authors institute the sound pressure coefficient as a method of correction of flexural vibration.

From Eq. (12) which is transformed from Eq. (4)

$$p_e^2 = \nu \left\{ \frac{\rho a^2}{2r} \sqrt{\sum_n \left(\frac{1}{\sqrt{2}} \alpha_{n\max} \right)^2} \right\}^2 \quad (14)$$

Expressing the above equation logarithmically

$$10 \log_{10} \left(\frac{p_e}{p_0} \right)^2 = 10 \log_{10} \nu + 20 \log_{10} \left(\frac{\rho a^2 \alpha_0}{2r p_0} \right) + 10 \log_{10} \frac{\sum_n \left(\frac{1}{\sqrt{2}} \alpha_{n\max} \right)^2}{\alpha_0^2} \quad (15)$$

Taking the reference value p_0 of the sound pressure level to be 2×10^{-5} Pa and the reference value α_0 of the vibration acceleration level to be 1 Gal

$$\text{SPL} = 10 \log_{10} \nu + \text{VAL} + 20 \log_{10} \frac{a^2}{r} + 50 \quad (16)$$

The value of ν obtained by substituting the measured values of SPL, VAL, r and into Eq. (16) will be as shown in Fig. 8.

According to Fig. 8, the coefficient of the sound pressure ν will be nearly $-10 < 10 \log \nu < 10$, regardless of the frequency, and $\nu \approx 1$.

4.6.2 SPL and PWL

Suppose that there is a sound source of a circular surface, as shown in Fig. 4, with a radius of a m and an area of s m². When the acoustic power per unit area is taken to be W , the distance from the center of the disc to a point on the disc is to be a_s , and the sound intensity at a point on the vertical line at a distance r from the center of the disc is to be I , the following formula can be derived:

$$I = \int_s \frac{W}{2\pi(r^2 + a_s^2)} ds \quad (17)$$

By expressing both sides logarithmically, the relationship between PWL or power level and SPL is obtained from the following:

$$\text{PWL} = \text{SPL} + 10 \log_{10} s - 10 \log_{10} \left\{ \log_e \left[1 + \left(\frac{a}{r} \right)^2 \right] \right\} + 3 \quad (18)$$

If $a \ll r$

$$\text{PWL} = \text{SPL} + 10 \log_{10} s - 20 \log_{10} \frac{a}{r} + 3 \quad (19)$$

If the coefficient of radiation k is used as the power ratio of the radiated sound of inflexion vibration to that of piston vibration, as defined by Gölsele,⁽⁵⁾

$$W_r = k \rho c u_s^2 s \quad (20)$$

where, u_s is the vibration velocity (effective value). The above equation is expressed logarithmically as follows:

$$10 \log_{10} \frac{W_r}{10^{-12}} = 10 \log_{10} k + 10 \log_{10} \rho c s + 20 \log_{10} \frac{u_s}{\alpha_0} - 20 \log_{10} \omega + 20 \log_{10} \alpha_0 + 10 \log_{10} 10^{12} \quad (21)$$

In the above equation, $10 \log_{10} W_r / 10^{-12} = \text{PWL}$ and $20 \log_{10} u_s / \alpha_0 = \text{VAL}$, and hence PWL will be as in the following, according to Eq. (21).

$$\begin{aligned} \text{PWL} &= 10 \log_{10} k + \text{VAL} + 10 \log_{10} (\rho c s a_0^2 \omega^{-2} \times 10^{12}) \\ &= 10 \log_{10} k + \text{VAL} - 20 \log_{10} f + 10 \log_{10} s + 90 \quad (22) \end{aligned}$$

When Eq. (18) is substituted into Eq. (22), SPL will be

$$\begin{aligned} \text{SPL} &= 10 \log_{10} k + \text{VAL} - 20 \log_{10} f \\ &\quad + 10 \log_{10} \left[\log_e \left\{ 1 + \left(\frac{a}{r} \right)^2 \right\} \right] + 87 \quad (23) \end{aligned}$$

If $a \ll r$

$$\begin{aligned} \text{SPL} &= 10 \log_{10} k + \text{VAL} \\ &\quad - 20 \log_{10} f + 20 \log_{10} \frac{a}{r} + 87 \quad (24) \end{aligned}$$

The value of k obtained by substituting the measured values of SPL, VAL, r and a into Eq. (23) will be as shown in Fig. 9.

Fig. 8 Coefficient of sound pressure ν

($N = 5,000$ rpm, with an engine vibration isolating suspension)

Fig. 9 Coefficient of radiation k

($N = 5,000$ rpm, with an engine vibration isolating suspension)

4.6.3 Comparison between ν and k

According to Eqs. (16) and (24), the following relation can generally hold between $10\log \nu$ and $10\log k$

$$\begin{aligned} 10\log_{10} k &= 10\log_{10} \nu + 20\log_{10} f + 10\log_{10} s - 42 \\ &= 10\log_{10} \nu + 20\log_{10} f + 20\log_{10} a - 37 \end{aligned} \quad (25)$$

In this case

$$10\log_{10} k = 10\log_{10} \nu + 20\log_{10} f - 52 \quad (26)$$

This relation agrees with the results shown in Figs. 8 and 9.

According to Eq. (25), the formula expressing the relation between ν and k will be

$$k = 2a^2 f^2 \nu \times 10^{-4} \quad (27)$$

In this case

$$k = 6.5 f^2 \nu \times 10^{-6} \quad (28)$$

4.6.4 Proposal as a new coefficient for radiation

The coefficient of radiation, which has been in use so far, is one brought forward by Gösele and it can be obtained by Eq. (20). However, this is inconvenient as a coefficient, since k is a function of frequency. Therefore, the authors

would like to propose that μ in lieu of k be used as a coefficient of radiation.

$$W_r = \mu \rho c a_s^2 s \quad (29)$$

where, a_s is the vibration acceleration (effective value) and μ is

$$\mu = k/\omega^2 \quad (30)$$

It would be convenient if μ in the above equation is used, since it is independent of frequency.

4.6.5 Comparison between the estimated value and the measured value of sound pressure level

It is possible to estimate SPL by measuring the value of VAL when the coefficient of sound pressure ν and the coefficient of radiation k or μ are substituted into Eq. (16) or (23). Fig. 10 shows a comparison between the estimated values of the sound pressure level thus obtained (broken line) and the measured values of the sound pressure level (solid line).

Thus, it can be seen that the measured value practically agrees with the estimated (i.e. calculated) value within an error of approximately 5dB in each frequency band except for two or three bands, and within 3dB with regard to the overall value.

Fig. 10 Comparison between estimated values and measured values of sound pressure level

($N = 5,000$ rpm, with an engine vibration isolating suspension)

5. Conclusions

In this report, the relationship between vibrations and radiated sounds of packages applied to a small engine was examined.

The noticeable points in this report can be summarized as follows.

(1) When an engine is enclosed within a cubic package, radiated vibration sound at a lower frequency increases with an increase of the surface area of the package.

(2) With regard to the overall value in a cylindrical package when the engine speed of revolution is changed, the radiated sound pressure of a disc is in proportion to the 0.88 th power or the 1.0 st power of the vibration acceleration of the plate of the package.

(3) The frequency response of sound pressure coincides well to that of vibration acceleration in nearly all frequency bands.

(4) The relation between the radiation coefficient k and the sound pressure coefficient v is $k=2a^2f^2v \times 10^{-4}$.

(5) The value of the coefficient of sound pressure v is $0.1 < v < 10$, but it can be presumed to be nearly $v \approx 1$.

(6) An estimated value of sound pressure level with the use of a coefficient k or v nearly agrees with a measured value.

(7) A proposal of a new coefficient of radiation μ is made in this paper.

(8) The relation mentioned in this paper can be applied to the examination of the relationship between vibrations and noises of a frame used for a small size common machine.

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