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Study on the Mechanism of Train Noise and its Countermeasure*

(Part I; Characteristics of Wheel Vibration)

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Recently, train noise has become a great social problem. One of its main causes has been reported to be the vibration of the wheels, but only a few studies in this area have been reported. The authors made a wheel noise and vibration testing machine which had two wheels simulating the train wheel and the rail. The relationship between the noise and the vibration was studied, and the following results were obtained.

- 1) The noise is caused by wheel axial vibration.
- 2) Wheel axial vibration can be theoretically modeled by the vibration of a circular plate with the boundary conditions that the inside circumference (axis) is fixed, the outside is free and one point on it is simply supported.

1. Introduction

In Japan, high-speed railway have been developed since 1964, and studies concerning the noise produced by interaction of wheel and rail are urgently needed⁽¹⁾

The authors aim to develop a noise decreasing system by studying the vibration and noise generating mechanism. In this first report, the relationship between the vibration and the noise of a wheel disk is considered, and the characteristics of vibration of a wheel disk are experimentally and theoretically studied. The characteristics of the vibration and the noise of train wheels were studied by Remington⁽²⁾ and others^{(3)~(5)}, but a complete theoretical analysis of the vibration has not been made. Arai⁽⁶⁾ reported an analysis carried out on the assumption that the inside circumference of a wheel was fixed, and the outside was free. Nakazawa⁽⁷⁾ and Irie⁽⁸⁾ studied vibration of a circular plate. However, a wheel is supported not only by an axis but by a rail, and the noise is mainly caused by the interaction of the two, and so the problem under these boundary conditions must be analyzed.

The authors built a rolling experimental apparatus, which has two wheels simulating a wheel and a rail. With this experimental apparatus, the relationship between the vibration and the noise, especially frequency distributions, are studied. Theoretical analysis of the vibration is carried out, assuming that the wheel's inside circumference is fixed by an axis, and the outside circumference is

free and one point on it (the point contacting the rail) is simply supported. The results of the theoretical analysis are compared with the experimental results.

2. Experiment

2.1 Experimental apparatus

The experimental apparatus is shown in Fig.1. From the left side there are a press device, a small disk simulating a wheel, a big disk simulating a rail and a motor. The press device presses the small disk against the big disk by a coil spring, and the load is changeable up to 200kg, which is measured by a strain gage attached to a plate between the coil spring and the small disk. The small disk which is used in the experiment is shown in Fig.2, and referred to as the wheel disk. As shown in Fig.2, the outside diameter is 150mm, the inside diameter is 34.5mm and the thickness is 5mm. These dimensions are 1/6 of the actual wheel used in the high-speed railways. The dimensions of the big disk are 300mm outside diameter and 30mm thickness, and the outside circumference has 1/20 inclination and is referred to as the rail disk. Both disks

Fig.1 Experimental apparatus

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are supported by plain bearings with forced lubrication. The motor is a 1HP variable-speed drive type, separated from the common bed to avoid vibration, connected with the rail disk by a plane belt, and housed in a sound-proofing box. The rotational frequency of the wheel disk is changeable up to 1500r.p.m..

The measuring system is shown in Fig.3. The vibration is measured by acceleration pick-ups weighing 2g made by Bruel & Kjaer. The frequency spectrum is analyzed by a real time type frequency analyzer made by Nicolet.

Fig.2 Wheel disk

Fig.3 Measuring system

2.2 Experimental procedure

Non-rolling experiment

(1) The wheel disk was pressed against the rail disk with a load of 50 ~ 150kg. For a given axial white noise vibration at the center of the wheel disk shaft, the axial acceleration of vibration of the wheel disk was measured and the frequency distribution was obtained.

(2) The wheel disk was pressed against the rail disk, and the same procedure as (1) was done.

(3) In the pressed position as in the experiment (1), vibrating the wheel disk harmonically with the peak frequencies, which are the results of the experiment (1), two pick-ups were moved on the wheel disk and phases of every two points were detected to find the modes of the peak frequencies. Then keeping the wheel disk plane horizontal, sand was scattered on it, and vibrating it with the peak frequencies, Chladni's pictures of all the modes were obtained.

(4) In the pressed position, the center of the rail disk shaft was vibrated with white noise and the vibrational characteristics of the rail disk were investigated as mentioned in the experiment (1).

Rolling experiment

The relationship between the noise and the vibration of the wheel disk was investigated when the disk was rolling. The load was changed from 50 to 150kg and the rotational frequency was increased from 400 to 1500r.p.m.. The vibration was measured by the acceleration pick-up which was attached to the wheel disk and the electric signal was passed out through a slip ring. The noise was measured by a microphone located at ($r=b-2\text{cm}$, $\phi=150^\circ$, $z=5\text{cm}$) in Fig.9, z indicating the perpendicular direction. The rotational frequency was measured by photoelectrically detecting the reflection by reflecting tape on the wheel disk.

2.3 Experimental results

Non-rolling experiment

The frequency distribution of the vibrational acceleration is shown in Fig.4, its upper part showing the result of the non-rolling pressed experiment (1), and the lower part showing the non-rolling free experiment (2). The resolving power of the analyzer is 25Hz. The main peak frequencies in the non-rolling pressed examination are 1000, 1750, 1900, 2625, 3125, 4200, 5100, 6300, 7575 and 8300Hz.

The non-rolling free examination has main peaks at 1400, 1900, 3125, 5100 and 7575Hz. The peak values change according to the position of the pick-up; the nearer the pick-up is to the nodal lines, the smaller the peak values are.

Experiment (3) shows that the modes for each of the resonant frequencies 1750, 1900, 2625, 3175, 4200, 5100, 6300, 7500 and 8300Hz are almost the same as the modes shown in Fig.10, which are obtained in the theoretical calculation which will be mentioned later. Chladni's pictures of resonant frequencies 1750 and 2625Hz are shown in Fig.5 and Fig.6.

Experiment (4) shows that the peak frequencies of the rail disk are 1825, 2250, 3725, 4700, 6050, 6900 and 9125Hz.

Rolling experiment

Fig.7 shows the frequency distributions of the vibrational acceleration and the sound pressure caused by rolling wheel disk. Both distributions resemble each other, and the main peak frequencies are 1400, 1750, 2600, 3125, 4250, 5050, 6050, 6300, 7500 and 8300Hz. It can be concluded that the noise caused by the axial vibration of the wheel disk. At the frequency range below 1000Hz, the sound pressure has high peaks, while the vibrational acceleration does not. This is explained by the vibration of the bed whose frequency characteristics is shown in Fig.8.

As shown in Fig.7, the peak levels decrease as the frequency increases, especially above 10kHz. Therefore, peaks above 10kHz are not discussed here.

A comparison of Fig.4 and Fig.7 shows that the rolling vibration and the non-rolling pressed vibration have almost the same frequency distributions. The peak levels at the 3725 and 6050Hz frequencies which appear in the rolling vibration of the rail disk. When rolling, both disks act reciprocally and the contacting point of the wheel disk has a deflection. The deflection is small in comparison with other points, and the contacting point can be assumed to be a simply supported point.

Fig. 4 Frequency distributions of vibrational acceleration of the stationary pressed and separated wheel disk

Fig.7 Frequency distributions of vibrational acceleration and sound pressure of the rolling wheel disk

Fig. 8 Frequency distribution of the bed's acceleration

Fig.5 Chladni's figure ($f=1750\text{Hz}$)

Fig.6 Chladni's figure ($f=2625\text{Hz}$)

3. Theoretical analysis

3.1 Theoretical analysis

The eigenfunction is obtained by assuming that the wheel disk is a circular plate whose inside circumference is fixed and outside circumference is free. By making use of the eigenfunction, Green's function is calculated. By taking account of the boundary condition, that is, the outside circumference is simply supported at only one point, the solution of the equation for the deflection is obtained.

The equations are derived by using the polar coordinate shown in Fig.9. On the assumption that the supporting force is a distributed harmonic force $Q(r, \phi) \sin \omega t$, the differential equation of the deflection $W(r, \phi)$ becomes as follows⁽⁹⁾

$$D \nabla^4 W - \frac{\gamma h}{g} \omega^2 W = Q(r, \phi) \quad \dots\dots\dots(1)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2}, \quad D = \frac{Eh^3}{12(1-\nu^2)}$$

and, E : modulus of elasticity

ν : Poisson's ratio

γ : specific weight

ω : angular frequency of external force

W : deflection

Suppose $w(r, \phi)$ is the solution of Eq.(1) when $Q=0$.

$$w(r, \phi) = \{AJ_m(kr) + BY_m(kr) + CI_m(kr) + DK_m(kr)\} \times \cos m(\phi - \alpha) \quad \dots\dots\dots(2)$$

$$\text{where } k^4 = \frac{12(1-\nu^2)\gamma\omega^2}{Eh^2g} \quad \dots\dots\dots(3)$$

and, J_m : Bessel function of the first kind of order m

Y_m : Bessel function of the second kind of order m

I_m : modified Bessel function of the first kind of order m

K_m : modified Bessel function of the second kind of order m

Because of the boundary condition that the inside circumference is fixed, the deflection and the slope must be zero, and the following equations hold at $r=a$:

$$w = 0 \quad \dots\dots\dots(4)$$

$$\frac{\partial w}{\partial r} = 0 \quad \dots\dots\dots(5)$$

Due to the boundary condition that the outside circumference is free, the bending moment and the shear force must be zero at $r=b$:

$$M_r = -D \left\{ \frac{\partial^2 w}{\partial r^2} + \nu \left(\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \phi^2} \right) \right\} = 0 \quad \dots\dots(6)$$

$$V = -D \left\{ \frac{\partial}{\partial r} \nabla^2 w + \frac{1-\nu}{r^2} \frac{\partial^2}{\partial \phi^2} \left(\frac{\partial w}{\partial r} - \frac{w}{r} \right) \right\} = 0 \quad \dots\dots(7)$$

Substitution of Eq.(2) into Eqs.(4)-(7) gives Eqs.(8)-(11).

$$AJ_m(ka) + BY_m(ka) + CI_m(ka) + DK_m(ka) = 0 \quad \dots\dots(8)$$

Fig.9 Coordinate

$$\begin{aligned} &A \{ (m/ka) J_m(ka) - J_{m+1}(ka) \} \\ &+ B \{ (m/ka) Y_m(ka) - Y_{m+1}(ka) \} \\ &+ C \{ (m/ka) I_m(ka) + I_{m+1}(ka) \} \\ &+ D \{ (m/ka) K_m(ka) - K_{m+1}(ka) \} = 0 \quad \dots\dots\dots(9) \end{aligned}$$

$$\begin{aligned} &A [J_m(kb) - \{ (1-\nu)/k^2 b^2 \} \\ &\quad \times \{ m(m-1) J_m(kb) + kb J_{m+1}(kb) \}] \\ &+ B [Y_m(kb) - \{ (1-\nu)/k^2 b^2 \} \\ &\quad \times \{ m(m-1) Y_m(kb) + kb Y_{m+1}(kb) \}] \\ &- C [I_m(kb) + \{ (1-\nu)/k^2 b^2 \} \\ &\quad \times \{ m(m-1) I_m(kb) - kb I_{m+1}(kb) \}] \\ &- D [K_m(kb) + \{ (1-\nu)/k^2 b^2 \} \\ &\quad \times \{ m(m-1) K_m(kb) + kb K_{m+1}(kb) \}] = 0 \quad \dots\dots(10) \end{aligned}$$

$$\begin{aligned} &A [m J_m(kb) - kb J_{m+1}(kb) + \{ (1-\nu)/k^2 b^2 \} \\ &\quad \times \{ m^2(m-1) J_m(kb) - m^2 kb J_{m+1}(kb) \}] \\ &+ B [m Y_m(kb) - kb Y_{m+1}(kb) + \{ (1-\nu)/k^2 b^2 \} \\ &\quad \times \{ m^2(m-1) Y_m(kb) - m^2 kb Y_{m+1}(kb) \}] \\ &- C [m I_m(kb) + kb I_{m+1}(kb) - \{ (1-\nu)/k^2 b^2 \} \\ &\quad \times \{ m^2(m-1) I_m(kb) + m^2 kb I_{m+1}(kb) \}] \\ &- D [m K_m(kb) - kb K_{m+1}(kb) - \{ (1-\nu)/k^2 b^2 \} \\ &\quad \times \{ m^2(m-1) K_m(kb) - m^2 kb K_{m+1}(kb) \}] = 0 \quad \dots\dots(11) \end{aligned}$$

By setting the determinant of the set of Eqs.(8)-(11) for A , B , C and D equal to zero, the eigenvalues ($\lambda=kb$) are obtained.

In Eq.(2), when $m\alpha = \pi/2$, $\cos m(\phi - \alpha) = \sin m\phi$, the line connecting the origin and the supporting point becomes a nodal line. Therefore, the deflection W at the supporting point becomes zero, and the reacting force at that point is zero. And Eq.(2) becomes the solution under the boundary conditions.

When $m\alpha \neq \pi/2$, the line connecting the origin and the supporting point is not a nodal line, and the reacting force works to keep $W=0$ at the contacting point. We introduce Green's function $G(r, \phi; n, \theta)$, which is the summation of the eigenfunction $\Psi_{ms}(r, \phi)$:

$$G = \sum_{ms} \sum_{ms} \Psi_{ms} \quad \dots\dots\dots(12)$$

$$\begin{aligned} \Psi_{ms} = & \{ J_m(k_{ms} r) + f_{ms} Y_m(k_{ms} r) + g_{ms} I_m(k_{ms} r) \\ & + h_{ms} K_m(k_{ms} r) \} \cos m(\phi - \theta) \quad \dots\dots\dots(13) \end{aligned}$$

where $f_{ms} = B_{ms}/A_{ms}$, $g_{ms} = C_{ms}/A_{ms}$,

$$h_{ms} = D_{ms}/A_{ms}$$

The deflection $W(r, \phi)$ is given by

$$W(r, \phi) = \int_0^{2\pi} \int_a^b G(r, \phi; \eta, \theta) Q(\eta, \theta) \eta d\eta d\theta \quad \dots\dots\dots(14)$$

The point (r_0, ϕ_0) being simply supported, the reacting force is given by Dirac's delta function.

$$Q(r, \phi) = R_0 \delta(r - r_0) \delta(\phi - \phi_0) \quad \dots\dots\dots(15)$$

From Eq.(14) and Eq.(15), we obtain

$$W(r, \phi) = R_0 G(r, \phi; r_0, \phi_0) \quad \dots\dots\dots(16)$$

The eigenfunction ψ_{ms} satisfies the following differential equation.

$$D\nabla^4 \psi_{ms} - \frac{\gamma h}{g} \omega_{ms}^2 \psi_{ms} = 0 \quad \dots\dots\dots(17)$$

Eq.(17) is reformed to Eq.(18).

$$\nabla^4 \psi_{ms} = \frac{\gamma h}{Dg} \omega_{ms}^2 \psi_{ms} = k_{ms}^4 \psi_{ms} \quad \dots\dots\dots(18)$$

Substitution of Eqs.(15) and (16) into Eq.(1) gives

$$D\nabla^4 G - \frac{\gamma h}{g} \omega^2 G = \delta(r - r_0) \delta(\phi - \phi_0) \quad \dots\dots\dots(19)$$

Substituting Eqs.(12) and (18) into Eq.(19), the following Eq.(20) is obtained.

$$\sum_{ms} \sum_{ls} F_{ms} k_{ms}^4 \psi_{ms} - \sum_{ms} \sum_{ls} F_{ms} k_{ms}^4 \psi_{ms} = \delta(r - r_0) \delta(\phi - \phi_0) \frac{1}{D} \quad \dots\dots\dots(20)$$

ψ_{ms} 's are orthogonal functions, and when $m \neq l$ or $s \neq k$, the following equation is satisfied.

$$\int_0^{2\pi} \int_a^b \psi_{ms} \psi_{lk} r dr d\phi = 0 \quad \dots\dots\dots(21)$$

By multiplying ψ_{lk} to Eq.(21), and making a double integration Eq.(22) holds for $m=l$ and $s=k$.

$$F_{ms} (k_{ms}^4 - k^4) \int_0^{2\pi} \int_a^b \psi_{ms}^2 r dr d\phi = \frac{1}{D} \psi_{ms}^2(r_0, \phi_0) \quad \dots\dots\dots(22)$$

The double integration can be rewritten as:

$$\int_0^{2\pi} \int_a^b \psi_{ms}^2 r dr d\phi = \{b^2 L(k_{ms} b) - a^2 L(k_{ms} a)\} \frac{2\pi}{\epsilon_m} \quad \dots\dots\dots(23)$$

where, $\epsilon_m = 1$; $m=0$, $\epsilon_m = 2$; $m \geq 1$, and

$$L(z) = (1/2) \{ J_m^2(z) + J_{m+1}^2(z) - (2m/z) J_m(z) J_{m+1}(z) \} + (1/2) f_{ms}^2(z) \{ Y_m^2(z) + Y_{m+1}^2(z) - (2m/z) Y_m(z) Y_{m+1}(z) \}$$

$$\begin{aligned} & + (1/2) g_{ms}^2(z) \{ I_m^2(z) - I_{m+1}^2(z) - (2m/z) I_m(z) I_{m+1}(z) \} \\ & + (1/2) h_{ms}^2(z) \{ K_m^2(z) - K_{m+1}^2(z) + (2m/z) K_m(z) K_{m+1}(z) \} \\ & + f_{ms}(z) [J_m(z) Y_m(z) - (m/z) \{ J_m(z) Y_{m+1}(z) + J_{m+1}(z) Y_m(z) \} + J_{m+1}(z) Y_{m+1}(z)] \\ & + (1/z) g_{ms}(z) \{ J_m(z) I_{m+1}(z) + J_{m+1}(z) I_m(z) \} \\ & + (1/z) h_{ms}(z) \{ -J_m(z) K_{m+1}(z) + J_{m+1}(z) K_m(z) \} \\ & + (1/z) f_{ms}(z) g_{ms}(z) \{ Y_m(z) I_{m+1}(z) + Y_{m+1}(z) I_m(z) \} \\ & + g_{ms}(z) h_{ms}(z) [I_m(z) K_m(z) + (m/z) \{ I_m(z) K_{m+1}(z) - I_{m+1}(z) K_m(z) \} + I_{m+1}(z) K_{m+1}(z)] \\ & + (1/z) h_{ms}(z) f_{ms}(z) \{ -Y_m(z) K_{m+1}(z) + Y_{m+1}(z) K_m(z) \} \quad \dots\dots\dots(24) \end{aligned}$$

From Eqs.(12), (22) and (23),

$$G = \sum_{ms} \frac{\epsilon_m b^2}{2\pi D} \frac{\psi_{ms}(r_0, \phi_0) \psi_{ms}(r, \phi)}{L(\lambda_{ms}) - (a/b)^2 L(\lambda_{ms} a/b)} \frac{1}{\lambda_{ms}^4 - \lambda^4} \quad \dots\dots\dots(25)$$

where $\lambda = kb$, $\lambda_{ms} = k_{ms} b$

In this problem, we have only one supporting point $(b, 0)$, where the deflection W becomes zero, and from Eq.(16)

$$G(b, 0; b, 0) = 0 \quad \dots\dots\dots(26)$$

Therefore, a new eigenvalue λ can be obtained.

Setting R_0 equal to 1, calculation of $W(r, \phi) = G(r, \phi; b, 0)$ at each point on the wheel disk gives the mode.

3.2 Result of numerical calculation

Numerical calculations are carried out according to the theoretical analysis. In the calculation the following dimensions are used.

Outside radius: $b=75.0\text{mm}$

Inside radius: $a=25.0\text{mm}$

Thickness: $h=5.0\text{mm}$

Modulus of elasticity: $E=2.1 \times 10^4 \text{kg} \cdot \text{mm}^{-2}$

Poisson's ratio: $\nu=0.3$

Specific weight: $\gamma=7.8 \text{g} \cdot \text{cm}^{-3}$

Here, although the inside diameter is not defined clearly as shown in Fig.2, we assume it to be 25.0mm.

First, the eigenvalues λ_{ms} 's are obtained, as listed in Table 1.

Secondly, by use of λ_{ms} 's, the eigenvalues λ 's are obtained by setting Eq.(25), $G(75.0, 0; 75.0, 0) = 0$. In this process, we have to pay attention to the convergence of λ . When λ is less than 5, λ converges sufficiently by taking 150 λ_{ms} 's, and when

Table 1 Eigenvalues λ_{ms}

s	m=0	m=1	m=2	m=3	m=4	m=5	m=6	m=7	m=8
1	2.655	2.673	2.948	3.694	4.712	5.797	6.886	7.966	9.040
2	6.727	6.872	7.305	8.006	8.923	9.987	11.130	12.307	13.490
3	11.443	11.542	11.841	12.340	13.030	13.891	14.891	15.994	17.161

λ is less than 10, λ converges by taking 250 λ_{ms} 's.

Fig.10 shows the modes, the eigenvalues and the eigenfrequencies. Here, the eigenfrequencies f 's are calculated by

$$f = \frac{\omega}{2\pi} = \lambda^2 \frac{h}{b^2} \sqrt{\frac{Eg}{12(1-\nu^2)\gamma}} \frac{1}{2\pi} \dots\dots\dots (27)$$

To compare the calculated modes with Chladni's figures shown in Fig.5 and Fig.6, the theoretical modes and contour lines for amplitude ($\lambda=2.833$ and $\lambda=3.442$) are shown in Fig.11 and Fig.12.

Fig. 11 Mode ($\lambda=2.833$)

Fig. 12 Mode ($\lambda=3.442$)

$\lambda_{11}=2.673$ $f_{11}=1565$	$\lambda_{21}=2.948$ $f_{21}=1904$	$\lambda_{31}=3.694$ $f_{31}=2989$	$\lambda_{41}=4.712$ $f_{41}=4864$	$\lambda_{51}=5.797$ $f_{51}=7361$

$\lambda=2.661$ $f=1551$	$\lambda=2.833$ $f=1758$	$\lambda=3.442$ $f=2595$	$\lambda=4.359$ $f=4162$	$\lambda=5.344$ $f=6255$	$\lambda=6.231$ $f=8504$

Fig.10 Eigenvalues, eigenfrequencies and their modes

4. Discussion on the results

Table 2 shows the results of the experiment and the theoretical calculation. In the experiment, modes of $\lambda_{11}=2.677$ (1551Hz) and $\lambda=2.611$ (1565Hz) are not found because both frequencies are too close to be devided.

From Table 2, we can see that the theoretical calculation is in good agreement with the experimental result. Also, the theoretical modes are in good accordance with the modes obtained by the non-rolling experiment, i.e. modes checked by the two pick-ups and Chladni's figures. Therefore, we can say that the vibration of the rolling wheel disk is theoretically

represented by the vibration of the circular plate with the boundary conditions that the inside circumference is fixed, the outside is free and one point on it is simply supported.

Considering the theoretical analysis, when the ratio of the inside and the outside diameter is constant, the four boundary conditions are equivalent and λ_{ms} has the same value. Therefore, when the ratio is constant, the eigenfrequencies are proportional to the square of the inside and the outside diameters. When the thickness is proportional to the diameters, the eigenfrequencies are inversely proportional to the size of the wheel. Therefore, we can estimate the vibrational characteristics of the real size.

Table 2 Theoretical eigenfrequencies and experimental peak frequencies

Mode									
Theoretical calculation	1758	1904	2595	2989	4162	4864	6255	7361	8504
Rolling ex.	1750	1900	2600	3125	4275	5050	6300	7500	8300
Non-rolling pressed ex.	1750	1900	2625	3175	4200	5100	6300	7575	8300
Non-rolling separated ex.	—	1900	—	3175	—	5100	—	7575	—

5. Conclusions

In this paper, the vibrational characteristics are studied by use of a rolling experimental apparatus which has two disks simulating a rail and a wheel. The wheel disk has a simple shape which is different from the actual one, but useful suggestions are obtained for the future studies.

- (1) The noise is caused by the axial vibration of the wheel, and the vibration is caused by the interaction of the wheel and the rail.
- (2) The frequency distributions of the vibrations of the rolling wheel have the same characteristics as the non-rolling pressed wheel.
- (3) The vibration of the wheel can be theoretically analyzed with boundary conditions that the inside circumference is fixed, the outside is free and one point on it is simply supported.

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Reference

- (1) Matsui, M., et al, *Noise Control and its Examples* (in Japanese), (1974), p.105, Fuji Technosystem.
- (2) Remington, P.J., *Jour. Sound and Vibration*, Vol.46, No.3, (1976), p.359.
- (3) Rudd, M.J., *Jour. Sound and Vibration*, Vol.46, No.3, (1976), p.381.
- (4) Vér, I.L., et al, *Jour. Sound and Vibration*, Vol.46, No.3, (1976), p.395.
- (5) Remington, P.J., *Jour. Sound and Vibration*, Vol.46, No.3, (1976), p.419.
- (6) Arai, H., et al, *Proceeding of Japan Soc. Mech. Engrs.* (in Japanese), Vol.760, No.14, (1976-10), p.92.
- (7) Nakazawa, A., *Trans. Japan Soc. Mech. Engrs.* (in Japanese), Vol.33, No.256, (1967-12), p.1951.
- (8) Irie, T., et al, *Proceeding of Japan Soc. Mech. Engrs.* (in Japanese), Vol.760, No.10, (1976-10), p.127.
- (9) Sezawa, K., *Vibration* (in Japanese), (1971), p.252, Kokusai Riko Kenkyusha.