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On the Effect of a Variable Stiffness-type Dynamic Absorber

with Eddy-current Damping *

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The paper presents a new dynamic absorber which consists of a variable stiffness-type spring, a mass, and a magnetic damper using the damping effect of eddy-currents. It has advantages that the absorber is able to use for improving the damping property of a main vibration system where changes of the natural frequency take place, and it is stable in damping characteristics under varying environment.

To examine its practical applications, the damping performance of the absorber is studied in both experimental and theoretical aspects with a specific view to improving the dynamic stiffness of the ram structure. Further, the relation between viscous damping coefficient and intermediate factors of the magnetic damper is deduced in order to realize the best damping optionally.

1. Introduction

The most favorable conditions for the dynamic absorber, wellknown as a typical passive absorber, have been adequately studied in the past.⁽¹⁾ This absorber has been frequently utilized as a compensating element for improving the damping performance of a mechanical vibration system since it is of simple structure but, it develops high performance when these conditions are satisfied.

However, this absorber has the following drawback in practical applications. Even if the absorber has been set in the most favorable conditions in advance, when the natural frequency of the main vibration system changes as in vertical lathes where the ram moves vertically in cutting operations, or damping characteristics of the absorber system lack stability, the damping performance deteriorates markedly. A lack of stability is caused by a change of damping characteristics due to temperature variations and changes in properties of matter, since damping elements depend on the use of an oil damper and viscous and elastic materials. Thus the absorber has insufficient reliability. On the other hand, it is pointed out that the changes of the natural frequency, can be remedied by the introduction of a structure⁽²⁾ permitting the change of the spring constant of the absorber externally.

In the previous report⁽³⁾ we proposed a method of tuning to altered proper frequency of the main vibration system through a variable stiffness type dynamic absorber consisting of an oil damper, a cantilever spring fixed at one end and a mass movable along the spring. It was shown that this arrangement produces favorable damping characteristics.

However, the lack of reliability in damping characteristics and the impossibility of quantitatively computing the damping term during the stage of designing remained as an unresolved question.

In the present report, it is attempted to obtain a solution of this problem through the introduction of a magnetic damper instead of the oil damper. We authors shall explain the significance of using a magnetic damper, study the damping characteristics of the newly conceived absorber in both experimental and theoretical aspects with a specific view to improving the damping performance of the ram structure, and examine its practical applications. Furthermore, we intend to clarify the relationship between the viscous damping coefficient and interactive factors so as to deduce formulas applicable in order to realize the best damping optionally.

2. Significance of using a magnetic damper

The so-called magnetic damper using the braking effect of eddy currents, which is not suitable for producing a large damping effect compared with an oil damper, has not been applied to the damping of a mechanical vibration system previously except in such restricted cases as the combination with a magnetic bearing.⁽⁴⁾ However, it has the advantage that its damping characteristics are stabilized under varying environment and that it is easily installed without mechanical contacts. Therefore, it appears that the shortcomings inherent in a dynamic absorber can be compensated by a magnetic damper, if necessary amount of damping can be produced. Hence, we shall examine the viscous damping coefficient demanded of a dynamic absorber by comparing it with that of a viscous damper singly producing the same damping effect.

Fig. 1 shows a one degree of freedom system consisting of a spring with spring constant K and an inertial term with mass M , when (a) a viscous damper with a viscous damping coefficient D is attached, and (b) a dynamic absorber with viscous damping coefficient c is attached, respectively.

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Fig. 1. Damping of system with one degree of freedom; (a) Damping with a viscous damper (b) Damping with a dynamic absorber

Resonance peak value M_p under forced vibration is related to $\bar{c}$ and D in the following manner:

For (a)

$$D = 2\sqrt{MK} \sqrt{\frac{1}{2} \left(1 - \frac{\sqrt{M_p^2 - 1}}{M_p} \right)} \quad (1)$$

under the condition that the damping factor $\zeta = D/(2\sqrt{MK}) < 0.707$

For (b), M_p is unequivocally determined by the mass ratio $\mu = m/M$

$$M_p = \sqrt{1 + 2/\mu} \quad (2)$$

if Den Hartog's optimum tuning condition⁽¹⁾ is satisfied.

The relation of C_{opt} (the optimum value of c) to M_p is expressed by the following equation:

$$C_{opt} = 2\sqrt{MK} \sqrt{\frac{3}{(M_p^2 + 1)^3}} \quad (3)$$

Taking the ratio of (1) to (3) we obtain a function of M_p :

$$\frac{C_{opt}}{D} = \sqrt{\frac{6}{(M_p^2 + 1)^3 (1 - \sqrt{M_p^2 - 1}/M_p)}} \quad (4)$$

The formulas (4) and (2) plotted against M_p are shown in Fig. 2, which permits a simple comparison of D and c as referred to the resonance peak value M_p . For example, we read $M_p \approx 3.3$ for installing a dynamic absorber when mass ratio $\mu = 0.2$. The value of c/D for $M_p = 3.3$ is found to be 0.23. Hence, to achieve the damping effect corresponding to $M_p = 3.3$, the viscous damping coefficient of a dynamic absorber may be only 0.23 of that of a viscous damper. Usually $\mu < 0.2$ is adopted for a dynamic absorber, so the value of c/D may become still smaller.

We shall now define the damping factor of the absorber system $\zeta = c/2\sqrt{mk}$ and plot the relation between ζ_{opt} , the optimum damping factor supposing that c takes C_{opt} , and μ , which turns out as Fig. 3. This graph also shows that ζ_{opt} may be less than 0.25 for $\mu < 0.2$.

The above consideration shows that a large damping factor is not needed in the dynamic absorber. A very effective damping is achieved with a rather small damping factor. This, together with already mentioned advantages, makes the magnetic damper a damping element suitable for the dynamic absorber.

Fig. 2. Relation of M_p to C_{opt}/D and μ

Fig. 3. Relation between ζ_{opt} and μ

3. Analysis of a damping effect

Various suggestions^{(5) (6)} have been made to improve the damping performance of the so-called ram structure characterized by very low dynamic stiffness, which consists of a protruding arm with a tool post at its extremity as in vertical lathes, boring machines and similar machine tools.

Now we shall explore the possibility of improving dynamic stiffness of the ram structure by means of a dynamic absorber with the magnetic damping through the comparison of theory and experiment. The damping characteristics of this absorber are analyzed in detail, including those of higher vibration modes.

3.1 Analysis

Fig. 4 represents a mechanical system to be analyzed. The main vibration system consists of a ram of uniform cross-section with length l_1 and area inertia of moment I_1 , whose one end is fixed and another end carries an inertial term with mass m_1 . The damper system consists of a cantilever spring whose one extremity is fixed to mass m_1 of the main vibration system, a mass m_2 at the extremity of the spring, which can be moved along the axis, and a magnetic damper which generates viscous damping between m_2 and the main vibration system. The spring constant of the

Fig. 4. Mechanical system used for analysis

cantilever spring, whose area inertia of moment is I_2 , is adjustable optionally by varying the position of m_2 externally. So we may call the absorber of this type a variable stiffness type dynamic absorber.

In order to deduce the relation between the exciting force F_{in} and the resultant displacement in Fig. 4, we shall divide this mechanical system into 5 basic elements of the ram, the concentrated mass m_1 , the cantilever spring, the concentrated mass m_2 and the damper, and define the variables at each terminal as represented in Fig. 5. For analysis we use an 8-terminal signal flow graph which the authors have prepared beforehand ⁽⁷⁾⁽⁸⁾ as a dynamic model of the ram

structure. The transmittance of such signal flow graph is usually approximated by a lumped parameter system, but we employ here the distribution parameter system in its original form since we want to examine the behavior of higher vibration modes. After setting up boundary conditions consisting of the condition of continuity at joints, equilibrium of forces, $Y_0 = \phi_0 = 0$ at the terminal 0, and $M_7 = 0$ at the terminal 7, we obtain the signal flow graph of Fig. 6, resulting from 8-terminal signal flow graphs for the main vibration system and the absorber system, respectively, after rearrangement as to the joints of both. Each transmittance is expressed as follows:

$$\begin{aligned} Z_1 &= \frac{1}{E_1 I_1 \lambda_1^2} (\sinh \beta_1 \sin \beta_1), & A_1 &= \frac{1}{E_1 I_1 \lambda_1^3} (\sinh \beta_1 \cos \beta_1 - \cosh \beta_1 \sin \beta_1) \\ B_1 &= \frac{1}{(E_1 I_1 \lambda_1^2)^2} (\cosh \beta_1 \cos \beta_1 - 1) \\ C_1 &= -(\cosh \beta_1 \cos \beta_1 + 1) + \frac{m_1 S^2}{E_1 I_1 \lambda_1^3} (\sinh \beta_1 \cos \beta_1 - \cosh \beta_1 \sin \beta_1) \\ D_1 &= \frac{-1}{E_1 I_1 \lambda_1} \left\{ \sinh \beta_1 \cos \beta_1 + \cosh \beta_1 \sin \beta_1 - \frac{m_1 S^2}{E_1 I_1 \lambda_1^3} (\cosh \beta_1 \cos \beta_1 - 1) \right\} \\ Z_2 &= -E_2 I_2 \lambda_2^3 \left\{ \sinh \beta_2 \sin \beta_2 - \frac{m_2 S^2 + cS}{E_2 I_2 \lambda_2^3} (\sinh \beta_2 \cos \beta_2 + \cosh \beta_2 \sin \beta_2) \right\} \\ A_2 &= E_2 I_2 \lambda_2^3 \left\{ \cosh \beta_2 \sin \beta_2 + \sinh \beta_2 \cos \beta_2 - \frac{m_2 S^2 + cS}{E_2 I_2 \lambda_2^3} (2 \cosh \beta_2 \cos \beta_2) + \frac{cS}{E_2 I_2 \lambda_2^3} (\cosh \beta_2 + \cos \beta_2) \right\} \\ B_2 &= (\cosh \beta_2 \cos \beta_2 + 1) - \frac{m_2 S^2 + cS}{E_2 I_2 \lambda_2^3} (\sinh \beta_2 \cos \beta_2 - \cosh \beta_2 \sin \beta_2) \\ C_2 &= (E_2 I_2 \lambda_2^2)^2 \left\{ (1 - \cosh \beta_2 \cos \beta_2) - \frac{m_2 S^2 + cS}{E_2 I_2 \lambda_2^3} (\cosh \beta_2 \sin \beta_2 - \sinh \beta_2 \cos \beta_2) - \frac{cS}{E_2 I_2 \lambda_2^3} (\sinh \beta_2 - \sin \beta_2) \right\} \\ D_2 &= E_2 I_2 \lambda_2 \left\{ (\cosh \beta_2 \sin \beta_2 - \sinh \beta_2 \cos \beta_2) - \frac{m_2 S^2 + cS}{E_2 I_2 \lambda_2^3} (2 \sinh \beta_2 \sin \beta_2) \right\} \end{aligned}$$

where

$$\lambda_1 = \sqrt[4]{\frac{\rho_1}{E_1 I_1}} \omega, \quad \lambda_2 = \sqrt[4]{\frac{\rho_2}{E_2 I_2}} \omega, \quad \beta_1 = \lambda_1 l_1, \quad \beta_2 = \lambda_2 l_2$$

where S is Laplace operator, a_1 and a_2 are cross-sections of rams, ρ_1 and ρ_2 are their densities, and E_1 and E_2 are their Young's moduli.

Setting $S = j\omega$, the frequency transfer function between F_{in} and Y_3 , in which we are interested, takes the following form according to Fig. 6:

$$\frac{Y_3}{F_{in}}(\omega) = \frac{Z_1 Z_2 (B_1 D_2 - A_1 B_2)}{Z_1 Z_2 (A_1 A_2 - B_2 C_1 - C_2 B_1 + D_1 D_2 - Z_1 Z_2) + (A_1 D_1 - C_1 B_1)(B_2 C_2 - A_2 D_2)} = \frac{Y + j\bar{Y}}{X + j\bar{X}} \quad (5)$$

where X , $\bar{X}$ and Y , $\bar{Y}$ represent real and imaginary parts in the denominator and numerator of Eq. (5), respectively.

Fig. 5. Definition of variables and coordinates

Fig. 6. Signal flow graph

m_1 g	800	m_2 g	100
l_1 cm	45	l_2 cm	1~15
t_1 cm	0.8	t_2 cm	0.1
b_1 cm	6.4	b_2 cm	2.0
E_1 dyn/cm ²	2.1×10^{12}	E_2 dyn/cm ²	2.1×10^{12}

Table 1. Data of analyzing mechanical system

Figs. 7 and 8 represent the Bode diagrams drawn with a plotter according to the computer evaluation of Eq. (5) substituting the values in Table 1. Fig. 7 is the Bode diagram of the receptance Y_3/F_{in} of the main vibration system without an absorber. Here the resonance peaks occur at 20 Hz, 160 Hz, 460 Hz and 920 Hz. Since the peak of the first vibration mode is chiefly responsible for the deterioration of dynamic stiffness, the absorber constants l_2 and c may be selected to produce optimum tuning at this frequency, namely: $l_2 = 8.75$ cm, and $c = 1.8 \times 10^3$ dyns/cm. The receptance apparently corresponding to the optimum tuning is obtained as Fig. 8. As a result, the peak value of receptance is reduced to 15 dB or about 5 times in amplitude. Incidentally, this absorber tuned to the first vibration mode leaves higher vibration modes unaffected.

Fig. 7. Receptance of main vibration system

Fig. 8. Receptance after the introduction of optimally adjusted absorber

Fig. 9. Measuring apparatus of receptance

3.2. Experiment

3.2.1. Experimental apparatus

The configuration and dimensions of the main vibration system and the absorber system used for the experiment were made to conform to the values in Table 1. The mass M of the main vibration system, reduced to one degree of freedom, is expressed by $m_1 + 0.23 m_2$, where m_2 is the mass of the ram. Hence, the mass ratio μ of this apparatus becomes 0.083.

Fig. 9 represents an apparatus assembled for measuring receptance. The vibrator operates electromagnetically. The force and acceleration sensed by the impedance head are separated into displacement amplitude and phase in the transfer function analyzer and recorded with a plotting machine.

The structure of the magnetic damper used in this experiment is shown in Fig. 10. The magnetic field was generated by a U-shaped magnetic path made of mild steel provided with a permanent magnet made of sintered ferrite. B , the magnetic flux density at the 2 mm air gap, was 1,950 gauss. Two such magnets, opposed to each other, were placed on both sides of the ram of the main vibration system. A copper conductor fixed to the absorber mass was adjusted to be able to move vertically without mechanical contact.

3.2.2. Experimental results and consideration

Fig. 11 shows the receptance when the absorber is not provided. The curve has an excellent resemblance to the theoretical one in Fig. 7.

Fig. 10. Structure of magnetic damper

Fig. 11. Measured receptance (without absorber)

Fig. 12 represents the experimental results when an absorber with a spring ($l_s=8.4\text{cm}$) is provided. They coincide well with the theoretical results of Fig. 8. When the absorber is tuned to the first vibration mode, the vibration characteristics of higher frequency modes remain unaffected. However, viscous damping coefficient c in the experiment is estimated to be about $0.8 \times 10^3 \text{ dyne} \cdot \text{sec/cm}$, differing from the optimum value predicted by theoretical analysis. Hence, the receptance peak stays at about 20 dB, or 10 times.

At any rate, the resonance peak, which stood at 100 times when an absorber was not mounted, has been reduced to 10 times with the addition of such a simple absorber. Hence, we have arrived at the conclusion that the damping effect as predicted by the theory can be achieved by suitably setting the magnetic damper.

Fig. 12. Measured receptance (with absorber)

Fig. 13. Motion of a conductor in the uniform magnetic field

Fig. 14. Measuring apparatus of damping coefficient

4. Analysis of a magnetic damper

4.1. Theoretical analysis

A formula expressing the relation between the structure of the magnetic damper and the damping coefficient must be deduced in order to obtain the coefficient demanded for limited space. The formula for a conductor short-circuited in a ring form is known already.⁽⁹⁾ However, a practical formula for the plate has not been developed yet as far as we know. We must deduce it afresh.

Joule's heat P , generated by a current density i or electric field vector E in a conductor with a proper resistance ρ , can be expressed by the following formula:

$$P = \int \int \int \rho i^2 dv = \int \int \int \frac{1}{\rho} E^2 dv \quad (6)$$

According to the theory of electromagnetic induction, an electric power $\omega x_0 B \cos \omega t$ is generated through induction in a sufficiently wide conductor with a thickness t moving along the X-axis at a velocity of $\dot{x} = \dot{x}_0 \sin \omega t$ in a uniform magnetic field with density B directed along Z-axis (Fig. 13). This power acts as a distribution of electric field. When the projected area of magnetic flux acting on the conductor is expressed by bt , Joule's heat generated per one period becomes:

$$P = \int_0^{2\pi/\omega} \int_0^l \int_0^t \frac{1}{\rho} E^2 dx dy dz dt = \frac{\pi}{\rho} V \omega x_0^2 B^2 \quad (7)$$

where $V = btl$ (the volume of the conductor in the magnetic field).

On the other hand, an element having viscous damping coefficient c loses an energy amount of

$$W = \int_0^{2\pi/\omega} f \dot{x} dt = c x_0^2 \omega^2 \int_0^{2\pi/\omega} \cos^2 \omega t dt = c \omega \pi x_0^2 \quad (8)$$

per period through work when it is moving with a velocity $\dot{x} = x_0 \omega \cos \omega t$.

From $W = P$, viscous damping coefficient c is expressed as follows:

$$c = 10^{-14} \frac{V}{\rho} B^2 (\text{dyns/cm}) \quad (9)$$

where the units are taken as $\rho (\Omega \text{m})$, $V (\text{mm}^3)$ and $B (\text{G})$.

4.2. Experiment

4.2.1. Experimental apparatus and method

The damping coefficient of the magnetic damper c is measured by the apparatus shown in Fig. 14. The damping coefficient can calculate according to the following equation, when the damping factor ζ and the period T are found from wave forms of damping free vibration.

$$c = 2m \frac{\zeta}{\sqrt{1-\zeta^2}} \frac{2\pi}{T} \quad (10)$$

Fig. 15 represents a general aspect of the magnetic damper incorporating electro-magnets. This structure permits one to study the influence of the gap g , the magnetic flux density B , the volume V of the conductor (thickness: t) in the magnetic field, and the proper resistance on the damping coefficient. The input current i of the electromagnets is related to g and B in the following manner:

$$B = 2.45 \times 10^4 \frac{i}{g} \text{ (G)} \quad \dots\dots\dots(11)$$

where the units are taken as i A and g mm. An experiment has been carried out with parameters of Table 2.

The width of the conductor plate was 65 mm, the same as the width of the electro-magnet pole.

Fig. 15. Magnetic damper using a electromagnet

g	3, 4.85, 6.8	mm
i	0.1 ~ 0.8	A
t	1 ~ 6	mm
ρ	1.69×10^{-8} (Cu), 2.62×10^{-8} (Al)	Ωm
A	65×22	mm^2

Table 2. Measuring range of the experimental apparatus

4.2.2. Experimental results and consideration

Measured wave form of damping free vibration is shown in Fig. 16. Figs. 17 and 18 represent the relation between viscous damping coefficient and various parameters. The same units are employed as in the theoretical formula.

Figs. 17(a) and (b) correspond to different c - i correlations when copper and aluminum, respectively, are used as conductors. c is found to be proportionate to t , the plate thickness. Since A , the area of the conductor moving across a uniform magnetic field, is assumed to be constant, these diagrams correspond to the relation between c and V . Other relations excepting that between c and t not being proportionate, logarithmic scales are employed for their representation. Fig. 18(a) represents c - ρ relation, taking as parameter the proper resistance ρ of the material. This shows that c and ρ vary in inverse proportion. Since c and g^2 also vary in inverse proportion according to (b), c is found to vary proportionately to i^2 . These results are summarized by the following equation:

$$c = K_1 \rho^{-1} t i^2 g^{-2} = K_2 \frac{V}{\rho} B^2 \text{ (dyn s/cm)} \quad \dots(12)$$

Fig. 16. One example of the wave form of damping free vibration

Relation between c and i for two conductor materials

Relation between c and t

Fig. 17. Relations between intermediate factors and viscous damping coefficient

Fig. 18. Relations between intermediate factors and viscous damping coefficient in logarithmic scales

K_2	3	4.85	6.8
Cu	0.297	0.279	0.283
Al	0.257	0.279	0.282

$K_2 \times 10^{-14}$

Table 3. Values of K_2 under different experimental conditions

where the proportional constants K_2 are taken as in Table 3. They cluster around 0.28×10^{14} although they differ somewhat according to g and materials. The discrepancy between this result and the theoretical value $K_2 = 1 \times 10^{-14}$ is accounted for by the fact that, whereas in the theoretical analysis the field is assumed to act locally on a very wide conductor, in the experiment the width of the conductor and that of the magnet pole are made identical from the practical standpoint, by which apparently the loop of the eddy current is restricted and the work efficiency deteriorates. In order to confirm this hypothesis, we conducted another experiment with a conductor of larger width, and obtained the result $K_2 = 0.44 \times 10^{-14}$ when the width was 120 mm. Fig. 19 shows the effect of enlarged conductor width. Thus it seems possible to make K_2 closer to 1×10^{-14} if we take the conductor width sufficiently large. In practice, Eq. (12) seems to be more realistic since the width and, consequently, the area of the conductor are finite.

The viscous damping coefficient of the magnetic damper shown in Fig. 10 was computed by Eq. (12). $\bar{c}$ deduced from the combination of theory and practice took the value $0.837 \times 10^3 \text{ dyne} \cdot \text{sec}/\text{cm}$, coinciding well with the experimental result.

Fig. 19. Effect of varying conductor width on $\bar{c}$

5. Conclusions

In this paper, the variable stiffness type dynamic absorber incorporating a magnetic damper as the damping element has been studied with the following conclusions:

(1) The dynamic absorber permits effective utilization of the damping effect of comparatively small magnitude. Hence, a magnetic damper, which has stabilized characteristics although it does not produce so large damping effect as an oil damper, appears to be suitable as an element of a dynamic absorber.

(2) We have experimentally confirmed the theoretical damping effect predicted for a variable stiffness type dynamic absorber incorporating a magnetic damper, which has a permanent magnet as a principal element. Such absorber has been found practically feasible. It has been confirmed that higher vibration modes remain unaffected if the absorber is tuned to the first vibration mode. This means that the absorber can be designed in approximation on one degree of freedom.

(3) The relation between viscous damping coefficient and intermediate factors of a magnetic damper has been deduced from both theoretical and experimental sides. This has enabled a quantitative evaluation of the damping term, which was an obstacle to designing an optimum dynamic absorber in the past.

(4) The viscous damping coefficient evaluated through an appropriate formula for the magnetic damper used in the experiment has been found to coincide well with the experimentally obtained value.

We intend to use these results for practical purpose.

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