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Initial Behavior of Hysteresis Loop of Low Carbon Steels

under Repeated Load*

(Theoretical treatment on the multiplying model of dislocations)

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This is a second report of authors aiming to clarify the mechanism of the initial increase of the plastic strain amplitude $\Delta\epsilon_p$ of low carbon steels under fatigue load. Close observations showed that a sharp increase of $\Delta\epsilon_p$ took place after a gradual growth, and this gradual growing process had been analyzed in the first report by developing the interaction models between dislocations and solute carbon atoms under repeated load.

Therefore, in this report, a sharp increase of $\Delta\epsilon_p$ is theoretically dealt with by establishing a multiplying model of dislocation loops from the Frank-Read sources whose activation took place following the cease of the increase in the length of the movable dislocation lines upon reaching the strong pinning points. It is shown that the derived theory satisfactorily explains the experimental trend.

1. Introduction

Recently, much attention has been paid to the effect of strain aging due to cyclic stressing on the fatigue properties of steels such as the characteristic behavior of stress-strain hysteresis loops and the existence of clear fatigue limit[1]-[4].

The authors have also been interested in the aging effect on the hysteresis behavior of steels and had previously made careful observations on the initial growth of the plastic strain amplitude $\Delta\epsilon_p$ of low carbon steels during the fatigue process[5]. The observations showed that, in the first stage(stage I), $\Delta\epsilon_p$ increased at a very slow rate until it reached about 200 micro-strains, and then, in the second stage (stage II), the increase of $\Delta\epsilon_p$ was accelerated until $\Delta\epsilon_p$ reached its peak value depending on the applied stress. In the authors' previous report, the mechanism of the initiation and the slow growth of $\Delta\epsilon_p$ in the first stage was analyzed by means of an interaction model between dislocations and solute carbon atoms in the material. And it was shown that the results of the theoretical analysis satisfactorily explain the experimental trend of $\Delta\epsilon_p$ in the stage I. But a direct extension of this theoretical computation gave a considerable underestimation of $\Delta\epsilon_p$ in the stage II, which suggested that a different mechanism was underlying in this stage.

Therefore, it is the purpose of the

present study to analyze the behavior of $\Delta\epsilon_p$ in the stage II by establishing a theoretical model which is different from the one used but compatible with the theory in the previous report. This purpose is successfully attained by taking the multiplication of dislocation loops by the activation of Frank-Read sources into consideration, and it is shown that the derived theory is sufficient to explain the characteristic behavior of $\Delta\epsilon_p$ in the stage II.

2. Experimental procedure and results

Since the experimental results used in this report are all those obtained in the previous study and their descriptions are found in the previous report along with the experimental procedures[5], here is given a brief summary of the experimental procedures and the results, which will be necessary in the later discussions. Cylindrical

Table 1 Chemical compositions of the materials

Materials	Chemical composition (%)						
	C	Si	Mn	P	S	sol Al	total N
Rimmed steel	0.10	0.001	0.42	0.027	0.028	<0.001	0.0025
Si-killed steel	0.10	0.20	0.43	0.024	0.016	0.001	0.0023
Al-killed steel	0.097	0.009	0.47	0.011	0.017	0.028	0.0042

Table 2 Heat treatment and mechanical properties

Materials	Heat treatment	Upper yield point σ_{Uy} kg/mm ²	Lower yield point σ_{Ly} kg/mm ²	Tensile Strength σ_b kg/mm ²	Elongation ϵ %	Reduction of area ψ %
Rimmed steel	920°Cx1h, A.C 720°Cx1h, W.Q	34.0	33.0	49.8	30.6	66.9
Si-killed steel	920°Cx1h, A.C	32.4	25.4	42.1	38.5	70.1
Al-killed steel	920°Cx1h, F.C 350°Cx3h, F.C	27.1	21.0	33.3	55.6	75.2

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drical specimens of a rimmed steel and Si- and Al-killed steels were tested under push-pull fatigue load with frequencies of 0.2 and 0.02 Hz, and the initial behavior of the hysteresis loops was recorded on an X-Y recorder, from which the value of the plastic strain amplitude $\Delta\epsilon_p$ was obtained. The stress amplitude σ_a was selected from the range of the usual long life fatigue. The chemical compositions and the mechanical properties after heat treatments are listed in Tables 1 and 2, and the geometry of specimen is shown in Fig.1.

The experimental results are shown in Figs.2, 3 and 4. It is seen that $\Delta\epsilon_p$ grows from a small initial value to a certain maximum and then decreases gradually. These trends of $\Delta\epsilon_p$ of annealed steels under cyclic load have been found by many investigators[6], [7].

However, through a careful observation of the experimental results, the variation of $\Delta\epsilon_p$ can be classified into three stages as are schematically shown in Fig.5. Stage

Fig.1 Specimen

Fig.2 Variation of $\Delta\epsilon_p$ (Rimmed steel) Fig.3 Variation of $\Delta\epsilon_p$ (Si-killed steel)

I is the initiation process of the hysteresis loop or $\Delta\epsilon_p$ based on the increase of the movable length of dislocations due to their successive unpinning at weakly pinned points by solute carbon atoms, and its analysis was the major subject of the previous report. In the stage II, however, the increase of $\Delta\epsilon_p$ is much faster than that in the stage I and it clearly deviates from a mere extension of the curve from the stage I which is indicated by the broken line. This sharp increase of $\Delta\epsilon_p$ in the stage II is certainly caused by a catastrophic multiplication of dislocation loops, and the analysis of this behavior is the subject of the present study. In the stage III, a gradual decrease of $\Delta\epsilon_p$ is observed and it is certain that a cyclic hardening takes place in this stage. The analysis of this process is the next subject and will be treated in the near future.

3. Discussion

3.1 Plastic strain produced by the activation of Frank-Read sources

As a preliminary work, discussion is made in this section on the plastic strain produced by the activation of the Frank-Read sources under given external stress.

If there are N active Frank-Read sources per unit volume of material, and an area S is swept by dislocation loops generated from one Frank-Read source, then the induced plastic strain ϵ_p is given by

$$\epsilon_p = NSb, \quad (1)$$

Fig.4 Variation of $\Delta\epsilon_p$ (Al-killed steel) Fig.5 Three stages of the variation of $\Delta\epsilon_p$

where b is the magnitude of Burger's vector. Accordingly, the computation of ϵ_p fully depends on the derivation of S as a function of applied stress, grain size and the other physical parameters.

Now, supposing a square grain model of length $2a$ as is depicted in Fig.6, for simplicity, and assuming that a unit Frank-Read source is located at the center O of the grain, then, by an external shear stress τ_a which is assumed to exceed a given frictional stress τ_f , a certain number of edge and screw dislocations are generated from the Frank-Read source and they form the dislocation arrays $D_e(x)$ and $D_s(y)$, respectively, as are also shown in Fig.6. Here, $D_e(x)$ and $D_s(y)$ are given by the well-known formulas [8],

$$\left. \begin{aligned} D_e(x) &= \frac{\tau_a - \tau_f}{\pi A_e} \frac{x}{\sqrt{a^2 - x^2}}, \\ D_s(y) &= \frac{\tau_a - \tau_f}{\pi A_s} \frac{y}{\sqrt{a^2 - y^2}}, \\ A_e &= \mu b / 2\pi(1 - \nu), \quad A_s = \mu b / 2\pi, \end{aligned} \right\} \quad (2)$$

where μ is the shear modulus and ν is Poisson's ratio. Consequently, the total numbers of edge and screw components of the dislocations, m_e and m_s , are given by integrating $D_e(x)$ and $D_s(y)$ from $-a$ to a with respect to x and y , respectively. And we obtain

$$\left. \begin{aligned} m_e &= \int_{-a}^a D_e(x) dx = \frac{(\tau_a - \tau_f)a}{\pi A_e}, \\ m_s &= \int_{-a}^a D_s(y) dy = \frac{(\tau_a - \tau_f)a}{\pi A_s}. \end{aligned} \right\} \quad (3)$$

Fig.6 Distribution of dislocations

Fig.7 Substitution of the model grain

Here the inequality of m_e and m_s is an obvious contradiction, since the same number of both components must be produced by unit Frank-Read source and this fact suggests the necessity of modification of Eq.(2), whenever the two dimensional dislocation arrays are dealt with. The modification is made here by replacing the above square grain model by a rectangular grain one with the length of $2\alpha \times 2\alpha'$, as shown in Fig.7, where α and α' are determined by the equation $\int_{-a}^a D_e dx = \int_{-a'}^{a'} D_s dy$ which means both components are equal. Moreover, by assuming the areas of the square and rectangular grains in Fig.7 are equal, the relation $a^2 = \alpha \alpha'$ is obtained. These considerations lead to the relationship $\alpha = \sqrt{A_e/A_s} a$, $\alpha' = \sqrt{A_s/A_e} a$, and the total number of edge dislocations contained in the modified grain is given by

$$m_e = \int_{-a}^a \frac{\tau_a - \tau_f}{\pi A_e} \frac{x}{\sqrt{a^2 - x^2}} dx = \frac{(\tau_a - \tau_f)a}{\pi \sqrt{A_e A_s}} \quad (4)$$

A comparison of Eq.(3) with (4) indicates that A_e is replaced by $\sqrt{A_e A_s}$. Similarly, for screw components, A_s is replaced by $\sqrt{A_e A_s}$ and it is evident that $m_e = m_s$. Thus, the equilibrium distribution of the two dimensional dislocation arrays is attained by putting

$$\left. \begin{aligned} D(x) &= \frac{\tau_a - \tau_f}{\pi A} \frac{x}{\sqrt{a^2 - x^2}}, \\ D(y) &= \frac{\tau_a - \tau_f}{\pi A} \frac{y}{\sqrt{a^2 - y^2}}, \\ A &= \sqrt{A_e A_s} = \frac{\mu b}{2\pi\sqrt{1-\nu}}. \end{aligned} \right\} \quad (5)$$

However, in the following discussions, the grain is again treated as a square of length $2a$ for convenience's sake.

Next step is to compute the area S swept by the dislocation loops whose distribution is given by Eq.(5). First of all, one dimensional case is dealt with, and it is assumed that edge dislocations of unit length are distributed along x -axis with the distribution $D(x)$ as shown in Figs.8 (a) and (b). Then, since the number of

Fig.8 One dimensional array of dislocations

dislocations located between x and $x+dx$ is $D(x)dx$ and the area swept by them is $xD(x)dx$, the total area S swept by all the dislocations between $-a < x < a$ is given by

$$S = 2 \int_0^a x D(x) dx. \quad (6)$$

Therefore, introducing a new function $\mathcal{D}(x)$ defined by

$$\mathcal{D}(x) = \int_x^a D(\xi) d\xi = \frac{\tau_a - \tau_f}{\pi A} \sqrt{a^2 - x^2}, \quad (7)$$

S is written as follows,

$$S = 2 \int_0^a \mathcal{D}(x) dx = \int_{-a}^a \mathcal{D}(x) dx, \quad (8)$$

where $\mathcal{D}(x)$ is an even function of x and is drawn in Fig.8(c).

The above derivation in one dimensional case is immediately extended to a two dimensional case. That is, by introducing a function $\mathcal{D}(x, y)$ defined by

$$\mathcal{D}(x, y) = \begin{cases} \frac{\tau_a - \tau_f}{\pi A} \sqrt{a^2 - x^2}, & (|x| \geq |y|) \\ \frac{\tau_a - \tau_f}{\pi A} \sqrt{a^2 - y^2}, & (|x| \leq |y|) \end{cases} \quad (9)$$

the area swept by all dislocation loops is given by

$$S = \int_{-a}^a \int_{-a}^a \mathcal{D}(x, y) dx dy. \quad (10)$$

As shown in Fig.9, the contour for $\mathcal{D} = \text{const.}$ is a square. Now, by substituting this

Fig.9 Two-dimensional distribution of $\mathcal{D}(x, y)$

Fig.10 Relation between $\Delta \epsilon_{pmax}$ and σ_a

equation into Eq.(1), the plastic strain ϵ_p is given by

$$\epsilon_p = Nb \int_{-a}^a \int_{-a}^a \mathcal{D}(x, y) dx dy = \frac{8Na^3b}{3\pi A} (\tau_a - \tau_f). \quad (11)$$

This is the functional relation between the applied stress τ_a and the plastic strain ϵ_p due to the dislocations generated from N Frank-Read sources.

In the above derivation, the grain is assumed to be a square of length $2a$. But this assumption is not essential, since a similar derivation shows that, in the case of a circular grain of radius a , ϵ_p is given by an equation similar to Eq.(11) with the coefficient $2/3$ instead of $8/3\pi$ of Eq.(11).

According to Eq.(11), the plastic strain should be a linear function of the applied stress, and this is confirmed by the experimental data as indicated in Fig. 10, where $\Delta \epsilon_{pmax}$, the maximum plastic strain amplitude corresponding to the point P in Fig.5, is plotted against applied stress σ_a . Moreover the appropriateness of Eq.(11) is also proved through the following order evaluation of ϵ_p ; that is, in the case of the rimmed steel, for example, by assuming $N=10^6/\text{mm}^3$, $\sigma_a=2\tau_a=25\text{kg/mm}^2$, $\sigma_f=2\tau_f=16.9\text{kg/mm}^2$, $\mu=8200\text{kg/mm}^2$, $\nu=0.3$ and $b=2.48 \times 10^{-7}\text{mm}$, the value of $\epsilon_p=2200$ micro-strains is obtained, which is the same order as the experimental value. Incidentally, under these assumptions, about 30 dislocation loops are to be generated from a single Frank-Read source.

Finally it should be noted that τ_f in Eq.(11) is a parameter representing the frictional resistance to the motion of dislocation, but its physical meaning is not clear at present.

3.2 Theoretical derivation of the plastic strain amplitude during the fatigue process

3.2.1 The fundamental models

Among the behaviors of the plastic strain amplitude $\Delta \epsilon_p$ in Fig.5, the analysis of the stage II is the main purpose of the present study as previously mentioned. But, since most of the parameters used in the analysis of the stage I are necessary in order to discuss the stage II, the theory already derived on the stage I[5] is briefly illustrated first, upon which the theoretical model on the stage II is constructed.

(a) Summary of the theory on the stage I

Fig.11 illustrates the theoretical model used in the previous report to explain the gradual increase of $\Delta \epsilon_p$ in the stage I. It is appropriately assumed that, in the matrix of annealed steels, solute carbon atoms are clustered around dislocation lines and form Cottrell's atmosphere. A simple calculation shows[5] that there are more than 10^3 carbon atoms per atomic plane along a dislocation, and they are assumed to gather just below the dislocation line from the stress field of the dislocation. Another assumption is that the concentration of

these carbon atoms, c , along a dislocation line is not uniform, but random. This random distribution of c is reduced to the stepwise distribution A_0PBS along a straight line dislocation located on x -axis as shown in Fig.11(a). Then, by assuming that c_0 is the value of c corresponding to the critical pinning force to counterbalance the applied peak stress σ_a , the part of dislocation with the length l_0 bows out by the first application of σ_a (Fig.11(b)).

Now the underlying idea is that, since the concentration c becomes almost zero along the bowed out part, which is shown by A'ABS, a flow of carbon atoms rapidly occurs from the adjacent to the bowed out part of the dislocation owing to the pipe diffusion, whose diffusion coefficient is about 5000 times that of the volume diffusion through the regular matrix. The distribution c after a half cycle then becomes such as given by A'Q'RS in Fig.11(a). Therefore, the initial point Q giving the concentration c_0 moves to the point Q', and the movable length of the dislocation l_0 increases by Δl when the reversed peak $-\sigma_a$ is applied, where dislocation is assumed, for simplicity, to move rapidly only at the time of application of the peak stress. By further assuming that the distribution A'A"QRS along the dislocation with bowed out length $l_0 + \Delta l$ can be approximated by A'A"B"S, the same length Δl is added after next half cycle in exactly the same way, and, as a result, the movable length l increases linearly with the number of cycles n . Since this linearity between l and n successfully coincided with the result of the elaborate calculation based on the diffusion model assuming a random distribution of the concentration c , the adoption of a reduced simple model shown in Fig.11 was justified, the detail of which was mentioned in the previous report[5].

The derivation of the length of the movable dislocation, l , is now calculated by solving Fick's equation of diffusion, and is given by the following expression, as a first approximation,

$$l = 4n\sqrt{\pi Dt_1} \left(\frac{2c_0}{c_1} - 1 \right) + l_0, \quad (12)$$

(a) Distribution of the concentrations of carbon atoms along a dislocation line

(b) Bowed-out part of a dislocation line

Fig.11 Diffusion model for the behavior of $\Delta \epsilon_p$ in the stage I

where t_1 is a half period of the stress cycles, c_1 is the initial value of c and D is diffusion coefficient of carbon atoms flowing through a dislocation line. Here, by putting $2c_0/c_1 = \sigma_a/\sigma_0$ (σ_0 is a new parameter) and $t_1 = 1/2f$, the following expression is obtained.

$$l = \Xi n(\sigma_a - \sigma_0) + l_0, \quad \Xi = \frac{4}{\sigma_0} \sqrt{\frac{\pi D}{2f}}. \quad (13)$$

On the other hand, it is well-known that $\Delta \epsilon_p$ is given by[9]

$$\Delta \epsilon_p = \frac{\sigma_a l^3}{6\mu l_1}, \quad (14)$$

where μ is the shear modulus and l_1 is the average length of dislocation between the strong pinning points.

By combining Eqs.(13) and (14), the following expression is obtained for $\Delta \epsilon_p$ in the stage I,

$$\Delta \epsilon_p = K \sigma_a \{ \Xi n(\sigma_a - \sigma_0) + l_0 \}^3, \quad K = \frac{1}{6\mu l_1}. \quad (15)$$

The solid line in the stage I and its broken line extension in Fig.5 indicates the schematical trend given by Eq.(15).

(b) Multiplying model of dislocation for stage II

A multiplying model of dislocations for the stage II is illustratively shown in Fig. 12. Each square shows a grain containing originally one dislocation line of length l_1 , whose ends are assumed to be strong pinning points. At a number of cycles n_1 , a fraction of the dislocation line starts motion under cyclic load by the mechanism reviewed in the preceding article. This length l increases linearly with the number of cycles according to Eq.(13), and, at n_2 cycles, l reaches a certain length as shown in Fig.12 (a). These states at n_1 and n_2 are those

(a) Growth of movable length of a dislocation line

(b) Variation of $\Delta \epsilon_p$

Fig.12 Illustration for the mechanism in the stage II

of the stage I and $\Delta\epsilon_p$ increases along the curve A in Fig.12(b).

Now, it is a matter of course that the process of the stage I ceases at a certain number of cycles n_0 , when l reaches l_1 . Thus, it is understood that there is a limitation in the process of the stage I, and a further increase of $\Delta\epsilon_p$ beyond some 200 micro-strain level must be analyzed by introducing the multiplying mechanism of dislocations instead of the above string model.

Presence of the above limitation is also proved by a simple calculation, as follows. By assuming the dislocation density $\rho=10^6/\text{cm}^2$, Burger's vector length $b=2.48 \times 10^{-7}$ mm, and $\epsilon_p=200$ micro-strains, the average moving distance of the dislocation string, r , becomes about 100 μm using the formula $\epsilon_p=\rho br$. This means that dislocations move further beyond the grain boundaries, and this is unrealistic.

Now, going back to Fig.12, after the movable length of the dislocation l reaches the fixed length l_1 at the number of cycles n_0 , a multiplication of the dislocation loops occurs under applied stress in exactly the same way as the multiplication from the Frank-Read source. Therefore, if the mass effect and viscous resistance to the motion of the dislocation are ignored, $\Delta\epsilon_p$ increases catastrophically until it reaches the value given by Eq.(11), that is,

$$\Delta\epsilon_p = \Lambda(\sigma_a - \sigma_f), \quad \Lambda = \frac{4a^3b}{3\pi A}, \quad (16)$$

where σ_a and σ_f are used instead of τ_a and τ_f from the relation $\sigma=2\tau$. The vertical line at $n=n_0$ in Fig.12(b) represents an instantaneous increase of $\Delta\epsilon_p$ in the operation of multiplying mechanism of dislocation loops, and the horizontal line B indicates the strain level given by Eq.(16). The case of $n=n_3$ in Fig.12(a) illustrates the configuration of dislocation loops after n_0 .

From the above model concerning the characteristic behavior of $\Delta\epsilon_p$ indicated in Fig.12(b), the overall behavior of $\Delta\epsilon_p$ against n can be expressed by

$$\Delta\epsilon_p = NK\sigma_a \{ \Xi n(\sigma_a - \sigma_0) + l_0 \}^3 \{ 1 - u(n - n_0) \} + N\Lambda(\sigma_a - \sigma_f) u(n - n_0), \quad (17)$$

where $u(n)$ is a step function of n . Here, by assuming, for simplicity, that the length l_0

Fig.13 Graphic presentation of three statistical distributions

which is the part of dislocation weakly pinned at the beginning (Fig.11) is located at the center of the string, n_0 is given by

$$n_0 = \frac{l_1 - l_0}{\Xi(\sigma_a - \sigma_0)} \quad (18)$$

Eq.(17) supplemented by Eq.(18) provides a fundamental formula explaining the behavior of $\Delta\epsilon_p$ including the stages I and II in the case of only one multiplication source present in a grain. The remaining problem is, therefore, to extend the above discussion to the case where a grain has many multiplication sources with different l_1 and l_0 , as it is naturally supposed for an actual grain.

For this aim, it is reasonable to assume that l_1 and l_0 are random variables having certain distribution functions, and to sum up all the contributions from the sources of different l_1 and l_0 to obtain the actual $\Delta\epsilon_p$. However, it is assumed here, for simplicity, that all dislocation sources have the same length l_1 , the distance between strong pinning points, and only l_0 is a random variable having the distribution function $f(l_0)$. Then the following expression can be written for $\Delta\epsilon_p$,

$$\Delta\epsilon_p = N \int_0^\infty \Delta\epsilon_p(l_0) f(l_0) dl_0, \quad (19)$$

where $\Delta\epsilon_p(l_0)$ is given by Eq.(17) for a certain value of l_0 .

Putting Eq.(17) into Eq.(19), the plastic strain amplitude $\Delta\epsilon_p$ which includes the contributions from all sources is given by

$$\Delta\epsilon_p = NK\sigma_a \int_0^{l_0^*} \{ \Xi n(\sigma_a - \sigma_0) + l_0 \}^3 f(l_0) dl_0 + N\Lambda(\sigma_a - \sigma_f) \int_{l_0^*}^\infty f(l_0) dl_0, \quad (20)$$

Table 3 Parameters common to three steels

Shear modulus	μ	8200	kg/mm ²
Poisson's ratio	ν	0.3	
Burger's vector length	b	2.48×10^{-7}	mm
Grain size	a	10	μm
Diffusion coefficient	D	3.57×10^{-12}	mm ² /sec

Table 4 Parameters depending on the materials

	O_0 kg/mm ²	O_f kg/mm ²	l_1 mm	N 1/mm ³
Rimmed steel	19.5	16.9 (0.2Hz) 19.0 (0.02)	6.84×10^{-3}	3.75×10^5
Si-killed steel	21.1	18.9 (0.2) 16.6 (0.02)	3.93	7.51
Al-killed steel	14.1	12.9 (0.2) 12.9 (0.02)	3.96	6.54

Table 5 Parameters of the distributions

	Parameters	Mean $\bar{l}_0$ μm	(Variance) ^{1/2} s μm
Exponential distribution	$\beta = 0.65 \times 10^3$	1.54	1.54
Gamma distribution	$\lambda = 1.3 \times 10^3$	1.54	0.901
Normal distribution	—	1.54	0.901

where

$$l_0^* = l_1 - \Xi n(\sigma_a - \sigma_0).$$

This equation can be simplified by using an approximation $l_0 \approx \bar{l}_0$ for l_0 in the bracket in the integrand, where $\bar{l}_0$ is the mean of l_0 . Thus, $\Delta \epsilon_p$ is written as

$$\Delta \epsilon_p = \frac{N\sigma_a}{6\mu} \{ \Xi(\sigma_a - \sigma_0) + \bar{l}_0 \}^3 \int_0^{l_0^*} f(l_0) dl_0 + \frac{4Na^3b(\sigma_a - \sigma_f)}{3\pi A} \int_{l_0^*}^{\infty} f(l_0) dl_0. \quad (21)$$

Eq.(21) gives the theoretical expression for the behavior of $\Delta \epsilon_p$ including the stage II. But for further discussion, and, especially for the calculation of $\Delta \epsilon_p$, it is necessary to know the expression of the function $f(l_0)$, which is unknown at present except an exponential distribution concerning the pinning points along a dislocation line proposed by Granato and Lücke[10]. Therefore, in the present study, the following three distributions are assumed,

exponential distribution:

$$f(l_0) = \beta e^{-\beta l_0}$$

gamma distribution:

$$f(l_0) = \lambda^2 l_0 e^{-\lambda l_0}$$

normal distribution:

$$\left. \begin{aligned} f(l_0) &= \frac{1}{\sqrt{2\pi}s} \exp \left\{ -\frac{(l_0 - \bar{l}_0)^2}{2s^2} \right\} \quad (l_0 > 0), \\ f(l_0) &= \int_{-\infty}^0 \frac{1}{\sqrt{2\pi}s} \exp \left\{ -\frac{(l_0 - \bar{l}_0)^2}{2s^2} \right\} dl_0 \quad (l_0 = 0), \end{aligned} \right\} \quad (22)$$

Among these, the exponential and gamma distributions belong to skew-symmetric distributions, and the normal distribution is a symmetric one, though it is modified in the part of the negative l_0 , since the region of the negative l_0 is meaningless. It is further assumed that the three distributions have the same mean, and that the variance of the normal distribution is equal to that of gamma distribution. In Fig.13, the three distributions thus defined are presented.

Numerical calculations based on the above derivations and the comparisons with the experimental results are carried out in the next article by using the suitable values of parameters.

3.2.2. Numerical calculations and the comparisons with the experimental results

The numerical values assigned for the parameters involved in the equations derived in the preceding article are listed in Tables 3, 4 and 5. The parameters in Table 3 are those common to three steels, but those in Table 4 are different depending on the materials. The distribution parameters relating to Eq.(22) are all listed in Table 5. It should be noted that there are few parameters which are exactly known or can be theoretically derived, and therefore, the most values in these tables are computed from the experimental results, or determined to fit the experimental trends. In this respect, some explanations will be given for better understanding.

First of all, the grain size a in Table 3 is put as 10 μm , for the grains of the three steels are controlled approximately to this size by the heat treatments. D is the diffusion coefficient of pipe diffusion of carbon atoms along dislocation lines and

Fig.14 Comparison of theoretical trends with experimental data (Rimmed steel)

Fig.15 Comparison of theoretical trends with experimental data (Si-killed steel)

Fig.16 Comparison of theoretical trends with experimental data (Al-killed steel)

the value in Table 3 is adopted, since it is about 5000 times that of the volume diffusion.

The parameters in Table 4 are those independent of the stress amplitude and the testing frequency, but depend on the materials. σ_0 is a parameter related to the concentration c_0 of carbon atoms and is determined so as not to conflict with the experimental results. σ_f is the frictional stress as discussed in the section 3.1, which is determined to the stress level at which each straight line in Fig.10 cuts across the abscissa. L_1 is also determined so as to fit the experimental data, but the value itself is of the same order suggested by Friedel [11] as the mean distance between nodal points of dislocations. N is the number of Frank-Read sources per unit volume, and, if the dislocation density $\rho=10^6/\text{cm}^2$ and $L_1=5\mu\text{m}$ are assumed, N becomes $2 \times 10^6/\text{mm}^2$ from the formula $N=\rho/L_1$. Further by assuming ten percent of them act as active Frank-Read sources, the obtained value turns to be of the same order as the value of N in Table 4. Finally, the distribution parameters in Table 5 are chosen rather arbitrarily to satisfy the previously mentioned restrictions on the means and the variances and to be compatible with the other parameters such as L_1 .

The curves in Figs.14, 15 and 16 indicate the numerically calculated trends of $\Delta\epsilon_p$ for the three steels using Eqs.(21) and (22) with the above parameters. Only in the case of the rimmed steel (Fig.14), an additional result of the calculations is also drawn by the fine line which is obtained by the strict expression of Eq.(20) combined with the gamma distribution. It is observed that there is little difference between the curves of fine and solid lines both for the gamma distribution, so that the approximation made in the derivation of Eq.(21) is justified. Moreover, an observation of these three figures shows that the trends calculated from the assumed three distributions have no significant difference from each other, although the calculations based on the gamma distribution show slightly better agreement with the experimental results.

Consequently, it is concluded that the theory derived above is a satisfactory one explaining the experimental trends of $\Delta\epsilon_p$ particularly in the stage II, and that the choice of the distribution function $f(L_0)$ has no serious effect on the results, if the mean and the variance of the distribution are not altered.

4. Conclusions

The characteristic behavior of the plastic strain amplitude $\Delta\epsilon_p$ in the rapidly increasing stage observed during the fatigue test of low carbon steels was theoretically analyzed using a multiplying model of dislocations, and the following conclusions were obtained.

(1) The plastic strain caused by dislocation loops generated from the Frank-Read source is given by Eq.(11), and has a linear relation with the applied stress. This linearity is proved by the experimental results.

(2) It is confirmed that the rapidly increasing stage of $\Delta\epsilon_p$ is the multiplying process of dislocation loops from Frank-Read sources, which takes place after unpinned lengths of the dislocation lines reach a critical value.

(3) This rapid increase of $\Delta\epsilon_p$ following the gradual growth is theoretically expressed by Eqs.(20) and (21) combined with a suitable distribution function, $f(L_0)$, of the weakly pinned initial length L_0 of the dislocation line.

(4) The calculated trend of $\Delta\epsilon_p$ is little affected by the type of the distribution functions, if they have the same mean and variance.

Finally, there are still various problems arising in the course of analysis such as the physical meaning of $f(L_0)$, and these problems will be studied in the future work.

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