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Free Vibration of Clamped Polygonal Plates*

By

Toshihiro IRIE**, Gen YAMADA*** and Yoshihiro NARITA****

This paper presents an analytical method to study free vibration of polygonal plates clamped at the edges. A polygonal plate is formed by setting some rigid line supports on a simply supported rectangular plate. Regarding the reaction force and moment acting on all edges of the polygonal plate as unknown harmonic exciting loads, the stationary response of the plate to these loads is obtained. The force and moment distributed along the edges are expanded into a Fourier sine series with unknown coefficients, and homogeneous linear equations are derived with use of restraint conditions at the edges. The natural frequencies and the mode shapes of the polygonal plate are determined by calculating the eigenvalues and eigenvectors of the equations.

From the numerical calculation carried out on an equilateral triangular plate through a regular octagonal plate, it was clarified that the mode shapes of clamped polygonal plates can be classified into several groups of various types.

1. Introduction

The present paper studies free vibration of clamped polygonal plates with small uniform thickness. This problem has been treated by some other authors. Waller⁽¹⁾ obtained Chladni's figures of free polygonal plates experimentally. With use of point-matching method, Conway⁽²⁾ studied the fundamental vibration of simply supported polygonal plates, and Walkinshaw et al.⁽³⁾ obtained the frequencies of axisymmetrical vibration of simply supported plates and clamped plates. Laura et al.⁽⁴⁾ and Yu⁽⁵⁾ analyzed the same problem by complex variable method. By the Ritz method, Young⁽⁶⁾ studied free vibration of a clamped square plate and Ota et al.⁽⁷⁾ did that of a triangular plate. However, only a limited number of papers are available on free vibration of clamped polygonal plates up to higher modes.

This paper presents a theoretical method to analyze free vibration of clamped polygonal plates. For this purpose, we consider a rectangular plate which is simply supported at the edges and clamped on an internal segment. Regarding the reaction force and bending moment acting on the segment as unknown harmonic force and moment, the stationary response to these loads is obtained. The unknown force and moment distributed along the segment are

expanded into a Fourier sine series with unknown coefficients, and homogeneous linear equations concerning the coefficients are written in a matrix form with use of restraint conditions on the segments. By obtaining the eigenvalues and eigenvectors of the matrix equation, the natural frequencies and the mode shapes of the plate are determined.

In the application of the method, several clamping segments of equal lengths are located to form a regular polygon on a simply supported plate (extended plate). This theory is applied to an equilateral triangular plate through a regular octagonal plate in this paper, but it is applicable to the study of variously shaped plates by appropriately locating the segments

2. Free vibration of rectangular plate simply supported at the edges and clamped on some segments

Figure 1 shows a rectangular plate which is simply supported at the edges and clamped along some segments C_p ($p=1,2,\dots,P$) located inside the plate or at the edges. Denoting the length of two edges

Fig.1 Rectangular Plate simply supported at the edges and clamped on an internal segment

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** Professor, Faculty of Engineering, Hokkaido University, Sapporo.

*** Associate Professor, Faculty of Engineering, Hokkaido University.

**** Graduate Student, Faculty of Engineering, Hokkaido University.

a, b , the xy plane of a rectangular coordinate $Oxyz$ is taken in the neutral surface of the plate. Regarding the unknown reaction force and moment acting along the segment as unknown harmonic force and moment $\{Q(x, y), M_r(x, y)\} \sin \omega t$ acting on the plate, the deflection of the plate is written as

$$W(x, y) = \int_0^a \int_0^b \left\{ Q(u, v) G(x, y; u, v) + M_r(u, v) \frac{\partial}{\partial r} G(x, y; u, v) \right\} du dv \quad (1)$$

with use of Green's function of the plate

$$G(x, y; u, v) = \frac{1}{\rho h} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\omega_{mn}^2 - \omega^2} W_{mn}(x, y) W_{mn}(u, v) \quad (2)$$

where ρ is the mass per unit volume, h is the plate thickness and r is the length in the direction normal to the segment. ω_{mn} and $W_{mn}(x, y)$ are the natural frequency and the normalized eigenfunction of a simply supported rectangular plate, respectively, which are given by

$$\omega_{mn}^2 = \frac{D}{\rho h} \left\{ \left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right\}, \quad W_{mn}(x, y) = \frac{2}{\sqrt{ab}} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (3)$$

where D is the flexural rigidity of the plate expressed by $D = Eh^3/12(1-\nu^2)$ (E : Young's modulus, ν : Poisson's ratio).

When external force $Q^{(p)}(s) \sin \omega t$ and bending moment $M_r^{(p)}(s) \sin \omega t$ act on the segment

$$C_p: u_p(s) = a_p + s \cos \theta_p, \quad v_p(s) = b_p + s \sin \theta_p \quad (p=1, 2, \dots, P) \quad (4)$$

the deflection of the plate is written as

$$W(x, y) = \sum_{p=1}^P \int_0^{l_p} \left\{ Q^{(p)}(s) G(x, y; u_p(s), v_p(s)) + M_r^{(p)}(s) \left(\sin \theta_p \frac{\partial}{\partial u_p} - \cos \theta_p \frac{\partial}{\partial v_p} \right) G(x, y; u_p(s), v_p(s)) \right\} ds \quad (5)$$

where l_p is the length of the segment C_p and s denotes the length measured along it and θ_p does the angle between the segment and x axis. Expanding the force and moment distributed along the segment into a Fourier sine series with unknown coefficients

$$Q^{(p)}(s) = \sum_{i=1}^{\infty} Q_{i,i}^{(p)} \sin \frac{i\pi s}{l_p}, \quad M_r^{(p)}(s) = \sum_{i=1}^{\infty} M_{r,i}^{(p)} \sin \frac{i\pi s}{l_p} \quad (6)$$

Eq.(5) is written as

$$W(x, y) = \frac{1}{\rho h} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\omega_{mn}^2 - \omega^2} W_{mn}(x, y) \sum_{i=1}^{\infty} \sum_{j=1}^P \left\{ Q_{i,i}^{(p)} I_{mn,i}^{(p)} + M_{r,i}^{(p)} J_{mn,i}^{(p)} \right\} \quad (7)$$

with use of Eqs.(2) and (6). $I_{mn,i}^{(p)}$ and $J_{mn,i}^{(p)}$ denote definite integrals

$$\left. \begin{aligned} I_{mn,i}^{(p)} &= \int_0^{l_p} W_{mn}(u_p(s), v_p(s)) \sin \frac{i\pi s}{l_p} ds \\ J_{mn,i}^{(p)} &= \int_0^{l_p} \left(\sin \theta_p \frac{\partial}{\partial u_p} - \cos \theta_p \frac{\partial}{\partial v_p} \right) W_{mn}(u_p(s), v_p(s)) \sin \frac{i\pi s}{l_p} ds \end{aligned} \right\} \quad (8)$$

When the plate is clamped along the segments, the following restraint conditions must be satisfied.

$$W(u_p(s), v_p(s)) = 0, \quad \left(\sin \theta_p \frac{\partial}{\partial u_p} - \cos \theta_p \frac{\partial}{\partial v_p} \right) W(u_p(s), v_p(s)) = 0 \quad (9)$$

$W_{mn}(u_p(s), v_p(s))$ and $(\partial/\partial r)W_{mn}(u_p(s), v_p(s))$ are also expanded into a Fourier sine series

$$\left. \begin{aligned} W_{mn}(u_p(s), v_p(s)) &= \frac{2}{l_p} \sum_{j=1}^{\infty} I_{mn,j}^{(p)} \sin \frac{j\pi s}{l_p} \\ \left(\sin \theta_p \frac{\partial}{\partial u_p} - \cos \theta_p \frac{\partial}{\partial v_p} \right) W_{mn}(u_p(s), v_p(s)) &= \frac{2}{l_p} \sum_{j=1}^{\infty} J_{mn,j}^{(p)} \sin \frac{j\pi s}{l_p} \end{aligned} \right\} \quad (10)$$

By substituting Eqs.(10) into the equations obtained by the substitution of Eq.(7) into Eqs.(9), and equating all coefficients of $\sin(j\pi s/l_p)$ to zero, the following equation is derived.

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\omega_{mn}^2 - \omega^2} \left\{ \frac{I_{mn,i}^{(p)}}{J_{mn,i}^{(p)}} I_{mn,j}^{(p)} J_{mn,j}^{(p)} - \frac{Q_j^{(p)}}{M_{r,j}^{(p)}} \right\} = 0 \quad (p, q=1, 2, \dots, P; i, j=1, 2, \dots) \quad (11)$$

By introducing the dimensionless expressions

$$f_{mn}(\lambda) = (m^2 + \alpha^2 n^2)^2 - \lambda^4, \quad \lambda^2 = \rho h a^4 \omega^2 / \pi^4 D \quad (\alpha = a/b) \quad (12)$$

and

$$\left. \begin{aligned} g_{mn,i}^{(p)} &= \int_0^1 \sin m\pi \xi_p(z) \sin n\pi \eta_p(z) \sin i\pi z dz \\ h_{mn,i}^{(p)} &= m \sin \theta_p \int_0^1 \cos m\pi \xi_p(z) \sin n\pi \eta_p(z) \sin i\pi z dz - \alpha n \cos \theta_p \int_0^1 \sin m\pi \xi_p(z) \cos n\pi \eta_p(z) \sin i\pi z dz \end{aligned} \right\} \dots\dots\dots (13)$$

$$\xi_p(z) = \frac{1}{a}(a_p + l_p z \cos \theta_p), \quad \eta_p(z) = \frac{1}{b}(b_p + l_p z \sin \theta_p) \quad (z = s/l_p) \dots\dots\dots (14)$$

equation (11) is rewritten in the convenient form

$$\left[\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{f_{mn}(\lambda)} \begin{Bmatrix} g_{mn,i}^{(p)} \\ h_{mn,i}^{(p)} \end{Bmatrix} \begin{Bmatrix} g_{mn,j}^{(q)} \\ h_{mn,j}^{(q)} \end{Bmatrix} \right] \begin{Bmatrix} (a/\pi) Q_j^{(p)} \\ M_{r,j}^{(p)} \end{Bmatrix} = 0 \quad (p, q = 1, 2, \dots, P; i, j = 1, 2, \dots) \dots\dots\dots (15)$$

where $\begin{Bmatrix} \end{Bmatrix}$ denotes a column vector, and the product of the column vector and the transposed vector in the bracket $\begin{bmatrix} \end{bmatrix}$ gives a square coefficient matrix. The natural frequencies ω of the plate are obtained by calculating the eigenvalues λ of Eq.(15), and the mode shapes are determined by

$$W(x, y) \propto \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{f_{mn}(\lambda)} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \begin{Bmatrix} g_{mn,i}^{(p)} \\ h_{mn,i}^{(p)} \end{Bmatrix} \begin{Bmatrix} (a/\pi) Q_j^{(p)} \\ M_{r,j}^{(p)} \end{Bmatrix} \dots\dots\dots (16)$$

from the calculation of the eigenvectors $\begin{Bmatrix} (a/\pi) Q_j^{(p)} \\ M_{r,j}^{(p)} \end{Bmatrix}^T$. In the numerical calculation, the number of terms (m, n) of infinite series and terms (i, j) of Fourier series may be truncated at appropriate finite number, considering the convergence and accuracy of the solution.

3. Free vibration of clamped polygonal plates

Regular polygonal plates can be formed on a simply supported rectangular plate by appropriately setting several clamping segments inside the plate or at the edges. For example, three segments C_1, C_2 and C_3 of equal lengths make an equilateral triangle on the extended plate of $\alpha = 2/\sqrt{3}$ as shown in the left figure of Table 1. A clamped square plate is also formed by setting four segments at the four edges of a simply supported square plate. According to the same procedure, other inscribed polygonal plates are formed on the extended plate and lateral deflection and rotation are clamped at all sides of the polygonal

plates. One of the segments is at least conveniently located on the x axis. By an appropriate location of some segments and application of this theory, free vibration of various polygonal plates can be calculated analytically.

As there are some geometrically symmetrical axes on regular polygonal plates, symmetrical and antisymmetrical vibrations arise about these axes. For the calculation of each type of vibration, it is sufficient to consider only one side of the plate divided by the axes, which reduces the size of matrix equation (15). The frequency equations derived by this proce-

Table 1 Equilateral triangular plate

S type	A type
$\left[\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{f_{mn}(\lambda)} \begin{Bmatrix} h_{mn,i}^{(1)} \\ g_{mn,i}^{(2)} \\ h_{mn,i}^{(2)} \end{Bmatrix} \begin{Bmatrix} h_{mn,j}^{(1)} \\ g_{mn,j}^{(2)} \\ h_{mn,j}^{(2)} \end{Bmatrix} \right] \begin{Bmatrix} M_{r,j}^{(1)} \\ \frac{a}{\pi} Q_j^{(2)} \\ M_{r,j}^{(2)} \end{Bmatrix} = 0 \dots\dots\dots (17)$	
$\xi_1 = z/2, \quad \eta_1 = 0 \quad (\theta_1 = 0); \quad \xi_2 = z/2, \quad \eta_2 = z/2 \quad (\theta_2 = \pi/3) \quad (\alpha = 2/\sqrt{3})$	
$m = 1, 3, \dots$	$m = 2, 4, \dots$
$n = 1, 2, 3, \dots$	

Table 2 Square plate

SSSS type	SSAA type	AASS type	AAAA type	SA type
$\left[\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{f_{mn}(\lambda)} \begin{Bmatrix} h_{mn,i}^{(1)} \\ h_{mn,i}^{(2)} \end{Bmatrix} \begin{Bmatrix} h_{mn,j}^{(1)} \pm h_{mn,j}^{(2)} \\ h_{mn,j}^{(1)} \pm h_{mn,j}^{(2)} \end{Bmatrix} \right] \begin{Bmatrix} M_{r,j}^{(1)} \\ M_{r,j}^{(2)} \end{Bmatrix} = 0 \dots\dots\dots (18)$				$\left[\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{f_{mn}(\lambda)} \begin{Bmatrix} h_{mn,i}^{(1)} \\ h_{mn,i}^{(2)} \end{Bmatrix} \begin{Bmatrix} h_{mn,j}^{(1)} \\ h_{mn,j}^{(2)} \end{Bmatrix} \right] \begin{Bmatrix} M_{r,j}^{(1)} \\ M_{r,j}^{(2)} \end{Bmatrix} = 0 \dots\dots\dots (19)$
$\xi_1 = z/2, \quad \eta_1 = 0 \quad (\theta_1 = 0); \quad \xi_2 = 0, \quad \eta_2 = z/2 \quad (\theta_2 = \pi/2) \quad (\alpha = 1)$				
sign	+	-	+	-
$m, n = 1, 3, \dots$	$m, n = 2, 4, \dots$	$m, n = 2, 4, \dots$	$m, n = 1, 3, \dots$	$m = 1, 3, \dots; n = 2, 4, \dots$

Table 3 Regular pentagonal plate

S type	A type

$\left[\sum_m \sum_n \frac{1}{f_{mn}(\lambda)} \begin{Bmatrix} h_{mn,i}^{(1)} \\ g_{mn,i}^{(2)} \\ h_{mn,i}^{(3)} \\ g_{mn,i}^{(4)} \\ h_{mn,i}^{(5)} \end{Bmatrix} \left\{ h_{mn,j}^{(1)} g_{mn,j}^{(2)} h_{mn,j}^{(3)} g_{mn,j}^{(4)} h_{mn,j}^{(5)} \right\} \right] \begin{Bmatrix} M_{y,j}^{(1)} \\ (a/\pi) Q_j^{(2)} \\ M_{x,j}^{(3)} \\ (a/\pi) Q_j^{(4)} \\ M_{r,j}^{(5)} \end{Bmatrix} = 0 \dots\dots\dots (20)$	
$\xi_1 = q_1 + (1/2 - q_1)z, \quad \eta_1 = 0 \quad (\theta_1 = 0); \quad \xi_2 = q_1(1-z), \quad \eta_2 = q_2z \quad (\theta_2 = 3\pi/5); \quad \xi_3 = z/2, \quad \eta_3 = q_2 + (1 - q_2)z \quad (\theta_3 = \pi/5)$	
$\left[\alpha = \left(1 + 2 \cos \frac{2}{5} \pi \right) / 2 \cos \frac{1}{5} \pi \cos \frac{1}{10} \pi \right], \quad q_1 = \cos \frac{2}{5} \pi / \left(1 + 2 \cos \frac{2}{5} \pi \right), \quad q_2 = 1/2 \cos \frac{1}{5} \pi$	
$m = 1, 3, \dots$	$m = 2, 4, \dots$
$n = 1, 2, 3, \dots$	

Table 4 Regular hexagonal plate

SS type	SA type	AS type	AA type

$\left[\sum_m \sum_n \frac{1}{f_{mn}(\lambda)} \begin{Bmatrix} h_{mn,i}^{(1)} \\ g_{mn,i}^{(2)} \\ h_{mn,i}^{(3)} \\ g_{mn,i}^{(4)} \\ h_{mn,i}^{(5)} \\ g_{mn,i}^{(6)} \end{Bmatrix} \left\{ h_{mn,j}^{(1)} g_{mn,j}^{(2)} h_{mn,j}^{(3)} g_{mn,j}^{(4)} h_{mn,j}^{(5)} g_{mn,j}^{(6)} \right\} \right] \begin{Bmatrix} M_{y,j}^{(1)} \\ (a/\pi) Q_j^{(2)} \\ M_{x,j}^{(3)} \\ (a/\pi) Q_j^{(4)} \\ M_{r,j}^{(5)} \\ (a/\pi) Q_j^{(6)} \end{Bmatrix} = 0 \dots\dots\dots (21)$			
$\xi_1 = (1+z)/4, \quad \eta_2 = 0 \quad (\theta_1 = 0); \quad \xi_2 = (1-z)/4, \quad \eta_2 = z/2 \quad (\theta_2 = 2\pi/3) \quad (\alpha = 2/\sqrt{3})$			
$m, n = 1, 3, \dots$	$m = 1, 3, \dots$ $n = 2, 4, \dots$	$m = 2, 4, \dots$ $n = 1, 3, \dots$	$m, n = 2, 4, \dots$

Table 5 Regular septagonal plate

S type	A type

$\left[\sum_m \sum_n \frac{1}{f_{mn}(\lambda)} \begin{Bmatrix} h_{mn,i}^{(1)} \\ g_{mn,i}^{(2)} \\ h_{mn,i}^{(3)} \\ g_{mn,i}^{(4)} \\ h_{mn,i}^{(5)} \\ g_{mn,i}^{(6)} \\ h_{mn,i}^{(7)} \end{Bmatrix} \left\{ h_{mn,j}^{(1)} g_{mn,j}^{(2)} h_{mn,j}^{(3)} g_{mn,j}^{(4)} h_{mn,j}^{(5)} g_{mn,j}^{(6)} h_{mn,j}^{(7)} \right\} \right] \begin{Bmatrix} M_{y,j}^{(1)} \\ (a/\pi) Q_j^{(2)} \\ M_{x,j}^{(3)} \\ (a/\pi) Q_j^{(4)} \\ M_{r,j}^{(5)} \\ (a/\pi) Q_j^{(6)} \\ M_{r,j}^{(7)} \end{Bmatrix} = 0 \dots\dots\dots (22)$	
$\xi_1 = q_1 + (1/2 - q_1)z, \quad \eta_1 = 0 \quad (\theta_1 = 0); \quad \xi_2 = q_1(1-z), \quad \eta_2 = q_2z \quad (\theta_2 = 5\pi/7) \quad (\alpha = e_2/e_1)$	
$\xi_3 = q_3z, \quad \eta_3 = q_2 + (q_4 - q_2)z \quad (\theta_3 = 3\pi/7); \quad \xi_4 = q_3 + (1/2 - q_3)z, \quad \eta_4 = q_4 + (1 - q_4)z \quad (\theta_4 = \pi/7)$	
$q_1 = e_1 \cos \frac{2}{7} \pi, \quad q_2 = e_2 \cos \frac{3}{14} \pi, \quad q_3 = e_1 \sin \frac{1}{14} \pi, \quad q_4 = 2e_2 \cos \frac{1}{2} \pi \cos \frac{1}{14} \pi$	
$\left[e_1 = 1 / \left(1 + 2 \cos \frac{2}{7} \pi \right), \quad e_2 = 1 / \left(\cos \frac{1}{14} \pi + \cos \frac{3}{14} \pi + \cos \frac{5}{14} \pi \right) \right]$	
$m = 1, 3, \dots$	$m = 2, 4, \dots$
$n = 1, 2, 3, \dots$	

ture are presented for an equilateral triangular plate through a regular octagonal plate in Tables 1~6. These tables also provide the way of locating the segments and taking integers (m, n) for each type of vibration, and give the side ratio α of the extended plate where a regular polygon is formed. The S type vibration denotes symmetrical vibration about a symmetrical axis (broken line) and the A type vibration does antisymmetrical one with a nodal line (solid line) on the axis. In these tables, inscribed regular polygons are formed so that, at least, one of the edges might be located on the x axis on the extended plate. But a square plate and an

octagonal plate can be also formed by locating the segments so that the symmetrical axis of the plates is parallel to the edges of an extended square plate.

The coefficient matrices in Eqs.(17)-(24) are expressed by double infinite series on m and n . The summation of one of the series can be written by a function, but this expression is not adopted here, because it is rather complicated except for a square plate and inconvenient for the computer programming. In the numerical calculations of this study, 48×48 terms of double infinite series and several terms of Fourier series are used by taking the accuracy of solution into con-

Table 6 Regular octagonal plate

SSSS type	SSAA type	AAAA type	SA type

$\left[\sum_{m,n} \frac{1}{f_{mn}(\lambda)} \begin{pmatrix} h_{mn,i}^{(1)} \\ g_{mn,i}^{(2)} \\ h_{mn,i}^{(2)} \end{pmatrix} \begin{pmatrix} h_{mn,j}^{(1)} \\ g_{mn,j}^{(2)} \\ h_{mn,j}^{(2)} \end{pmatrix} \pm \begin{pmatrix} h_{mn,j}^{(1)} \\ g_{mn,j}^{(2)} \\ h_{mn,j}^{(2)} \end{pmatrix}^T \right] \times \begin{Bmatrix} M_{y,j}^{(1)} \\ (a/\pi) Q_j^{(2)} \\ M_{x,j}^{(2)} \end{Bmatrix} = 0 \quad (23)$			$\left[\sum_{m,n} \frac{1}{f_{mn}(\lambda)} \begin{pmatrix} h_{mn,i}^{(1)} \\ g_{mn,i}^{(2)} \\ h_{mn,i}^{(2)} \end{pmatrix} \right] \times \begin{Bmatrix} h_{mn,j}^{(1)} \\ g_{mn,j}^{(2)} \\ h_{mn,j}^{(2)} \end{Bmatrix} \begin{Bmatrix} M_{y,j}^{(1)} \\ (a/\pi) Q_j^{(2)} \\ M_{x,j}^{(2)} \end{Bmatrix} = 0 \quad (24)$
$\xi_1 = q_1 + (1/2 - q_1)z, \eta_1 = 0 \quad (\theta_1 = 0)$			$q_1 = 1 - 1/\sqrt{2} \quad (\alpha = 1)$
$\xi_2 = q_1(1 - z/2), \eta_2 = q_1 z/2 \quad (\theta_2 = 3\pi/4)$			$\xi_2 = q_1(1 - z), \eta_2 = q_1 z \quad (\theta_2 = 3\pi/4)$ $\xi_3 = 0, \eta_3 = q_1 + (1/2 - q_1)z \quad (\theta_3 = \pi/2)$
sign	+	-	-
	$m, n = 1, 3, \dots$	$m, n = 2, 4, \dots$	$m = 1, 3, \dots; n = 2, 4, \dots$

Table 7 Eigenvalues of clamped polygonal plates

(a) Fundamental mode						
	λ					
	Triangle	Square	Pentagon	Hexagon	Septagon	Octagon
present authors	3.173	1.913	2.296	2.283	2.155	2.002
Ota et al. ⁽⁷⁾	3.17	—	—	—	—	—
Young ⁽⁶⁾	—	1.910	—	—	—	—
Laura et al. ⁽⁴⁾	—	1.923	2.301	2.282	2.153	1.998
Yu ⁽⁵⁾	3.393	1.987	2.353	2.319	2.180	2.019
Walkinshaw et al. ⁽³⁾	3.167	1.910	2.291	2.277	2.149	1.997

(b) Second symmetrical mode						
	λ					
	Triangle	Square	Pentagon	Hexagon	Septagon	Octagon
present authors	—	3.676	4.481	4.487	4.242	3.946
Walkinshaw et al. ⁽³⁾	5.470	3.660	4.477	4.474	4.232	3.93

sideration. More terms are employed for the calculation of the case when accuracy is insufficient. The convergence characteristics with an increase of terms is almost the same as that shown in the previous paper⁽⁵⁾.

The frequencies of small pieces of plate made by the separation of a polygonal plate from the extended plate are calculated together, but these values can be easily removed, because they give the known values which are relatively higher than those of the actual plate. When they cannot be removed easily, they are distinguished by drawing the mode shapes of vibration.

Table 7 compares the present eigenvalues for the fundamental and the second mode of symmetrical vibration with those obtained by other authors. The present results agree well with those obtained by Laura et al.⁽⁴⁾, Young⁽⁶⁾ and Ota et al.⁽⁷⁾ for the fundamental mode. The values of Walkinshaw et al.⁽³⁾ obtained by point-matching method are slightly lower for all polygonal plates. It is pointed out by Laura et al.⁽⁴⁾ that Yu's results⁽⁵⁾ are several percent higher than those by other

authors.

Figure 2 shows the eigenvalues and the mode shapes of clamped regular polygonal plates inscribed in a circle of constant radius r_0 . The eigenvalues in this figure are converted by multiplying λ by $2r_0/a$. The eigenvalues and the mode shapes of a clamped circular plate are also shown for comparison. In this case, the eigenvalues of polygonal plates become higher with a decrease in the number of sides and the area. It is noted that there are certain similarities among the mode shapes of polygonal and circular plate, and they can be classified into some groups connected by broken lines. The close eigenvalues of the same group should take the same values (degeneration of frequencies), and slight errors between them are caused by the difference of equations used and the truncation of terms in the series. However, the errors are less than 0.4% for most polygonal plates, though the triangular plate has about 0.6% error because of insufficient convergence. Without the occurrence of degeneration, the eigenvalues take two different values for V-group vibration of hexagonal plate and VII-group of octagonal

plate. In this case, the mean values of them are adopted, considering that each set of them belongs to the same group.

Figure 3(a) and (b) show the converted eigenvalues of polygonal plates that are circumscribed over a circle of constant radius r , and that have the same area as a circle of mean radius $\bar{r}$, re-

spectively. The points marked with $\circ$ express the eigenvalues of the plates belonging to each group, which are connected by broken lines. As shown in these figures, the eigenvalues of circumscribed polygonal plates become larger monotonously with an increase in the number of sides. Meanwhile, the eigenvalues of polygonal

Fig.2 Eigenvalues and mode shapes of clamped polygonal plates inscribed in a circle

Fig.3 Eigenvalues of clamped polygonal plates

plates of mean radius r become smaller slightly with an increase in the number of sides, but they take almost equal values to those of a circular plate when the number of sides is over six. The effects of plate shape are found in polygonal plates whose sides are less than six. These plates have relatively higher frequencies than polygonal plates with many sides and have different mode shapes.

4. Conclusions

A theoretical method was presented for the analysis of free vibration of clamped polygonal plates and the following conclusions were obtained.

- (1) Fundamental equations are derived for studying free vibration of a rectangular plate which is simply supported at all edges and clamped on the segments located inside the plate or at the edges.
- (2) A polygonal plate is formed on a rectangular plate simply supported at all edges by appropriately setting several clamping segments of equal lengths. The frequency equations are derived with use of this method for the analysis of an equilateral triangular plate through a regular octagonal plate.
- (3) The eigenvalues of polygonal plates obtained by the present method are compared with those by other authors, all of which are in good agreement.
- (4) The eigenvalues and the mode shapes of polygonal plates inscribed in a circle are presented in a figure, where it is found that the eigenvalues become larger with a decrease in the number of sides and the mode shapes can be clas-

sified into some groups.

- (5) The eigenvalues and the mode shapes of polygonal plates with more sides than six are similar to those of a circular plate with the same area, but triangular, square and pentagonal plates have relatively higher eigenvalues and different mode shapes due to the effect of plate shape.

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