Roughness Effects on the Flow along an Enclosed Rotating Disk*

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Roughness effects on the three-dimensional boundary layer flow along an enclosed rotating disk have been studied theoretically and experimentally as well as the admissible roughness.

The results show that an increase of roughness of the rotating disk results in an increase of core rotation and then in a considerable increase of pressure drop toward the disk center, that an increase of roughness of the chamber wall has reverse effects, and that the equal roughness of both walls makes the flow almost same as one along smooth surfaces.

The theoretical results based on the logarithmic velocity distributions show good agreement with the measurements.

The well-known formula of admissible roughness in the two-dimensional flow is confirmed to be effective even in the three-dimensional boundary layer flow, because the secondary flow has little effect on the flow in the immediate vicinity of the wall and then the viscous sublayer is not influenced so much.

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1. Introduction

For the performance estimation of large-size hydraulic turbo-machinery, model testing is usually used. When the results obtained by the model testing are applied to the prototype performances, the wall roughness of the impeller and the casing is one of the main influencing factors. Various formulae of efficiency scaling-up have been proposed for the cases where either the prototype or the model has a completely rough surface, where both have a hydraulically smooth surface, or where both have completely rough surfaces.

On the other hand, since the flow in the back spaces of a centrifugal impeller should also be strongly influenced by the surface roughness of the impeller and the casing wall, the axial thrust, the disk friction torque, and the leakage loss caused by this flow might accordingly be influenced by the surface roughness.

In the blade-to-blade channels and in the back spaces of an impeller the main flow is superimposed by the secondary flow perpendicular to it and a skewed boundary layer flow is formed. The roughness effects in such a three-dimensional flow might show a different behaviour from that in a two-dimensional flow along a flat plate, and the analytical estimation of roughness effects in that flow has been demanded in relation to the scaling-up effects.

A flow along an enclosed rotation disk is generally used to study a flow in the back spaces of an impeller. This flow has been studied theoretically and experimentally by many investigators for the case where both the rotating disk and the casing wall are hydraulically smooth. However, where either of the walls is rough or both walls are rough, the behaviour of the flow has scarcely been made clear, but only the roughness effects on the friction torque have been studied experimentally.

As for the admissible roughness $k_{adm}$ which gives the limitation of roughness effects, the well-known formula $k_{adm} = C V/U, C = 100$ ($V$: kinetic viscosity, $U$: main flow velocity) obtained for a two-dimensional boundary layer flow along a flat plate has been mainly examined for its application to a three-dimensional boundary layer flow. It has been the main problem what value should be given to the constant term $C$ in a real impeller flow. Cairney gave the value $C$ ranging $20 < C < 30$ for a centrifugal impeller, and Fay proposed $20 < C < 70$ for a turbine impeller. Fukuda obtained the same value $C = 100$ as that for a flat plate from the friction data of the enclosed rotating disk when $U$ was replaced with $u_{0}/2$ ($u_{0}$: peripheral velocity of the disk tip), but Fay pointed out that Fukuda's roughness description was inadequate.

The values of $C$ in the above mentioned studies were deduced from the measurements of the disk friction torques or the pump performances. However, a more detailed examination of the boundary layer flow might be of basic important, since the
admissible roughness is largely dependent upon the inter-relationship between the viscous sublayer and the roughness elements.

The authors have made clear the flow along a smooth rotating disk enclosed in a smooth chamber in the former report \(^{(1)}\). The present report deals with the flow along a rough disk in a smooth or a rough chamber and the flow along a smooth disk in a rough chamber, and the roughness effects are studied theoretically and experimentally as well as the admissible roughness.

2. Nomenclature

- \(a, a', c\): experimental constants
- \(C\): coefficient of disk friction torque, Eq. (22)
- \(C_p, C_p'\): pressure coefficients based on the pressure at \(r = 0.976 r_0\) and \(r = 0.303 r_0\), respectively
- \(C_T\): coefficient of axial thrust, Eq. (23)
- \(K\): core rotation, \(U/\omega\)
- \(K_s\): effective height of roughness protuberances \((m)\)
- \(P\): pressure \((kg/m^2)\)
- \(r\): radius \((m)\)
- \(R\): radius ratio, \(r/r_0\)
- \(Re\): Reynolds number, \(r_0^2 \omega / \nu\)
- \(s\): axial clearance between disk and casing wall \((m)\)
- \(U\): main flow velocity \((m/s)\)
- \(u, v\): peripheral and radial velocity components in the boundary layer, respectively \((m/s)\)
- \(v_s\): friction velocity, \(\sqrt{T/\rho}\) \((m/s)\)
- \(x\): axial distance from wall \((m)\)
- \(\alpha\): angle of wall shearing stress \((rad)\)
- \(\delta, \delta_0\): boundary layer thicknesses on disk and casing, respectively \((m)\)
- \(\zeta, \eta\): non-dimensional axial distances, Eq. (5)
- \(\epsilon\): radial clearance between disk tip and cylindrical wall \((m)\)
- \(\rho\): fluid density \((kg/m^3)\)
- \(\nu\): kinematic viscosity of fluid \((m^2/s)\)
- \(\tau\): wall shearing stress \((kg/m^2)\)
- \(\omega\): angular velocity of disk \((m/s)\)
- \(\phi\): non-dimensional velocity of fluid, Eq. (16)

Subscripts

- \(\theta, r\): peripheral and radial components, respectively
- \(R, S, C\): rotating disk, stationary end wall and cylindrical casing wall, respectively
- \(0, 1\): disk periphery and outer edge of boundary layer, respectively

3. Theoretical analysis

A flow along an enclosed rotating disk is illustrated in Fig. 1. This analysis is based on the assumption of a steady, incompressible, and fully developed turbulent flow with a core region in which the peripheral velocity component \(u\) is constant in the axial direction and the radial velocity component \(v\) is zero, as shown in Fig. 1. Later, this assumption will be shown to be almost reasonable by the measured results, when the axial spacing is not too small. Thus the core rotation is written as \(U = K \omega, K = K(r)\).

The equations of angular momentum balance and continuity in this flow field are written as follows:

\[ \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) = -\frac{\tau}{\rho} \]  

(1)

\[ 2\pi \int_{0}^{1} u \rho \, dr = \Phi \]  

(2)

where the velocity components \(u, v\) and the shearing stress components \(\tau_{xx}, \tau_{xy}\) are taken positive in the direction shown in Fig. 1. \(Q\) is the rate of the flow which flows through the chamber radially and corresponds to the leakage flow at the back of an impeller. In this study \(Q\) is assumed zero.

3.1 Assumptions of the velocity distributions

It is known that a turbulent flow along a rough flat plate with zero attack angle includes three different regions according to surface roughness, namely (A) hydraulically smooth region, (B) transition region and (C) completely rough region. A velocity distribution formula for each region has been proposed. But in the actual turbomachinery the problem is whether the wall surface is hydraulically smooth or not, and the transition region is rather out of the problem. So the regions (A) and (C) are analyzed in this report.

To analyze the flow in the regions (A) and (C) inclusively, the logarithmic velocity assumptions are best introduced. The basic form of the logarithmic velocity distribution \(w\) is written as follows: (19)

For a hydraulically smooth surface (A):

\[ w_{st} = (85 \log k/\nu) \]  

(3)

For a completely rough surface (C); \( \cdots \cdots (3) \)

In this study the velocity distribution in the skewed boundary layer is assumed as follows with the boundary conditions \(v = v' = 0\) at the outer edge of the boundary layer taken into consideration.

On a stationary end wall;

\[ w_{st} = (85 \log k/\nu) \]  

(4)

On a rotating disk;

\[ w_{st} = (85 \log k/\nu) \]  

(5)

where \(\delta, \delta_0, \zeta, \eta\) are defined as follows;

On a hydraulically smooth wall:

\[ \eta_{st} = (20 \log k/\nu) \]  

(6)

On a completely rough wall:

\[ \eta_{st} = (15 \log k/\nu) \]  

(7)

Fig. 1 Flow along an enclosed rotating disk
and are also written with values \( \eta, \) and \( \zeta, \) at the outer edge of the boundary layer

\[
s_{p}/s_{p} = \tan \eta, \quad \eta, \zeta, = s'_{p} = \zeta_{p} = \zeta_{p} \quad \text{at} \quad s'_{p} = 0. \quad \text{at} \quad s'_{p} = s'_{p} \quad \text{and} \quad s'_{p} = s'_{p} \quad \text{at} \quad s'_{p} = s'_{p} \quad \text{at} \quad s'_{p} = s'_{p} \quad \text{at} \quad s'_{p} = s'_{p}.
\]

The radial and tangential components of the friction velocity are written as follows with the angles \( \alpha \) and \( \alpha' \) of the shearing stresses on the stationary and the rotating walls, shown in Fig. 2.

\[
\begin{align*}
\tau_{r} &= \frac{\alpha}{\cos \alpha}, \quad \tau_{rr} = \frac{\alpha}{\cos \alpha}, \\
\tau_{r} &= \frac{\alpha}{\cos \alpha}, \quad \tau_{rr} = \frac{\alpha}{\cos \alpha}.
\end{align*}
\]

As there exists a viscous sublayer very near to the wall surface, it is difficult to apply the velocity assumptions to the immediate vicinity of the wall. But if we assume that the direction of the flow in the immediate vicinity of the wall is nearly equal to that of the wall shearing stress, we can write \( \tau_{r} = \tau_{r} \cdot \tan \alpha \) (or \( \tau_{r} = \tau_{r} \cdot \tan \alpha' \)).

Then according to Eq. (4), we obtain

\[
\tan \eta = \tau_{r}, \quad \tan \alpha = \tau_{r} \quad \text{and} \quad \tan \alpha = \tau_{r}.
\]

Moreover, we extend these velocity assumptions to the outer edge of the boundary layer with the same procedure as that for the two-dimensional flow. Namely, introducing the boundary conditions \( u = u' = K \) at \( z = 0 \) and \( z' = \delta \), we obtain the following friction velocities.

\[
\begin{align*}
\tau_{r} &= K \tau_{r} \cdot \cos \alpha \cdot \cos \alpha', \\
\tau_{r} &= K \tau_{r} \cdot \cos \alpha \cdot \cos \alpha',
\end{align*}
\]

On a completely rough wall; \( \tau_{r} = K \tau_{r} \cdot \cos \alpha \cdot \cos \alpha' \).

Finally we obtain the following velocity distributions by substituting Eqs. (5), (6), (7), and (9) into Eq. (4).

\[
\begin{align*}
\eta &= n_{r} = 1 - (1 - K) \left( \frac{\tau_{r}}{\tau_{r}} \right), \\
\eta &= n_{r} = 1 - (1 - K) \left( \frac{\tau_{r}}{\tau_{r}} \right),
\end{align*}
\]

On a hydraulically smooth disk; \( \eta = 2.5 \log(9.0 \Omega), \quad \zeta = 2.5 \log(9.0 \Omega), \)

\[
\begin{align*}
\eta &= n_{r} = 1 - (1 - K) \left( \frac{\tau_{r}}{\tau_{r}} \right), \\
\eta &= n_{r} = 1 - (1 - K) \left( \frac{\tau_{r}}{\tau_{r}} \right),
\end{align*}
\]

On a hydraulically smooth casing wall; \( \eta = 2.5 \log(9.0 \Omega), \quad \zeta = 2.5 \log(9.0 \Omega), \)

\[
\begin{align*}
\eta &= n_{r} = 1 - (1 - K) \left( \frac{\tau_{r}}{\tau_{r}} \right), \\
\eta &= n_{r} = 1 - (1 - K) \left( \frac{\tau_{r}}{\tau_{r}} \right),
\end{align*}
\]

On a completely rough disk; \( \eta = 2.5 \log(30.0 \Omega), \quad \zeta = 2.5 \log(30.0 \Omega), \)

\[
\begin{align*}
\eta &= n_{r} = 1 - (1 - K) \left( \frac{\tau_{r}}{\tau_{r}} \right), \\
\eta &= n_{r} = 1 - (1 - K) \left( \frac{\tau_{r}}{\tau_{r}} \right),
\end{align*}
\]

On a completely rough casing wall; \( \eta = 2.5 \log(30.0 \Omega), \quad \zeta = 2.5 \log(30.0 \Omega), \)

\[
\begin{align*}
\eta &= n_{r} = 1 - (1 - K) \left( \frac{\tau_{r}}{\tau_{r}} \right), \\
\eta &= n_{r} = 1 - (1 - K) \left( \frac{\tau_{r}}{\tau_{r}} \right),
\end{align*}
\]

Substituting the above velocity distributions into Eqs. (1) and (2), we obtain two ordinary differential equations in which the unknown variables are \( K, \zeta, \) and \( \eta, \) as a function of \( r. \) It is, therefore, necessary to assume one of these variables. Daily and others (13) have extended the theory proposed by Schultz-Grunow (12) and obtained the following expression of the boundary layer thickness \( \delta \) on the hydraulically

\[\text{Fig. 2 Direction of shearing stress}\]

smooth rotating disk in a smooth enclosure with the aid of measured data.

\[
\beta = 0, \quad 0 \leq r < r_{s}, \quad \beta = \beta_{s}, \quad 0 \leq r < r_{s}.
\]

Here we extend the above expression to a rough wall and assume that the surface roughness influences only on the core rotation \( K \) and the above expression is still effective for the case of a rough disk.

Moreover, when the same way of thinking as above mentioned is applied to the flow angles in the immediate vicinity of both rotating and stationary walls, the constants \( a, \) and \( a' \) in Eq. (8) are determined from the measured data (9) in the case of a smooth disk with a smooth enclosure, as follows.

\[
a = \sqrt{\tan a'} = 0.612, \quad a' = \sqrt{\tan a'} = 0.491.
\]

Experimental results in the case of rough surfaces will show later in (Fig. 5) that these assumptions are appropriate.

With these assumptions Eqs. (1) and (2) are transformed to one ordinary differential equation in which the variable is the core rotation \( K \) as a function of radius \( r. \) When the boundary value \( K_{s} \) of the core rotation is given, the radial distribution of \( K \) is determined with the aid of numerical integration. The integration of the velocity components in Eq. (3) diverges to negative infinity with \( \zeta - 0. \) To prevent the divergence of the calculation, the same procedure as that for the two-dimensional flow is introduced as follows. --- As the value of \( \zeta \) is about 5 at the outer edge of the viscous sublayer, the approximations of \( \zeta = 5, \zeta = 5, \zeta = 5, \) and \( \zeta = 5, \zeta = 5, \zeta = 5, \) make hardly any change over the whole range of the turbulent boundary layer. Thus we can approximate:

On a hydraulically smooth wall; \( \eta = 2.5 \log(9.0 \Omega) + 1 \)

On a completely rough wall; \( \eta = 2.5 \log(30.0 \Omega) + 1 \)

\[
\begin{align*}
\eta &= n_{r} = 1 - (1 - K) \left( \frac{\tau_{r}}{\tau_{r}} \right), \\
\eta &= n_{r} = 1 - (1 - K) \left( \frac{\tau_{r}}{\tau_{r}} \right),
\end{align*}
\]

3.2 Equation of boundary condition

The angular momentum balance in the control surface \( S \) at the outer periphery of the flow field, shown in Fig. 1, might be written approximately as follows:

\[
\begin{align*}
\frac{d}{dr}
\end{align*}
\]

\[
\text{The tangential component} \quad \tau_{t} \quad \text{of the wall shearing stress on the cylindrical casing wall is assumed as follows, when the wall roughness of the cylindrical wall is equal to that of the stationary endwall;}
\]

\[
\begin{align*}
\tau_{t} &= \frac{\beta}{\tau_{t}} \cdot \tau_{t}, \\
\tau_{t} &= \frac{\beta}{\tau_{t}} \cdot \tau_{t},
\end{align*}
\]

\( M \) is the angular momentum which the throughput takes into (\( M > 0 \)) or carries out of (\( M < 0 \)). The rest section at the outer periphery of the disk, and has already been examined in detail in the former report (4). In this study \( M \) is zero, as the case of no throughput flow is treated.

Substitution of Eqs. (10), (11), (12), (13) and (18) into Eq. (17) gives the boundary value \( K_{s} \) of the core rotation at the outer periphery, and accordingly the radial distribution of the core rotation \( K \) is obtained when the disk dimensions \( (r_{s}, \alpha, \epsilon) \) and the surface roughness \( (K_{s}, K_{s}') \) of both walls are given.

In addition to the above calculation procedure, when the radial and tangential
components of the friction velocities are simplified as \( v_\gamma = v_\eta = v_\phi = v_ρ = v_\alpha \). The calculated results show little change with this simplification, since this analysis is based on the momentum theory.

On the other hand, the pressure distribution is given by

\[
\frac{dp}{dr} = \frac{1}{r} \frac{\partial p}{\partial \rho} \nu_r^2
\]

4. Experimental equipment and procedure

General view of the test stand is shown in Fig. 3. The inner wall of the casing was chrome-plated, and had nine pressure holes \( \number{1} \) and four holes \( \number{2} \) for velocity measurements by Pitot tube. A smooth, bronze disk of 328 mm OD and 12 mm thickness was used. The annular gap \( c \) between the disk periphery and the cylindrical casing wall was 2 mm and the axial spacing \( s \) between the disk and the stationary endwall was 12.8 mm.

To examine the roughness effects on the flow, the surface roughnesses of the disk and the casing walls were varied, while the disk revolution was kept constant, \( n = 850 \text{ rpm} \). Surface roughnesses in these tests were obtained by cementing commercial waterproof grit papers onto the surface of the disk and/or the casing inner wall with a waterproof bonding cement.

The surface roughness used was measured by means of a roughness indicator employing a needle which travelled across the surface, the amplitude of the needle's axial displacement being recorded. The traces obtained were analyzed and the results are summarized in Table 1. The roughness profiles dimensions are shown in Fig. 4 as an example.

In the data reduction of random roughness, one of the most troublesome problems is how to determine the effective roughness \( R_z \). According to Japan Industrial Standard (JIS), the arithmetically-averaged roughness \( R_a \) and the ten-points-averaged roughness \( R_z \) have been used. The ten-points-averaged roughness \( R_z \) is defined as the height difference between the 3rd from the maximum height and the 3rd from the maximum depth in the specified length of the roughness. In other countries, \( R_z \) or the root-mean-square roughness \( R_m \) is generally used. On the other hand, from the viewpoint of hydraulics, the equivalent Nikuradse sand-grain diameter \( (\xi) \) has been generally used. This is based on the Prandtl-Schlichting resistance formula \( (\xi) \) for a completely rough region. But according to Nikuradse's pipe tests, lacquer coating (dünflüssigen Japanlack) was used to obtain better adherence of the grains to the pipe wall, and the effective roughness used in data reduction must be smaller than the actual sand-grain diameter. According to Young's experiments in which airplane surfaces were lacquer-coated with various methods of coating, the equivalent sand-grain diameters were about 1.6 times (1.31 \( \sim \) 1.96), the statistically obtained roughness \( (\xi) \). Nece and Daily \( (\xi) \) also measured the surface roughnesses of waterproof grit papers and obtained almost the same results as those of Young. That is to say, the mean-grit-particle-diameters were 1.58, 1.72 and 1.89 times the root-mean-square roughness \( R_m \).

In the present study, the mean-grit-particle-diameters were about 1.6 times (1.54 \( \sim \) 1.69) the root-mean-square roughness \( R_m \) and were also nearly equal to the ten-points-averaged roughness \( R_z \) measured in accordance with JIS. These values were adopted as the effective roughnesses \( R_z \) in this report, as shown in Table 1. The Symbols I, II, ... in this table will be taken in the subsequent roughness descriptions.

To realize No. VII roughness, the sand grains eliminated by the sieves of No. 42 and 43 meshes were cemented onto the disk surface, since this roughness was beyond the maximum roughness in the waterproof grit papers.

Measurements were performed in one side of the disk shown in Fig. 3. The axial distributions of flow angles, total pressures and static pressures in the test section were measured by an three-hole cobra probe and a Pitot static tube axially. Static pressure on the stationary endwall was also measured. These measurements were performed for the smooth-disk, smooth-enclosure case, for the smooth-disk, rough-enclosure case, for the rough-disk, smooth-enclosure case, and for the rough-disk, rough-enclosure case. When a grit paper was cemented onto a wall, the axial position of the disk was adjusted to maintain a constant axial spacing \( s \) by using the same grit paper as a spacer.

The fluid used was water. The Reynolds number \( R_e = \text{Re}_p \) based on the disk peripheral velocity \( v_p \) was \( 3.1 \times 10^6 \), which corresponded to a fully turbulent boundary layer flow over the whole disk but the central region, since the transition Reynolds number is of the order of \( 10^5 \) for the smooth-disk, smooth-enclosure case \( (\xi) \).

According to the measured data of disk friction torque by Fukuda \( (\xi) \), the relative admissible roughness of the disk \( (ks) \) at the Reynolds-number of \( 3.1 \times 10^6 \) was about \( 2.4 \sim 4.0 \times 10^{-3} \) which resulted in that the roughness effects emerged in all of the roughness cases tested in this study except for No. 0 roughness case (smooth).

5. Experimental results and comparison with theoretical results

5.1 Velocity distribution

Representative results of the velocity data at \( R_e = 0.79 \) are shown in Figs. 5(a) and (b). Fig. 5(a) shows the results when only the roughness of the disk is varied with the roughness of the casing wall kept constant. Fig. 5(b) shows the results when the surface roughness is kept constant for the smooth-disk, smooth-enclosure case, for the rough-disk, smooth-enclosure case, and for the rough-disk, rough-enclosure case in comparison with the results for the smooth-disk, smooth-enclosure case. The notations 0-IV, 1V-IV, and so on in these figures mean that the right of the hyphen(-) corresponds to the disk roughness and the left the roughness of the casing wall (refer to Table 1). The data on a rough disk with
smooth enclosure (Fig. 5(a)) show that the core rotation \( K \) increases with an increase of the disk roughness but the boundary layer thicknesses on both walls show a slight change with it. On the other hand, the velocity data on a smooth disk, with rough enclosure (Fig. 5(b)) show that the core rotation \( K \) decreases with an increase of the casing roughness, while the results with equal roughness on both the disk and the enclosure wall are almost the same as those with smooth walls on both the disk and the enclosure.

The variation of the core rotation \( K \) in the radial direction is shown in Figs. 6(a) and (b). Fig. 6(a) shows the results of a smooth disk with smooth enclosure, and Fig. 6(b) shows the results of the roughness being kept constant for 0-IV, IV-0, and IV-IV cases in comparison with 0-0 case.

Roughness effects are generally explained with the protuberance of the effective roughness \( ks \) in comparison with the thickness \( \delta_c \) of the viscous sublayer. In the two-dimensional boundary layer flow along a smooth flat plate, \( \delta_c \) is known to increase gradually in proportion to the one-tenth power of the flow-directional distance. On the other hand, in the three-dimensional boundary layer flow along a smooth rotating disk with no radial throughflow, the tangential velocity difference \((1-K)\delta \) between the fluid core velocity and the disk velocity becomes larger toward the outer radii, and accordingly, \( \delta_c \) along the disk becomes thinner toward the outer periphery. This can also be proved theoretically as follows:

\[
\delta_c = \frac{1}{\sqrt{\delta}} = \frac{1}{\sqrt{\delta_c}} (1 - K) \delta 
\]

where \( \delta_c \) is the product of \( \delta \) and \( (1-K) \delta \), and \( \delta_c \) is the thickness of the viscous sublayer. Since \( \delta_c \) varies little in the radial direction, and the core rotation \( K \) varies also little in the radial direction as shown in Fig. 6. As a result, \( \delta_c \) is nearly proportional to \( 1/r \) and becomes thinner toward the outer periphery. So is the viscous sublayer on the stationary endwall.

Accordingly, roughness effects appear firstly at the outer periphery of the disk and the range of influence by roughness becomes wider toward the inner radii with an increase of the relative roughness \( ks'/r_0 \). From Fig. 6(a) it is recognized that the core rotation \( K \) becomes larger from the outer radii with an increase of the disk roughness (see the curves No. II and III).

With further increase of the disk roughness (No. IV and V roughnesses), the curves of \( K-R \) relation (Fig. 6(b)) begin to depart from those for the smooth disk case and the boundary layer flow might be in the transition region \((\delta_c < \delta_r)\) at \( 0.8 \gtrsim r \gtrsim 0.4 \). In this case the flow near the outer periphery is considered to be a completely-rough flow.

When the relative roughness \( ks'/r_0 \) becomes large enough (No. VI), the boundary layer flow comes into the completely rough region almost all over the disk, and the core rotation becomes nearly constant in

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**Table 1** Surface characteristics of grit papers used in roughness tests

<table>
<thead>
<tr>
<th>Surface finish</th>
<th>Symbol</th>
<th>( K_s )</th>
<th>( R_{max} )</th>
<th>( R_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine grinds</td>
<td>0</td>
<td>0.6</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Grit paper #1200</td>
<td>1</td>
<td>19</td>
<td>12.0</td>
<td>9.6</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>22</td>
<td>13.0</td>
<td>10.4</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>24</td>
<td>15.0</td>
<td>12.0</td>
</tr>
<tr>
<td></td>
<td>8.5</td>
<td>28</td>
<td>16.8</td>
<td>14.0</td>
</tr>
<tr>
<td></td>
<td>8.6</td>
<td>36</td>
<td>36.0</td>
<td>32.0</td>
</tr>
<tr>
<td></td>
<td>#1000</td>
<td>95</td>
<td>45.6</td>
<td>52.0</td>
</tr>
<tr>
<td>Sand grains</td>
<td>VII</td>
<td>200</td>
<td>42.4 Mesh</td>
<td></td>
</tr>
</tbody>
</table>

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**Fig. 3** Test stand dimensions

**Fig. 4** Roughness protuberance dimensions

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**Fig. 5** Roughness effects on velocity distributions in the radial direction as shown in Fig. 6.

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**Fig. 6** Roughness effects on radial distribution of core rotation.
the radial direction, which shows that the fluid core rotates nearly as a forced vortex.

The transition Reynolds number \( r_o \omega / \nu \), based on the disk radius and obtained from the skin friction data \( \theta = 10 \) (9), is nearly equal to the Reynolds number based on the radius of the above mentioned transition point from the smooth region to the transition region. To show an example, in No. IV roughness the transition Reynolds number obtained by the disk friction data is about \( 5 \times 10^5 \), and this value corresponds to the Reynolds number at the radius of \( R = \sqrt{2/3} \times 10^6 \) (the denominator is the present Reynolds number based on the disk radius), which coincides approximately with the radius of the transition point obtained from the velocity data.

In Fig. 6(b) the core rotation \( K \) is smaller for the smooth-disk, rough-enclosure case, and is larger for the rough-disk, smooth-enclosure case than that for the smooth-disk, smooth-enclosure case. And for the rough-disk, rough-enclosure case of equal roughness, the core rotation is a little larger than that for the smooth-disk, smooth-enclosure case.

The comparison of the theory with the measured data of core rotation, shown in Fig. 7, shows a satisfactory agreement. Though this analysis is based on the assumption of the hydraulically smooth region or the completely rough region, the agreement is rather better for the smooth and the transition regions. This might be reasoned that the measured data on the smooth-disk, smooth-enclosure case are used in the assumptions of Eqs. (14) and (15).

5.2 Pressure distribution

Roughness effects on the pressure distribution are shown in Fig. 8. Pressure coefficient \( C_p \) is defined as the non-dimensional pressure difference based on the pressure \( P_o \) at the outermost measuring position of \( r = 0.976 r_o \); \( C_p = (p - p_o)/(1/2)p_o v^2/\rho \).

The curves are the theoretical results and are in a good agreement with the measurements. The pressure distribution for the case of equal roughnesses on both walls is nearly equal to that for the smooth-disk, smooth-enclosure case. But with an increase of the disk roughness the pressure drop toward the disk centre increases considerably, while it decreases with an increase of the casing-wall roughness.

From the above-mentioned roughness effects, it should be continued that the wall roughness would make the axial thrust and leakage flow change remarkably in a centrifugal turbomachine.

5.3 Admissible roughness

Schlichting (10) gave the roughness limitation in the hydraulically smooth region in the two-dimensional boundary layer flow as

\[ \frac{R_m}{U} = C (\theta = 100) \]  

(21)

In the three-dimensional flow along a rotating disk, the roughness effects appear at the outer radii and spread gradually toward the inner radii with an increase of the Reynolds number and the relative roughness. This tendency is remarkable especially in No. IV roughness, where a hydraulically-smooth flow might be realized in the inner radii of \( R < 0.4 \). But it is difficult to determine the transition point correctly.

Considering that the change of the flow pattern from the hydraulically-smooth region to the transition region might become evident in the radial pressure distribution, it would be possible to determine the transition point more correctly using the rearranged pressure data based on the pressure \( P_2 \) at the innermost measuring point of \( r = 0.393 r_o \) instead of \( P_o \). The pressure coefficient \( C_p' = (p - p_o)/(1/2)p_o v^2/\rho \) thus obtained, is illustrated in Figs. 9(a) and (b). Fig. 9(a) shows the results for No. IV roughness case in which the roughness effects upon the velocity data are

![Fig. 8 Pressure distributions in the radial direction](image)

(a) In case of No. IV roughness

(b) In case of No. II and No. III roughnesses

Fig. 9 Pressure coefficients \( C_p' \) based on the pressure at \( R = 0.303 \) and admissible roughnesses
remarkable. The values of the constant term C in Eq. (21) are also plotted in this figure, when the relative velocity (1-k)uw of the core to the disk velocity is adopted as the main flow velocity U. The radial position in which the pressure coefficient Cp' begins to depart from the curve of the hydraulically-smooth flow is at the radius of R = 0.39, which coincides approximately with the results obtained from the velocity data. Moreover, the value of C in this position is approximately 100 which agrees exactly with that in the two-dimensional flow.

Fig. 9(b) shows the case for No. II and III roughnesses, where transition occurs at the radius of R = 0.45 and the admissible roughness is also given when the value of C is 100.

In the three-dimensional boundary layer flow accompanied with a secondary flow perpendicular to the main flow, the boundary layer thickness is known to be much smaller than that in the two-dimensional boundary layer flow, because the secondary flow has the effect of boundary-layer suction. However, since the velocity component in the main-flow direction is much larger than that perpendicular to the main flow in the immediate vicinity of the wall surface, the secondary flow hardly influences the behaviour of the flow just near the wall. Accordingly, the thickness of the viscous sublayer δι might be little influenced by the secondary flow and the viscous sublayer along a rotating disk might show nearly the same behaviour as the one in the two-dimensional flow.

Above all, it should be concluded that the admissible roughness formula (Eq. (21)) proposed by Schlichting is still effective in the three-dimensional boundary layer flow caused by centrifugal force, if the relative velocity of the main flow to the wall velocity is used as the main flow velocity in Eq. (21).

6. Roughness effects on disk friction torque and axial thrust

The comparison of the analytical results with the measurements, stated above, shows a fairly good agreement. This analysis is applied to the estimation of disk friction torque coefficient Cb in comparison with the measured data by Nce and Daily.

As the shearing stress ηr in the inner radial has little influence upon Cb value, the integration is performed from r = 0.2r to r = r0. The analytical results are in satisfactory agreement with the measured data in the hydraulically-smooth and the completely-rough regions.

Fig. 11 shows the variation of Cb with the relative roughness of the disk or the casing wall. The admissible roughness calculated at Re = 10^6 is seen to be about r0/ks = 8000.

This figure gives nearly the same tendency as the equations

\[ C_b = \frac{C_b}{C_b} + \frac{C_b}{C_b} \] obtained from the measurements by Watabe. The reason why the Cb value is thus different according to the combination of the roughness of the disk and that of the casing wall, is owing to the existence of the cylindrical casing wall at the outer periphery of the flow field. If it were not for the cylindrical casing wall, the field of the core might rotate at a velocity about half the disk peripheral velocity, and CbO might be nearly equal to CbO^.

Lastly, the roughness effects on axial thrust are shown in Fig. 12. Axial thrust is defined here as the axial force working on one side of the disk, and its non-dimensional form, the axial thrust coefficient, is given as:

\[ c_T = \frac{2\eta r g - p_2 d_f}{\pi g \rho_0} \] (r = 0.2r) ............ (23)

The values of Cb for the case of equal roughnesses on both walls are seen to be nearly equal to those for the smooth-disk, smooth-enclosure case regardless of r0/ks value. But when the roughness of each wall is different, Cb value increases considerably with an increase of disk roughness and decreases with an increase of casing wall roughness.

7. Conclusions

The roughness effects on the flow along an enclosed rotating disk and the admissible roughness are studied theoretically and experimentally. The results obtained in this study are summarized as follows.

(a) Rough-disk, rough-enclosure wall

(b) Rough-disk, smooth-enclosure wall

Fig. 1 Disk friction torque—Comparison of theory with experiments by Nece and Daily.
(1) The viscous sublayer in the immediate vicinity of the disk or the casing wall becomes thinner toward the outer radii. When the roughness protuberance $k_s$ ($k_s'$) is smaller than the thickness of the viscous sublayer at the outer periphery, the flow behaves as a hydraulically-smooth flow and the fluid of the core region rotates like a forced vortex with the angular velocity of about 0.43.$\dot{u}$

(2) When either the disk or the casing wall is rough, the roughness effects appear firstly at the outer periphery and extend toward the inner radii with an increase of wall roughness, which results in a variation of the radial distribution of the core velocities. The core rotation $\dot{K}$ increases with an increase of the disk roughness, and decreases with an increase of the casing wall roughness. This tendency is remarkable at the outer radii. With further increase of the roughness, the flow becomes completely rough over the whole wall, and the core rotates with almost constant angular velocity in the radial direction, the flow pattern being a forced vortex type.

(3) While both the disk and the casing walls have equal roughnesses the core rotation $\dot{K}$ is approximately equal to that for the smooth disk, smooth-enclosure case, and the flow pattern is a forced vortex type.

(4) The radial pressure distribution is influenced considerably by wall roughness when the roughnesses of both the disk and the casing are not equal. The pressure drop toward the disk centre increases considerably with an increase of disk roughness and decreases with an increase of casing wall roughness. However, when the roughnesses of both walls are equal, the pressure distribution is nearly equal to that for the smooth disk, smooth-enclosure case.

(5) The admissible roughness in the three-dimensional boundary layer flow accompanied with a secondary flow perpendicular to the main flow, is considered to be subjected mainly to the main flow, if the flow-directional velocity component is large enough in the vicinity of the wall. Schlichting's formula proposed for the two-dimensional boundary layer flow is still effective in such a three-dimensional boundary layer flow, when the relative velocity of the main flow to the wall velocity is used as the main flow velocity $\dot{U}$ in Eq.(21).

(6) The analytical method based on the momentum theory and the logarithmic velocity assumptions gives satisfactory results, and makes it possible to estimate quantitatively the roughness effects upon disk friction torque, axial thrust and leakage loss.

References

(19) Kurokawa, J. and Toyokura, T., Discussion Note of Ref. (14), (1976), 21.