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Measurement of the First Normal Stress Differences of Water and Dilute Polymer Solutions*

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Small first normal stress differences of fluids are measured by a method which utilizes the reaction force of the jet issuing from a capillary tube. The fluids used in the experiment are water and dilute polymer solutions. The data and the following discussion show that the first normal stress difference is not zero even in a steady Poiseuille flow of water and that the first normal stress differences in various dilute polymer solutions are greater than that of water but not so strikingly different from that of water as expected from the dramatic reducing effect of drag in turbulent flow conditions. These experimental data will be fundamentally available for the construction of a constitutive equation or the analysis of more complicated flows.

1. Introduction

Flow of dilute polymer solutions, especially, turbulent flows with a drag reducing effect (Toms effect) have been widely studied. It is commonly admitted that the dramatic drag reducing effect is caused by the elasticity of polymer solution, since dilute polymer solutions of ppm order in weight concentration are almost the same as water in viscosity. From this point of view, analyzing the experimental data on turbulent or complicated flows or making a theoretical treatment of flow by a constitutive equation, one must beforehand know the value of the normal stresses in dilute polymer solutions particularly in a range of high shear rates of laminar flow conditions. In reality, however, the measurement has been too difficult to obtain reliable data on a wide range of solutions. Metzner et al.(1) devised a method to measure the normal stress by utilizing the swell of the jet (Barus effect) issuing from a capillary tube. Its utility, however, is limited to the case that the concentration of polymer solution is high and the normal stress is great. Harris(2) pointed out that the jet swell is not so advantageous but the jet thrust is important to know the normal stress. Following this suggestion, Shertzer and Metzner(3) measured the reaction force of a jet on a capillary tube and calculated the first normal stress difference. Metzner and Park(4), Patterson and Zakin(5), Meister and Biggs(6) made similar experiments, but solutions employed in these experiments are more concentrated than ones commonly used in drag reducing phenomena. Oliver(7) made

use of the thrust of the jet compacting a vertical plate positioned downstream at a distance from the tube exit, and measured the normal stress of rather dilute polymer solutions. In this case, too, the concentrations were higher than ppm order. Oliver's method may have the following demerits: the jet tends to become droplets downstream at a thrust measuring point; small perturbations enlarge as the jet travels downstream; and the jet has the tendency to make the plate rotate if its impacting position is incorrect. Furthermore there are some questions about how to correct the surface tension effect, which is generally a problem when jets issue in air and becomes a cause to decrease the reliability of this kind of measurement. The author(8) tried to get reliable data on first normal stress difference in a similar way to Oliver at the first stage of this study, but in vain for the reasons mentioned above.

In the present study the author employs the principle of utilizing the reaction force of a jet on a capillary tube, develops a new method and apparatus, and obtains the first normal stress difference in various dilute polymer solutions of ppm order in concentration. These experimental data will be of fundamental use to investigate constitutive equations or to study more complex flows. Mochimaru et al.(9) introduced a method similar but inferior to the present one and discussed a few factors which may affect the measured results.

2. Principle of measurement

Suppose that a fluid is issuing from a capillary tube into a large container filled with the same kind of fluid (Fig.1 (a)). Take the control surface which contains the tube and makes contact with the tube exit, and apply the momentum balance in the z direction. On this occasion we assume that the issuing jet has a velocity profile of a fully developed laminar flow

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Fig. 1(a)

Fig. 1(b)

at a large distance from the inlet and at the tube exit. Since the fluid flows on the control surface vertically from the downward direction, we have the following equation according to the momentum balance.

$$\int_0^R S_{zz} 2\pi r dr = T - T_M \quad (1)$$

where

$$T = 2\pi\rho \int_0^R v^2 r dr = \frac{\rho\pi V^2 D^2}{3} \quad (2)$$

R : the tube radius,

D : the tube diameter $= 2R$,

v : the flow velocity,

r : the radius at any position,

ρ : the fluid density,

V : the mean flow velocity,

S_{zz} : the normal stress in the z direction,

T_M : the reaction force exerted on the tube by the issuing jet in the inverse flow direction.

Now, if we substitute the integral variable r of Eq. (1) into the shear stress τ_{rz} by the relation $r = (\tau_{rz}/\tau_w)R$, where τ_w is the wall shear stress, and furthermore differentiate both sides of the equation by τ_w , then we have the following equation.

$$(S_{zz})_w = \frac{1}{2\pi R^2} \left[2(T - T_M) + \tau_w \frac{d(T - T_M)}{d\tau_w} \right] \quad (3)$$

where suffix w denotes the value at the wall surface. Assuming Newtonian viscosity for the shear stress,

$$\tau_{rz} = \mu \frac{dv}{dr} \quad (4)$$

and for the velocity distribution

$$v = 2V \left[1 - \left(\frac{r}{R} \right)^2 \right] \quad (5)$$

we have

$$\tau_w = -\mu \frac{8V}{D} \quad (6)$$

where μ is the viscosity coefficient of fluid and $8V/D$ is the absolute value of the wall shear rate. Substituting Eq. (6) into Eq. (3), we get

$$(S_{zz})_w = \frac{1}{2\pi R^2} \left[2 + d \log(T - T_M) / d \log \frac{8V}{D} \right] \times (T - T_M) \quad (7)$$

If we measure the reaction force T_M , we can know the normal stress at the wall $(S_{zz})_w$ by this equation, since T can be obtained from the measured flow rate by Eq. (2).

$(S_{zz})_w$ is the normal stress at the wall. When we want to relate it to the first normal stress difference $(\tau_{zz} - \tau_{rr})_w$, where τ_{zz} and τ_{rr} are the deviatoric normal stress in z and r directions respectively, we can do as shown below. That is, we consider two materials ① and ② adjoining each other through a boundary AB (see Fig. 1(b)). Now let $\vec{T}^{(1)}$ be the stress vector which the material ② exerts on the surface element of the material ① through the boundary AB, that is, the stress vector of the material ①, and similarly $\vec{T}^{(2)}$ be the stress vector of the material ②. Then we get

$$\vec{T}^{(1)} = -\vec{T}^{(2)} \quad (8)$$

where suffixes (1) and (2) mean the materials ① and ② respectively (10). Equation (8) reads in the form of the contravariant component,

$$T^{i(1)} = -T^{i(2)} \quad (9)$$

Applying Cauchy's formula $T^i = S^{ij}\nu_j$, we get

$$S^{ij(1)}\nu_j^{(1)} = -S^{ij(2)}\nu_j^{(2)} \quad (10)$$

where, S^{ij} is a contravariant component of the stress tensor and ν_j is a covariant component of the unit vector ν normal to the surface AB. Furthermore if it is assumed that the x^1 -axis (r -axis) points out of the surface AB (see Fig. 1(b)), then the next relations are obtained:

$$\left. \begin{aligned} \nu_1^{(1)} = -\nu_1^{(2)} = 1 \\ \nu_2^{(1)} = \nu_2^{(2)} = \nu_3^{(1)} = \nu_3^{(2)} = 0 \end{aligned} \right\} \quad (11)$$

Therefore,

$$S^{i(1)} = S^{i(2)}$$

or

$$S^{11(1)} = S^{11(2)}, \quad S^{21(1)} = S^{21(2)}, \quad S^{31(1)} = S^{31(2)}$$

and it is represented by the physical component as

$$S_{rr}^{(1)} = S_{rr}^{(2)}, \quad S_{r\theta}^{(1)} = S_{r\theta}^{(2)}, \quad S_{rz}^{(1)} = S_{rz}^{(2)} \quad (12)$$

Now we take the issuing jet as the material ① and the fluid around the jet as the material ②. Considering that at the tube exit the fluid around the jet is nearly static, we may put $S_{rr}^{(2)}=0$ at the tube exit and obtain the following relation by Eq.(12):

$$S_{rr}^{(1)} = (S_{rr})_w = 0 \quad (13)$$

On the other hand, the normal stress S_{rr} can be divided into two parts, the isotropic stress $-p$, where p is pressure, and the deviatoric normal stress τ_{rr} , namely,

$$(S_{rr})_w = (-p + \tau_{rr})_w$$

Hence we get

$$p_w = (\tau_{rr})_w \quad (14)$$

Finally the next relation is obtained at the tube exit,

$$(S_{zz})_w = (-p + \tau_{zz})_w = (\tau_{zz} - \tau_{rr})_w \quad (15)$$

namely, the normal stress in the z direction at the wall is equal to the first normal stress difference at the wall.

3. Experimental apparatus

The author devised the following experimental apparatus so as to measure the normal stress by Eq.(7). A schema of the apparatus is shown in Fig.2. Its outline is to convert the jet reaction force to the torsion angle of the steel wire. Namely, a steel wire of 1.5 mm diameter is stretched between the upper floor A and the lower one D which are about 556 cm apart, while the middle B is fixed but it can be rotated by the worm and worm wheel

Fig.2 Schema of the experimental apparatus

and there a mirror (I) is attached to know the deflection angle of rotation which is given by ΔS on the scale (I). At the lower point C of the wire the jet issuing device G is installed and the mirror (II) is attached there. These details are shown in Fig.3. When a fluid is supplied to the reservoir G vertically from the downward direction through the vinyl tubes K and it issues from the capillary tubes I as jets, the wire between B and C and the wire and vinyl tube between C and D are both torsioned by the jet reaction. Its torsion angle is detected by the telescope (II) by the deflection of the scale (II) from the zero point which is taken beforehand. (In the real measurement the torsion is made as small as possible by the stoppers which are not shown in the figure.) Next, in order to make zero the scale deflection in the telescope (II), namely, to match the point C to the zero point before issuing jets, the handle F is worked and through the shafts FE, EB and the worm wheel the point B is rotated and regulated. In this

Fig.3 Detail view of the part B

Fig.4 Jet issuing device (detail view of the part C)

4. Calibration

Let ΔS be the scale difference of the scale (I), and l, L be the lengths given in Figs.2 and 4 respectively. Then the torsion angle of the steel wire $\theta = \Delta S/l$ is proportional, within a range of small angles of θ , to the torque $T_M L$ by the force of jet reaction. That is,

$$\Delta S/l = k T_M L$$

where k is a constant decided from the modulus of rigidity of the steel wire, but in this experiment standard forces T_{M_0} 's are added by hanging weights and from the calibration graph k can be decided. Figures 6 and 7 show the system for calibration. Weights are hung by a thin string from R, and T_{M_0} 's are added through the string PQ by the gravity force mg , where m is the mass of the weight and g is the gravity acceleration. From the balance of forces on the point P, we have $T_{M_0} = \cot \alpha mg$ and $T_M L = T_{M_0} \overline{OS}$, where $\cot \alpha$ and $\overline{OS}$ can be obtained by measuring L_1, L_2, L_3, L_4, L_5 . Calibration curve is given in Fig.8, which shows good proportionality between ΔS and $T_M L$ and this gives the value of $1/kL$, which proved constant throughout this experiment regardless of the temperature variation of atmosphere between 5°C and 30°C.

Fig.9 First normal stress difference of water (27~30°C, 22~25°C)

5. Results

Figures 9-21 give the results obtained. It is a feature that the normal stress is not zero even for water in Poiseuille flow as seen from Figs.9 and 10, which is nearly ten times greater than the shear stress at the shear rate 10^4 1/sec. At the early stage of this study the author (11) (12) believed that the first normal stress difference of water in a steady Poiseuille flow vanishes and used the data of water as the calibration curve. However careful calibration and experiment brought the above mentioned result. Oliver and McSparran (13) presented a similar result for water at the higher shear rates, though the data are rather scattered (Fig.11). Figure 11 also shows an effect of temperature and that the first normal stress dif-

Fig.10 First normal stress difference of water (15~18°C, 5~10°C)

Fig.11 Temperature effect of the first normal stress difference of water and the data of Oliver and McSparran (13)

ference of water increases wholly as the temperature fall.

The normal stresses in various kinds of dilute polymer solutions are given in Figs.12-17, where those of water are given as a reference. The normal stresses in polymer solutions are higher than those of water, but not so dramatically different from water as expected from drag reducing phenomena in turbulent shear flows. Discussion should be made whether these slightly higher normal stresses can be a cause of Toms effect or not. Figures 18 and 19 depict the increase of normal stress of polymer solutions over water in terms of the concentration at the shear rate of 10^4 1/sec. From these graphs the increases are approximately given by the empirical relations:

$$\Delta(\tau_{zz}-\tau_{rr})_w = 540c^{0.55}$$

for PEO 18 solutions;

$$\Delta(\tau_{zz}-\tau_{rr})_w = 290c^{0.48}$$

for Separan AP-30 solutions;

where $\Delta(\tau_{zz}-\tau_{rr})_w$ means the difference between the first normal stress differences of polymer solution and water at the shear rate 10^4 1/sec, and C is the concentration (ppm). That is, $\Delta(\tau_{zz}-\tau_{rr})_w = (\text{First normal stress difference of polymer solution}) - (\text{First normal stress difference of water})$ at $8V/D = 10^4$ 1/sec.

6. Discussion

It will be important to investigate the fact that the first normal stress difference is not zero in the steady shear flow of water and to see whether it is a real fact or an experimental error. As the inner diameter of capillary tube and the distance L_1 , L_2 , etc. are measured precisely and repeatedly, the error may be small.

Fig.12 First normal stress differences of PEO 2ppm water solutions (25-27°C, 15-18°C)

Fig.14 First normal stress differences of PEO 10ppm water solution (4-6°C) and PEO 20ppm water solution (4-6°C)

Fig.13 First normal stress differences of PEO 2ppm water solution (3-5°C) and PEO 5ppm water solution (4-6°C)

Fig.15 First normal stress differences of PEO 40ppm water solution (3-5°C) and PEO 80ppm water solution (3-5°C)

Fig. 16 First normal stress differences of Separan 20ppm water solution (23~26°C, 16~18°C)

Fig. 17 First normal stress differences of Separan 40ppm water solution (14~16°C) and Separan 80ppm water solution (14.5~18°C)

Fig. 18 $\Delta(\tau_{zz} - \tau_{rr})_w$ vs. c ppm ($8V/D = 10^4$ 1/s) in the case of PEO water solutions

Fig. 19 $\Delta(\tau_{zz} - \tau_{rr})_w$ vs. c ppm ($8V/D = 10^4$ 1/s) in the case of Separan water solutions

Fig. 20 First normal stress differences of distilled water (2~3°C) and pure water (6°C)

Fig. 21 Measured data on the first normal stress difference of water when a brim was attached to the tube exit

Firstly, one may question if some ions or little particles contained in the city water play the same role as polymer additives and give the normal stress. To answer it the same experiment was made for pure water and for distilled water. The results are given in Fig.20. The data taken are almost the same as those on the city water, and therefore, ion and impurities in the city water, if they really exist, do scarcely affect the result.

Secondly some circulating flow caused by the submerged jet in the water tank may exert additional forces to the capillary tube and give an apparent normal stress effect on water. To check this, an experiment was made with the capillary placed at various positions in the water tank. As a result, however, there is no appreciable difference observed between the results. Furthermore to examine the secondary flow around the tube exit a brim or plate is attached to the tube exit (see Fig.21), in imitation of a very thick wall tube. In this case, too, the measured result is almost the same as one without a brim (Fig. 21). These additional experiments show that the secondary flow in the water tank eventually gives no appreciable effect on the result.

Thirdly, considerations should be made on the assumption of the velocity profile of Poiseuille flow at the tube exit. Reynolds number under experiment ranges from 170 to 1000, and the flow is laminar. The tube length is about 300 times greater than the tube diameter (Table 1) and at a large distance from the inlet the velocity profile is considered as fully developed, while at the tube exit, one may be unable to answer the question if Poiseuille flow is realized, without measuring a real velocity profile. In reality, however, its measurement is too difficult because of the small size of the tube used. Then, we consider the effect of a change from Poiseuille flow at the tube exit on the final results. Now, we represent the velocity profile of Poiseuille flow as

$$v=2V(1-t^2)$$

where $t=r/R$. Provided v changes by Δv and becomes v' at the tube exit,

Fig.22 Variation of velocity profile

$$v'=v+\Delta v$$

where, so as to satisfy the continuity equation $\int_0^1 \Delta v dt=0$ and the boundary condition $\Delta v|_{t=1}=0$, Δv is assumed as

$$\Delta v=C(3t^4-4t^2+1)$$

and C as a constant, then we get

$$v'=v+\Delta v=2V(1-t^2+\varepsilon(3t^4-4t^2+1))$$

in which $\varepsilon=C/2V$, and it gives the ratio of change of the center velocity (Fig.22). In this case the changed momentum T' under the velocity profile v' becomes

$$T'=\int_0^1 2\pi\rho v'^2 t dt = \frac{\pi R^2 (2V)^2}{30} (4\varepsilon^2 + 5\varepsilon + 10)$$

Taking ε as a parameter, we calculate $(\tau_{zz}-\tau_{rr})_w$ by the equation (7) in which T is replaced by T' . Table 2 shows at what value of ε , $T'-T_M=0$, namely, $(\tau_{zz}-\tau_{rr})_w=0$ under the data of T_M and V which were taken in the experiment of water. According to this, in the case $D=1.608$ mm a change of 3-4 % in the center velocity yields $T'-T_M=0$, whereas in the case $D=0.404$ mm $T'-T_M=0$ corresponds to the change of the center velocity of 40 % (negative value of ε means that the flow pattern approaches a turbulent one). However such a large change of velocity profile cannot be imagined at the tube exit. Therefore, the non-zero first normal stress difference of water in steady shear flow may not be explained by a change of velocity profile at the tube exit. In this connection, a recent measurement by laser reveals that the velocity profile of a fully developed

Table 2 Data of water (Fig.10, 5-10°C) and the value of ε

D cm	V cm/s	8V/D 1/s	$T-T_M$ dyn	$(\tau_{zz}-\tau_{rr})_w$ dyn/cm ²	$(T'-T_M)$ $\varepsilon=0$
0.0404	64.67	12.8×10^3	1.00	1.41×10^3	-0.423
	77.54	15.4	1.20	1.69	-0.311
	103.1	20.4	1.54	2.16	-0.202
	107.3	21.3	2.11	2.97	-0.275
	132.9	26.3	2.77	3.89	-0.223
	185.0	36.7	4.42	6.21	-0.176
	230.7	45.7	6.67	9.37	-0.170
	273.7	54.2	8.79	12.4	-0.157
0.0728	35.08	3.85	0.39	0.17	-0.127
	51.75	5.69	0.71	0.31	-0.104
	67.00	7.36	1.24	0.540	-0.109
	121.6	13.4	3.70	1.62	-0.0980
	181.9	17.9	6.70	2.94	-0.0778
	218.9	24.1	7.95	3.49	-0.0630
0.1064	294.5	32.3	13.8	6.05	-0.0602
	52.98	3.98	1.04	2.17×10^2	-0.0660
	62.71	4.72	1.37	2.37	-0.0618
	73.16	5.50	2.19	4.59	-0.0733
	85.78	6.45	2.39	5.00	-0.0574
	95.94	7.21	2.94	6.16	-0.0564
0.1608	105.5	7.93	3.31	6.93	-0.0524
	41.01	2.05	0.867	0.792	-0.0393
	48.07	2.40	1.14	1.04	-0.0376
	63.37	3.16	1.75	1.60	-0.0331
	70.31	3.51	2.00	2.00	-0.0306
	76.96	3.84	2.58	2.36	-0.0330
0.1608	86.51	4.32	3.09	2.82	-0.0313

laminar flow in tube is also attained near the tube exit(14).

From these discussion the measured non-zero value of the first normal stress difference of water in steady shear flow conditions seems unable to be regarded as an experimental error, but is considered as a real physical phenomenon. This means that the flow of water is not always described by Navier-Stokes equation, but should be taken as a non-Newtonian fluid flow in certain circumstances. Of course this is a very important result and the conclusion should not be drawn hastily by this research only. Theoretical and experimental investigations in future will give a final answer to this problem.

7. Conclusion

The data and the following discussion showed that the first normal stress difference is not zero even in the steady shear flow of water and that the first normal stress differences in various dilute polymer solutions are greater than that of water but not so strikingly different from that of water as expected from the dramatic drag reducing effect in turbulent shear flows. These experimental data will be of fundamental use for the construction of constitutive equation or the analysis of more complicated flows.

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