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Underwater Imaging System using Multi-frequency Ultrasonic Holography*

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This paper describes an underwater imaging system using multi-frequency ultrasonic holography. This imaging system has 11X11 receiving array and reconstructs the image by holographic calculation.

In conventional mono-frequency holographic system, undesirable ultrasonic refraction from non-target objects degrades the quality of image. To prevent this image degradation, the authors propose a method of multi-frequency ultrasonic holography. It prevents image degradation not by an increase in the number of receivers but by an increase in the number of holograms formed by different ultrasounds. A good quality image can be obtained from a synthesized hologram that is vector summation of the phase-corrected holograms. The reduction rate of undesirable ultrasound is estimated to be $1/\sqrt{M}$, where M is the number of different frequency ultrasounds. The upper and the lower limits of applicable frequency bandwidth are presented. Finally the prevention of image degradation is demonstrated by experiments in the water.

1. Introduction

Many ultrasonic imaging researches have been conducted for underwater observation, because acoustical imaging system can be utilized even in turbid water⁽¹⁾⁻⁽⁶⁾. This paper describes an acoustical holographic imaging system which has a receiving array and reconstructs the image using computer calculation.

By use of this imaging method, it will be not so difficult to get a good quality image, if the system has a large number of receivers. But this is not practical. When the number of receivers is limited for practical use, the undesirable ultrasound refracted from non-target objects degrades the quality of image.

To prevent this image degradation, the authors propose an imaging method of multi-frequency ultrasonic holography. The improvement of image quality by this method is analyzed and demonstrated by experiments in the water.

2. The ultrasound holographic imaging using computer image reconstruction

Figure 1 shows the imaging method proposed by authors. This system obtains the image with the following procedures. First, a target is illuminated with 250 kHz ultrasound, and the refracted sound is received by the receiving array which has 11X11 re-

ceiving elements. The receivers are placed 4 cm apart on the array. We wanted to use 16X16 receiving elements, but from economic consideration of research fund, the 11X11 receiving array has been used. Then the hologram is calculated from received signal. The image of target is displayed after the image reconstruction on a typewriter by star marks. This reconstructing calculation utilizes the fast Fourier transformation (FFT).

For the following reasons we selected this method. First, this method is able to reconstruct the image sensitively, because this system utilizes resonant piezo-

Fig. 1 Schematic diagram of imaging system

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electric elements for receiving array which have high sensitivity to signal detection and do not absorb a reflected sound signal like acoustical lenses. Second, the lateral resolving power can be easily improved by expanding the interval of receiving elements and changing the parameters of calculation algorithm. Third, the FFT enables us to reconstruct the image in a short time. Fourth, the technology of digital image processing is applicable to improvement of image quality, because in this method the image is reconstructed by computer software which can easily be changed.

The quality of image can be analyzed as follows. The lateral resolution (the resolution of x or y direction in Fig.1 which is determined by the minimum distance to separate two target placed close to each other) becomes⁽⁷⁾

$$\delta = \frac{\lambda D}{2a} \approx 1.5 \text{ cm} \quad (1)$$

The range resolution (the resolution along z-direction in Fig.1 which is determined at 80% level of maximum amplitude) is

$$\delta' = 2\lambda \left(\frac{D}{2a} \right)^2 \approx 7.5 \text{ cm} \quad (2)$$

The range of view L (the repeating period of the higher diffracted image caused by the spacial sampling on the receiving array) is

$$L = \frac{\lambda D}{\Delta} \approx 15 \text{ cm} \quad (3)$$

The figures given in Eqs. (1), (2) and (3) are the values at 1 m distance from the imaging system. The symbols are determined as follows.

- λ ; the wavelength of ultrasound (0.6 cm, at about 250 kHz)
- D ; the distance between the imaging system and the target (100 cm)
- $2a$; the length of aperture (40 cm)
- Δ ; the interval of receiving elements (4 cm)

R represents the ratio of undesirable ultrasonic maximum amplitude to that of reflected signal.

Fig. 2 Results of computer simulations; degradation of reconstructed image caused by undesirable ultrasound.

3. Undesirable ultrasound and degradation of the image

Figure 2 shows the degradation of the reconstructed image caused by undesirable ultrasound in conventional holographic imaging. In this computer simulation, the target has the shape of "H". The value R is the ratio of undesirable ultrasound amplitude to maximum detected sound signal amplitude without undesirable ultrasound. The amplitude and the phase angle of undesirable ultrasound are assumed to be constant over receiving elements.

Figure 2 shows that the image is heavily degraded under the condition of R being over 20%, and by decreasing this value to below 10%, the quality of image can be improved. In this paper, we propose the method of "multi-frequency ultrasonic holography" which realizes good images by diminishing the influence of undesirable ultrasound.

4. Multi-frequency ultrasonic holography

The multi-frequency ultrasonic holography is an imaging method which reconstructs the image from a single hologram obtained by superimposing phase-corrected holograms formed different frequency ultrasounds.

The imaging procedures of this method are as follows. Considering the reduction rate of undesirable ultrasound and the easiness of experiments, we utilize three holograms to obtain the image in this paper. The algorithm of this method consists of the following steps. To avoid the complexity, we explain it by the one dimensional model of Fig.3.

(1) 1st step

The target is illuminated by three kinds of different ultrasounds i.e. f_0 , $f_1 = (1+\alpha)f_0$ and $f_{-1} = (1-\alpha)f_0$, where α is a small shift of frequency ($\alpha \ll 1$) and D is the distance between the imaging system and the target T(x). Three kinds of reflected ultrasounds are received by receiving elements and from these received signals the computer calculates three holograms $H_j(\xi)$ ($j=-1,0,1$). Let $k_j = 2\pi f_j/c$ be the phase constant of frequency f_j , and the hologram is expressed as follows.

Fig. 3 One-dimensional imaging model

$$H_j(\xi) = C \int_{-\infty}^{\infty} T(x) \exp\{-ik_j \sqrt{(x-\xi_0)^2 + D^2}\} \\ \times \exp\{-ik_j \sqrt{(x-\xi)^2 + D^2}\} dx \\ (j = -1, 0, 1) \dots \dots \dots (4)$$

In Eq.(4), C is a constant and two exponential terms express the phase shift caused by ultrasound transmitted between target and receiving array. Considering the fact that $\alpha \ll 1$, $k_j = (1+j\alpha)k_0$, Eq.(4) can be approximated by Eq.(5).

$$H_j(\xi) \approx C \exp(-2ik_0 D) \exp(2ijk_0 \alpha D) \\ \times \int_{-\infty}^{\infty} T(x) \exp\left\{-\frac{ik_0}{2D} (x-\xi_0)^2\right\} \\ \times \exp\left\{-\frac{ik_0}{2D} (x-\xi)^2\right\} dx \dots \dots \dots (5)$$

The second term of Eq.(5) $\exp(2ijk_0 \alpha D)$ indicates a phase shift that is caused by the frequency shift from standard ultrasonic frequency f_0 . This phase shift is corrected before image reconstruction. Equation (5) shows that the amplitude distribution of hologram is the Fresnel Transformation of target $T(x)$.

(2) 2nd step

After the phase correction, three holograms obtained are superimposed into a single hologram. Equation (6) shows his procedure.

$$H(\xi) = \exp(ik_0 \alpha D) H_1(\xi) + H_0(\xi) \\ + \exp(-ik_0 \alpha D) H_{-1}(\xi) \dots \dots \dots (6)$$

Target, object, transmitter and receiver are represented by the points on z-axis, T, T', and H.

Fig. 4 Position of target, object, transmitter and receiver

$$\approx 3C \exp(-2ik_0 D) \\ \times \int_{-\infty}^{\infty} T(x) \exp\left\{-\frac{ik_0}{2D} (x-\xi_0)^2\right\} \\ \times \exp\left\{-\frac{ik_0}{2D} (x-\xi)^2\right\} dx \dots \dots \dots (7)$$

Equation (7) also shows that the amplitude distribution of hologram $H(\xi)$ is the Fresnel Transformation of target $T(x)$. So, the calculation of inverse transformation of this hologram reconstructs the image of the target.

(3) 3rd step

After the inverse Fresnel Transformation of $H(\xi)$, the following calculation is performed to get the amplitude of the image $|T'(p)|$.

$$|T'(p)| = \left| \int_{-\infty}^{\infty} H(\xi) \exp\left\{i \frac{k_0}{2D} (\xi-p)^2\right\} d\xi \right| \dots \dots \dots (8)$$

$$= |3CT(p) \exp(-2ik_0 D) \\ \times \exp\left\{\frac{ik_0}{2D} (p-\xi_0)^2\right\}| \\ = |3CT(p)| \dots \dots \dots (9)$$

Equation (9) shows that the amplitude of image is proportional to the reflexivity of target $T(p)$.

The steps stated above are the algorithm of multi-frequency ultrasonic holography.

5. Reduction of undesirable ultrasound and the limitation of frequency shift

5-1 Reduction rate of undesirable ultrasound

The reduction rate of undesirable ultrasound by use of the multi-frequency ultrasonic holography can be expected as follows (see Fig.4).

Let D be the distance between the receiving array H and a target indicated by T, and D' be the distance between the receiving array and a non-target object, where the object is supposed to be expressed by a point on the z-axis. Let $(1-\alpha)k_0$, k_0 and $(1+\alpha)k_0$ be the phase constants of three kinds of ultrasounds, and the holograms formed from these different frequency ultrasounds are superimposed after phase-correction by Eq.(6), and the image is computed by Eq.(8).

Tab. 1 Phase shift in multi-frequency ultrasonic holography

(a) Reflected ultrasound from target						
k_j	path	H to T, T to H	phase correct.	image rech.	summation	diff.
$(1+\alpha)k_0$		$-2(1+\alpha)k_0 D$	$2\alpha k_0 D$	$k_0 D$	$-k_0 D$	> 0
k_0		$-2k_0 D$	0	$k_0 D$	$-k_0 D$	> 0
$(1-\alpha)k_0$		$-2(1-\alpha)k_0 D$	$-2\alpha k_0 D$	$k_0 D$	$-k_0 D$	
phase deviation $\phi_T = 0$						
(b) Reflected ultrasound from non-target object						
k_j	path	H to T, T to H	phase correct.	image rech.	summation	diff.
$(1+\alpha)k_0$		$-2(1+\alpha)k_0 D'$	$2\alpha k_0 D$	$k_0 D$	$-2(1+\alpha)k_0 D' + (2\alpha+1)k_0 D$	$> \phi_{T'}$
k_0		$-2k_0 D'$	0	$k_0 D$	$-2k_0 D' + k_0 D$	$> \phi_{T'}$
$(1-\alpha)k_0$		$-2(1-\alpha)k_0 D'$	$-2\alpha k_0 D$	$k_0 D$	$-2(1-\alpha)k_0 D' + (-2\alpha+1)k_0 D$	
phase deviation $\phi_{T'} = 2\alpha k_0 (D - D')$						

Considering only the change of phase of ultrasound, this procedure is expressed by Table 1. Table 1(a) shows that the image of the target on which we focus has no phase deviation because of phase-correction. On the other hand, the image of the unfocused object T' has a phase deviation, because the distance between this object and the imaging system is different from the one between the target and the imaging system.

In order to simplify the explanation, we neglect the range resolution and assume amplitude to be equal to 1 at any distance. Further consideration about this assumption will be made later.

The amplitude of the image n on which we focus become 3 by simply superimposing three images, because no phase deviation occurs. On the other hand, the amplitude of the image on which we do not focus becomes less than 3 because of phase deviation. Equation (10) represents this.

$$\begin{aligned} |n| &= |1 + (\cos \phi_{T'} + i \sin \phi_{T'}) \\ &\quad + \{\cos(-\phi_{T'}) - i \sin(\phi_{T'})\}| \\ &= |1 + 2 \cos \phi_{T'}| = |1 + 2 \cos \{2\alpha k_0(D-D')\}| \end{aligned} \quad \dots\dots\dots(10)$$

Varying $\phi_{T'}$ from 0 to 2π , the amplitude of reconstructed image changes like Fig. 5(b). The mean square value of amplitude of this image becomes

$$\int_0^{2\pi} |n|^2 d\phi_{T'} = \int_0^{2\pi} (1 + 2 \cos \phi_{T'})^2 d\phi_{T'} = \sqrt{3}$$

Therefore, if there are only one target and only one non-target object, the image from one hologram becomes 1 and the one from three holograms becomes $\sqrt{3}/3$. If we assume that probability of existence of non-target object is constant from 0 to 2π by phase, the reduction rate will be expected to be $\sqrt{3}/3$.

In case of multi-frequency ultrasonic holography utilizing M holograms, the reduction rate is expected to be $\sqrt{M}/M$.

In the consideration stated above, we assumed the amplitude of the image of target to be equal to 1 at any distance. But the amplitude of the image of an object placed out of focus becomes less than 1 as illustrated in Fig. 5(b). Therefore, the reduction rate of undesirable ultrasound becomes less than $\sqrt{M}/M$. In addition, The range resolution given by Eq.(2) is the distance between the position of maximum amplitude and that of 80% level in Fig. 5(a)

5-2 Improvement of range resolution

Equation (10) illustrated by Fig. 5(b) shows that n becomes zero when $\phi_{T'} = 2\alpha k_0(D'-D) = (2/3)\pi$. At point B the condition is satisfied. This fact means that the procedure of image reconstruction focused on D erases the image of an object placed at the distance D' [$D + (\lambda/6\alpha)$]. The reverse is also true. That is, the focusing on an object at D' erases the image of an object placed at the distance D . This fact enables us to separate two images of the objects placed at the distances D and D' . Even if the two objects are placed closer than the system range resolution, it is possible to separate their images by setting $\alpha = \lambda/6(D'-D)$. Therefore, it can be said that the imaging method of multi-frequency ultrasonic holography can also improve the range resolution.

(a) Range resolution of conventional holographic imaging

(b) Range resolution of multi-frequency ultrasonic holography imaging

Fig. 5 Range resolution of reconstructed image

5-3 Limitation of frequency shift

In the multi-frequency ultrasonic holography, the image is reconstructed from one hologram by use of standard frequency, although this hologram is constructed by superimposing many holograms of different frequencies. This approximation limits the range of frequency shift.

From Eqs.(5) and (7), Eq.(11) which represents the phase error caused by the approximate calculation of Eq.(7) can be derived neglecting the higher order errors.

$$\Delta\phi = \alpha \frac{k_0}{2D} (x - \xi)^2 + \frac{\alpha k_0}{2D} x^2 \dots\dots\dots(11)$$

This phase error becomes maximum when the right/left end point of the view field reflects the ultrasound transmitted from the transmitter placed at the center of receiving array and left/right end point of the receiving array receives it. This condition is given by Eq.(12).

$$\begin{cases} x = \frac{\lambda D}{2a}, & \xi = -\frac{\lambda N}{2} \\ x = -\frac{\lambda D}{2a}, & \xi = \frac{\lambda N}{2} \end{cases} \dots\dots\dots(12)$$

Substituting Eq.(12) into Eq.(11), the upper and the lower limits of frequency shift are given as Eq.(13)

$$|\alpha| < \frac{\beta \lambda D}{\pi \left\{ \left(\frac{\lambda D}{2a} + \frac{\lambda N}{2} \right)^2 + \left(\frac{\lambda N}{2} \right)^2 \right\}} \dots\dots\dots(13)$$

where β is allowable limit of phase error.

If we determine the value of β equal to π , where the sign of received signal amplitude is reversed, we obtain $|\alpha| < 5.3\%$ from Eq.(13).

Figure 6 shows the change of image obtained by computer simulation when we vary the frequency shift of utilizing ultrasound. In Fig.6, the abscissa represents the coordinate of target and the ordinate represents the amplitude of the image.

The amplitude of the target which is placed at 1 m distance from the imaging system has a square shape described by a solid line. Dot line represents the image of this system derived from computer simulation. The images are fairly distorted from the shape of target when α is larger than 5.3%. In the above computer simulations, it is decided that the target lies at the place where the amplitude of reconstructed image is over the threshold level represented by a horizontal chain line. From Fig. 6, the upper and the lower limitations given by Eq.(13) are reasonable.

Fig. 6 Results of computer simulation : Frequency shift and the image

6. Underwater imaging experiments

6-1 Preliminary experiment

The receiving array is composed of piezo-electric lead zirconate elements of 13 mm diameter the resonant frequency of which is about 250 kHz. Among many variances of the characteristics of receiving elements, the phase variance degrades the image most heavily. In order to evaluate the image degradation, the following preliminary experiment and computer simulation are performed.

The transmitter placed two meters ahead of the center of receiving array projects a sinusoidal wave of 249.49 kHz to the center of the receiving array. And each receiving element detects the phase of the signal. Then the differences between detected phase and theoretical phase are calculated. Figure 7 illustrates this difference (errors). In Fig. 7, the ordinate represents the number of elements. Assuming this error distribution to be normal distribution illustrated by a solid line, the standard deviation is estimated from this figure to be $\sigma=25^\circ$. In order to evaluate the degradation of image caused

Fig. 7 Phase error distribution of receiving elements

Fig. 8 Results of computer simulation: Reconstructed image using the imaging system which has the same phase detection error as this experimental system.

by the phase detection error, the computer simulation is performed. In this simulation, the distribution of the phase errors of receiving elements is assumed to be a normal distribution of 25° standard deviation. The image of shape "H" obtained from computer simulation placed 1 m ahead of the receiving array is illustrated in Fig. 8(b). Figure 8(a) is the target of this image.

In this paper, the images are typed by the following procedure. First, the image obtained is normalized so that the maximum amplitude of image becomes 1000. Then the pixels over 433 are represented by the characters "****", and the pixels below 433 are represented by space code.

6-2 Underwater imaging experiment using multi-frequency ultrasonic holography

Figure 9 shows a schematic diagram of our experimental system. A sinusoidal wave generated by the oscillator is amplified and converted to ultrasonic sound wave by the transmitter. The reflected sound wave from a foamed styrol target placed 1 m ahead of the imaging system is detected by the receiving array. The transmitter is placed at the center of the receiving array. And Fig. 8(a) illustrates the shape of the target. The amplitude and the phase of received signal are measured by a voltmeter and a phase detector. The outputs from these instruments are converted to digital number by an 8 bit A/D converter and stored in a mini-computer (Hitac 10 II) sequentially. After repeating this procedure three times varying the frequency of ultrasound (243.77 kHz, 246.49 kHz, 247.21 kHz) each time, the calculation of image reconstruction is performed. Figure 10(a) represents the image obtained from only one ultrasound (246.49 kHz) hologram (conventional holographic imaging). The improvement of image quality can be seen by comparing these two images.

Fig. 9 Schematic diagram of experimental underwater imaging system

6-3 A separation experiment of
two objects placed within
the range resolution

To confirm the improvement of range resolution, a separation experiment is performed. In this experiment two objects (targets) are placed at distances 110 cm and 120 cm ahead of the imaging system respectively. The distance between two targets is within the range resolution of this experimental imaging system. Figure 11 represents the results. Applying the multi-frequency ultrasonic holography, we can obtain only the image of the target placed at the focused distance separately. On the other hand, a conventional holographic image reconstruction cannot separate these two objects as shown in Fig. 11(c).

(a) Reconstructed image using conventional holography

(b) Reconstructed image using multi-frequency ultrasonic holography

Fig. 10 Results of underwater imaging experiments

(a) Reconstructed image focused on the left "I" shaped target using multi-frequency ultrasonic holography.

(b) Reconstructed image focused on the right "I" shaped target using multi-frequency ultrasonic holography.

(c) Reconstructed image focused on the right "I" shaped target using conventional ultrasonic holography.

Fig. 11 Separation experiments of two targets placed within the range resolution of conventional ultrasonic holography

7. Conclusions

An underwater imaging method by use of a multi-frequency ultrasonic holography has been proposed. From experiments, we have reached the following conclusions.

- (1) This method prevents image degradation caused by undesirable ultrasound through vector summation of many phase-corrected holograms.
- (2) The method also decreases the range resolution.
- (3) The upper and the lower limits of applicable frequency shift are presented.
- (4) The prevention of image degradation and the improvement of resolution are demonstrated by experiments in the water and computer simulation.

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