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Turbulent Lubrication Theory Using the Frictional Law*

(2nd Report, Its Application to Journal Bearings)

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In this paper, an application of the turbulent lubrication theory developed in the first paper to the finite width journal bearings is presented.

Bearing performances such as film pressure, load capacity, frictional force and rate of flow in turbulent flows are evaluated by the numerical analysis. The effects of pressure flows on the performances not previously encountered are considered from the analytical results, and are discussed comparing with the published papers.

Also, an experiment in a range of high eccentricities and of the mean Reynolds numbers up to 5000 is conducted to compare with the theoretical results. From agreement of the experimental results with the analytical ones, the validity of the theory is confirmed.

1. Introduction

A number of studies on turbulent lubrication have been published to data, but there are few papers⁽¹⁾⁻⁽⁴⁾ discussing in detail the effect of turbulent lubricant film on the performances of finite width journal bearings such as film pressure, load capacity and frictional force. The effect of pressure gradients in the lubricant film on the viscosity of lubricants in the transition from laminar to turbulent flows has not been presented except in the report by Elrod⁽⁵⁾ discussing the effect of pressure gradients on the load capacity.

The authors have derived a generalized turbulent lubrication equation based on the friction law of fluid and clarified the effect of pressure gradients on the turbulent coefficients in the first paper⁽⁶⁾.

In this paper, applying the above mentioned lubrication equation to finite width journal bearings, the turbulent effects on the bearing performances are investigated theoretically, and comparing the theoretical results of film pressure with the experimental ones, it is confirmed that this theory is available for the turbulent lubrication.

2. Nomenclature

C : radial clearance
D : diameter of journal
F : frictional force
 G_θ , G_z : turbulent coefficients

L : bearing width
N : revolution per unit time
P : bearing pressure
Q : rate of flow
R : radius of journal
 R_e : mean Reynolds number
 R_h : local Reynolds number
S : Sommerfeld number
U : circumferential velocity of journal
W : load capacity
e : eccentricity
f : frictional coefficient
h : film thickness
p : film pressure
 u_m , w_m : mean velocities in x, z-directions
x, y, z : coordinates
 α_i , β : inherent constants for a range of Reynolds numbers
 ϵ : eccentricity ratio
 θ : bearing angle
 λ : slenderness ratio
 μ : viscosity
 ν : kinematic viscosity
 τ : shear stress
 ϕ : attitude angle
Subscripts
s, e, k : refer to initial, final and iterative values in computing
e : refer to equivalent pressure flow
c, p : refer to shear flow and pressure flow
j, b : refer to journal and bearing
- : nondimensional quantity

3. Theoretical analysis

By applying the turbulent lubrication equation to journal bearings, the bearing performances such as film pressure, load capacity and frictional force in turbulent flows are analyzed theoretically. The nondimensional quantities used in the analysis are listed in Table 1.

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Table 1 Nondimensional quantities

$\bar{\theta} = z/R$	$\bar{p} = C^2 \bar{p} / \mu UR$
$\bar{z} = z/L$	$\bar{\tau} = C\tau / \mu U$
$\bar{h} = h/C$	$\bar{W} = C^2 W / \mu ULR^2$
$\epsilon = e/C$	$\bar{F} = CF / \mu ULR$
$\lambda = L/D$	$f = \frac{R}{C} \frac{F}{W}$
$Re = CU/\nu$	$\bar{Q} = Q/CUR$
$R_h = \frac{2h}{\nu} \sqrt{u_m^2 + w_m^2}$	

3.1 Analysis of the lubrication equation

The turbulent lubrication equation for lubricant flows in finite width journal bearing as shown in Fig.1 is given by

$$\frac{\partial}{\partial \theta} (G_\theta \bar{h}^3 \frac{\partial \bar{p}}{\partial \theta}) + \frac{1}{4\lambda^2} \frac{\partial}{\partial \bar{z}} (G_z \bar{h}^3 \frac{\partial \bar{p}}{\partial \bar{z}}) = \frac{1}{2} \frac{\partial \bar{h}}{\partial \theta} \quad (1)$$

where $\bar{h} = 1 + \epsilon \cos \theta$, and the turbulent coefficients G_θ and G_z are indexes of flow mechanism in turbulent lubrication, which are given by the following expressions when both pressure and shear flows are considered.

$$\left. \begin{aligned} (1+aG_\theta) \{ (1+aG_\theta)^2 + (bG_z)^2 \}^{(\beta+1)/2} \\ - (1-aG_\theta) \{ (1-aG_\theta)^2 + (bG_z)^2 \}^{(\beta+1)/2} + c = 0 \\ G_z \{ (1+aG_\theta)^2 + (bG_z)^2 \}^{(\beta+1)/2} \\ + \{ (1-aG_\theta)^2 + (bG_z)^2 \}^{(\beta+1)/2} + d = 0 \end{aligned} \right\} \quad (2)$$

where

$$\begin{aligned} a &= 2\bar{h}^2 \frac{\partial \bar{p}}{\partial \theta}, \quad b = \frac{\bar{h}^2}{\lambda} \frac{\partial \bar{p}}{\partial \bar{z}}, \\ c &= -\frac{8}{\alpha_p} \bar{h}^{1-\beta} R_h^{-(\beta+1)} \frac{\partial \bar{p}}{\partial \theta}, \quad d = -\frac{4}{\alpha_p} (\bar{h} R_h)^{-(\beta+1)} \end{aligned}$$

Under an operating condition of low eccentricity, shear flows are dominant as compared with pressure flows, then Eq.(2) can be simplified as follows,

$$\left. \begin{aligned} a_1 G_\theta^3 + a_2 G_\theta^2 G_z + a_3 G_\theta + a_4 &= 0 \\ b_1 G_z^3 + b_2 G_\theta^2 G_z + b_3 G_z + b_4 &= 0 \end{aligned} \right\} \quad (3)$$

where

$$\begin{aligned} a_1 &= \frac{\beta(\beta+1)(\beta+2)}{3} \left(\bar{h}^2 \frac{\partial \bar{p}}{\partial \theta} \right)^2, \\ a_2 &= \beta(\beta+1) \left(\frac{\bar{h}^2}{\lambda} \frac{\partial \bar{p}}{\partial \bar{z}} \right)^2, \quad a_3 = \frac{\beta+2}{2}, \\ a_4 &= -\frac{1}{\alpha_p} (\bar{h} R_h)^{-(\beta+1)}, \quad b_1 = (\beta+1) \left(\frac{\bar{h}^2}{\lambda} \frac{\partial \bar{p}}{\partial \bar{z}} \right)^2, \end{aligned}$$

Fig.1 Geometry of a journal bearing

$$b_2 = \beta(\beta+1) \left(\bar{h}^2 \frac{\partial \bar{p}}{\partial \theta} \right)^2, \quad b_3 = \frac{1}{2},$$

$$b_4 = -\frac{1}{\alpha_p} (\bar{h} R_h)^{-(\beta+1)}$$

We can obtain the film pressure by solving Eq.(2) or (3) for the turbulent coefficients and the lubrication equation(1) simultaneously. The procedure of numerical analysis is shown in the following.

Since Eq.(1) is a nonlinear partial differential equation with respect to $\bar{p}$, it is rewritten as a difference equation for numerical solution. Then, we have

$$\begin{aligned} \bar{p}_{i,j}^{(K)} &= c_1 + c_2 \bar{p}_{i+1,j}^{(K-1)} + c_3 \bar{p}_{i-1,j}^{(K-1)} \\ &+ c_4 \bar{p}_{i,j+1}^{(K-1)} + c_5 \bar{p}_{i,j-1}^{(K-1)} \quad (4) \end{aligned}$$

where $\bar{p}_{i,j}$ is a film pressure at the i -th and j -th grid points in the θ - and z -directions respectively, and the coefficients C_{1-5} are the functions of $\lambda, \epsilon, G_{\theta i,j}^{(K-1)}, G_{\theta i+1,j}^{(K-1)}, G_{\theta i-1,j}^{(K-1)}, G_{\theta i,j+1}^{(K-1)}, G_{\theta i,j-1}^{(K-1)}, G_{\theta i,j}^{(K-1)}, G_{\theta i+1,j}^{(K-1)}, G_{\theta i-1,j}^{(K-1)}, G_{z i,j+1}^{(K-1)}$ and $G_{z i,j-1}^{(K-1)}$.

From Eq.(4), we can determine the k -th iterative value $\bar{p}^{(K)}$ by using the previously determined $(k-1)$ -th iterative value $\bar{p}^{(K-1)}$ and the coefficients C_{1-5} , and the turbulent coefficients $G_\theta^{(K)}$ and $G_z^{(K)}$ are calculated by using $\bar{p}^{(K)}$. The simultaneous equation(2) or (3) for the turbulent coefficients can be replaced in general by the following difference equations.

$$\left. \begin{aligned} f(G_{\theta i,j}^{(K)}, G_{z i,j}^{(K)}, \alpha_p, \beta, \lambda, \epsilon, R_h, \bar{p}_{i+1,j}^{(K)}, \\ \bar{p}_{i-1,j}^{(K)}, \bar{p}_{i,j+1}^{(K)}, \bar{p}_{i,j-1}^{(K)}) &= 0 \\ g(G_{\theta i,j}^{(K)}, G_{z i,j}^{(K)}, \alpha_p, \beta, \lambda, \epsilon, R_h, \bar{p}_{i+1,j}^{(K)}, \\ \bar{p}_{i-1,j}^{(K)}, \bar{p}_{i,j+1}^{(K)}, \bar{p}_{i,j-1}^{(K)}) &= 0 \end{aligned} \right\} \quad (5)$$

The coefficients α_p and β in the above equations vary with the turbulent flow conditions and take values as shown in Table 2 according to the following local Reynolds number.

$$\begin{aligned} R_h &= \frac{2h}{\nu} \sqrt{u_m^2 + w_m^2} \\ &= \bar{h} R_h \sqrt{\left(1 - 2G_\theta \bar{h}^2 \frac{\partial \bar{p}}{\partial \theta} \right)^2 + \left(\frac{1}{\lambda} G_z \bar{h}^2 \frac{\partial \bar{p}}{\partial \bar{z}} \right)^2} \quad (6) \end{aligned}$$

Therefore, it is necessary to solve Eq.(5) by applying the Newton-Raphson iterative method to evaluate the coefficients G_θ and G_z , in which α_p and β are revised to suit each condition of turbulent flows.

Based on such a procedure, solving iteratively the film pressure and the turbulent coefficients simultaneously to satisfy the following condition:

$$\sum_i \sum_j |\bar{p}_{i,j}^{(K)} - \bar{p}_{i,j}^{(K-1)}| / \sum_i \sum_j |\bar{p}_{i,j}^{(K)}| \leq 0.01\%$$

we can obtain the final values of film pressure and turbulent coefficients. In the calculations, half Sommerfeld boundary condition is used, and the number of grid points over the bearing surface is 12×12 . The flow chart of the iterative procedure mentioned

Table 2 Experimental constants α_i, β and κ

Constants	Flow regimes		
	$R_h < 10^3$	$10^3 \leq R_h < 1.35 \times 10^4$	$1.35 \times 10^4 \leq R_h$
α_p	24	0.4370	0.0791
α_c	8	0.3120	0.0637
β	-1	-0.43	-0.25
κ	3	1.40	1.24

above is shown in Fig.2.

3.2 Bearing performances

From the equilibrium of the film pressure and the applied load, we have

$$\left. \begin{aligned} \bar{W} \sin \phi &= \int_0^1 \int_0^{2\pi} \bar{p} \sin \theta d\theta dz \\ \bar{W} \cos \phi &= - \int_0^1 \int_0^{2\pi} \bar{p} \cos \theta d\theta dz \end{aligned} \right\} \dots\dots\dots(7)$$

The load capacity, the Sommerfeld number and the attitude angle are given by the following expressions.

$$\left. \begin{aligned} \bar{W} &= \sqrt{(\bar{W} \sin \phi)^2 + (\bar{W} \cos \phi)^2} \\ S &= \frac{\mu N}{P} \left(\frac{R}{C} \right)^2 = \frac{1}{\pi \bar{W}} \\ \phi &= \tan^{-1} \left(\frac{\bar{W} \sin \phi}{\bar{W} \cos \phi} \right) \end{aligned} \right\} \dots\dots\dots(8)$$

In order to calculate the shear stress on the surfaces of journal and bearing, the wall shear stresses due to the equivalent pressure flow presented in the first paper are quoted as follows,

$$\tau_{j,s} = \frac{\bar{h}}{2} \frac{\partial \bar{p}}{\partial \theta} + \kappa \bar{\tau}_{wc} \dots\dots\dots(9)$$

$$\tau_{b,s} = -\frac{\bar{h}}{2} \frac{\partial \bar{p}}{\partial \theta} + \kappa \bar{\tau}_{wc} \dots\dots\dots(10)$$

From the above equations, the wall shear stress $\bar{\tau}_{wc}$ according to shear flows is

$$\bar{\tau}_{wc} = \frac{\tau_{j,s} + \tau_{b,s}}{2\kappa} \dots\dots\dots(11)$$

Rewriting Eq.(11) by using the turbulent coefficients, we have

Fig.2 Procedure of calculation

$$\begin{aligned} \bar{\tau}_{wc} &= \frac{\alpha_s}{16} \bar{h}^2 R_e^{\beta+1} [(1+aG_\theta) \{ (1+aG_\theta)^2 \\ &\quad + (bG_z)^2 \}^{\beta+1/2} \\ &\quad + (1-aG_\theta) \{ (1-aG_\theta)^2 + (bG_z)^2 \}^{\beta+1/2}] \end{aligned} \dots\dots\dots(12)$$

In journal bearings, it seems that large pressure flows occur with thinner film and the shear flows dominate over the film. In such a case, the ratio of the wall shear stress in shear flows to that in pressure flows is $\kappa=1$, then the expression for the shear stresses on the surfaces of journal and bearing from Eqs.(9), (10) and (12) is as follows,

$$\begin{aligned} \tau_{j,s} &= \pm \frac{\bar{h}}{2} \frac{\partial \bar{p}}{\partial \theta} + \frac{\alpha_s}{16} \bar{h}^2 R_e^{\beta+1} [(1+aG_\theta) \{ (1+aG_\theta)^2 \\ &\quad + (bG_z)^2 \}^{\beta+1/2} \\ &\quad + (1-aG_\theta) \{ (1-aG_\theta)^2 + (bG_z)^2 \}^{\beta+1/2}] \end{aligned} \dots\dots\dots(13)$$

where the positive sign refers to the journal and the negative one to the bearing surface.

Considering that the film pressure disappears and the shear stress due to shear flows exists alone in the region $\pi \leq \theta \leq 2\pi$, the frictional force is given by integrating Eq.(13) as

$$F_{j,s} = \frac{\alpha_s}{8} R_e^{\beta+1} \int_{\pi}^{2\pi} \bar{h}^2 d\theta + \int_0^1 \int_0^{2\pi} \tau_{j,s} d\theta dz \dots\dots\dots(14)$$

The frictional coefficients is

$$f_{j,s} = \bar{F}_{j,s} / \bar{W} \dots\dots\dots(15)$$

The rate of flow out of both sides of the bearing is given by integrating the mean velocity in the z-direction, that is

$$\bar{Q} = -2 \int_0^{2\pi} (\bar{h} \bar{w}_m)_{z=0} d\theta = \frac{1}{\lambda} \int_0^{2\pi} \left(G_\theta \bar{h} \frac{\partial \bar{p}}{\partial z} \right)_{z=0} d\theta \dots\dots\dots(16)$$

The integration in Eqs.(7), (14) and (16) is performed by using numerical integrals.

3.3 Analytical results and discussions

Based on the analytical results obtained by the numerical procedure, the bearing performances in turbulent and laminar flows are discussed in which the turbulent coefficients evaluated from Eq.(2) and the slenderness ratio $\lambda=1$ are mainly considered.

The film pressure in the circumferential direction increases in transit from laminar to turbulent flows as shown in Fig. 3 where the rate of increase becomes pronounced as the mean Reynolds number increases. This is due to an increase of apparent

Fig.3 Circumferential pressure distribution

viscosity which is caused by a decrease of turbulent coefficients accompanied with an increase of the Reynolds number. The pressure distribution in the axial direction in turbulent flows is larger than that in laminar flows as shown in Fig.4. Also, the load capacity becomes larger in turbulent flows than in laminar flows as shown in Fig.5. Figure 6 shows the relationship between the Sommerfeld number and the eccentricity ratio. In Figs.5 and 6, the load capacity of journal bearings in turbulent flows is larger and the Sommerfeld number becomes smaller for the same eccentricity ratio.

The relationship between the eccentricity ratio and the attitude angle is shown in Fig.7, from which it follows that the attitude angle in turbulent flows is larger than that in laminar flows under the constant eccentricity ratio. However, the dependencies of the attitude angle on the mean Reynolds number are low.

In Fig.8 showing the relationship between the frictional force and the eccentricity ratio, the frictional force is remarkably larger than that in laminar flows. The frictional coefficient increases, as shown in Fig.9, under the same Sommerfeld number. Figure 10 indicates the rate of flow out of both sides of the bearing. The temperature rise of journal bearings in turbulent flows is more pronounced than in laminar flows because the heat-generation by

the frictional force is large and the rate of flow is small.

The relationship between the ratio of load capacity in turbulent flows to that in laminar flows and the slenderness ratio is shown in Fig.11. The frictional force is also given similarly. The load capacities increase as the slenderness ratio increases. The effect on the frictional force, however, is small.

Next, let us compare the calculated results of film pressure from Eq.(2) considering the effect of pressure flows with those obtained from Eq.(3) in which the effect of pressure flows is smaller than that of shear flows.

Figure 12 illustrates an example of circumferential pressure distribution at the central cross section of journal bearing, which is calculated on a case where the effect of pressure flows is pronounced, that is, the eccentricity ratio is large ($\epsilon=0.81$). In the figure, the thick line indicates the results from Eq.(2), the dashed line the results from Eq.(3) and the dot-dash line the results of laminar flows. In addition to this, the experimental data of film pressure measured by the methods mentioned later are presented. At the large eccentricity ratio shown in Fig.12, the effect of pressure flows is pronounced, then there is some difference between the results. Consequently, we can obtain accurate results by using Eq.(2) for a large eccentricity ratio and Eq.(3) for a small eccentricity ratio.

Figure 13 indicates a comparison between the results obtained by this theory

Fig.4 Axial pressure distribution

Fig.5 Load capacity versus eccentricity ratio

Fig.6 Eccentricity ratio versus Sommerfeld number

Fig.7 Eccentricity ratio versus attitude angle

and those by the other theories. The thick lines are the results from Eq.(2) and the dot-dash lines are the results from Eq.(3). The former agrees with the analytical results by Aoki and his co-worker which are derived by using the mixing length based on the law of wall and by modifying the velocity distribution to make it agree well with the experimental data. The latter agrees with the results by Ng and his co-worker which are derived by introducing the eddy diffusivity and by applying a linearized

method based on the assumption that the effect of pressure flows is small. The results from Eq.(3) assuming that the effect of pressure flows is small yield some errors unlike the results from Eq.(2) in which the full effect of pressure flows is considered, but good agreement between them is qualitatively shown.

4. Experiments

In order to confirm the validity of this theory, the turbulent effect on the bearing performances is investigated experimentally by using the testing equipment of journal bearings which can be operated in turbulent flows.

Fig. 8 Frictional force versus eccentricity ratio

Fig. 9 Frictional coefficient versus Sommerfeld number

Fig. 10 Rate of flow versus Sommerfeld number

Fig. 11 Effects of slenderness ratio on the bearing performances

Fig. 12 Circumferential pressure distribution

Fig. 13 Eccentricity ratio versus Sommerfeld number

4.1 Experimental apparatus and procedure

The testing equipment of journal bearings is illustrated schematically in Fig. 14. The rotating shaft is supported by four ball bearings to be rotatable continuously up to 12000 rpm by a speed converter with an electric motor, and the testing bearing is floating on the journal. The shaft, then, rotates within the accuracy of $1 \mu\text{m}$. The bore and width of the bearing are about 70 mm and the radial clearance ratio is about 8.8/1000 mm. The load can be changed from 0 to 50 kgf by a lever.

The displacement of the bearing is measured by a gap-sensor set at right angles to each other, in which the original point in measuring is the concentric point of the bearing and the journal. The film pressure is measured by the pressure transducers with strain-gauge set at the same intervals on the circumference of the bearing. When the pressure is small, the water-manometer is used.

In order to obtain turbulent flows, water is used as lubricant because the viscosity and the variation of temperature are small. Water is drop-fed through an inlet at the unload side of the bearing. The film pressure is measured varying the applied load, and the eccentricity ratio and the attitude angle are calculated from the displacement of the bearing.

4.2 Experimental results and discussions on them

The experimental results obtained from the testing equipment in laminar flows and turbulent flows are presented here and compared with the theoretical results obtained by Eq.(2). The experiments are conducted only for a range of high eccentricities ($\epsilon > 0.7$) where the stable running conditions can be obtained because the oil whirl arises

Fig.14 Journal bearing testing apparatus

at low eccentricity ratio and the testing is difficult under unstable conditions.

The pressure distribution is presented in Fig.15 to Fig.18 in which the plots indicate the experimental results and the thick lines are the theoretical ones. The pressure distribution for $Re=1000$ is shown in Fig.15, in which the lines are calculated by the laminar lubrication theory. It seems that the turbulent effect does not yet appear at such a Reynolds number.

A comparison between the theoretical results based on the turbulent lubrication equation and the experimental ones is shown in Figs.16 and 17. The experimental values almost coincide with the theoretical ones.

The film pressures for different Reynolds numbers are shown in Fig.18. Since the diffusivity of turbulence increases with an increase of the Reynolds number and the lubricant viscosity increases apparently,

Fig.15 Circumferential pressure distribution

Fig.16 Circumferential pressure distribution

Fig.17 Circumferential pressure distribution

Fig.18 Circumferential pressure distribution

Fig.19 Load capacity versus eccentricity ratio

the film pressure also increases.

The relationship between the load capacity and the eccentricity ratio is shown in Fig.19. The film pressure in turbulent flows becomes large, so the load capacity is also large. The tendency becomes more pronounced as the Reynolds number increases.

In Fig.20 showing the eccentricity ratio and the attitude angle, there are less effects of the Reynolds number on the attitude angle.

5. Conclusions

Comparing the results of analysis applying the turbulent lubrication equation in the first paper to finite width journal bearings with the experimental results, the following conclusions are obtained.

(1) The calculated results based on the

Fig.20 Eccentricity ratio versus attitude angle

turbulent lubrication equation agree well with those published previously.

(2) The bearing performances in turbulent flows are given by Eq.(2) for the high eccentricity ratio and by Eq.(3) for the low eccentricity ratio.

(3) The film pressure in turbulent flows is larger than in laminar flows, therefore the load capacity is larger and the Sommerfeld number is smaller for the same eccentricity ratio. The tendency is more pronounced as the Reynolds number increases.

(4) According to the analytical results of the frictional force, the frictional coefficient and the rate of flow, the frictional force increases and the rate of flow decreases in turbulent flow.

(5) The theoretical results agree well with the experimental ones concerning the bearing performances such as film pressure, load capacity and eccentricity ratio.

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