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Metal Forming by Underwater Wire Explosion

1. An Analysis of Plastic Deformation of Circular Membranes Under Impulsive Loading*

By Toshimi TOBE**, Masana KATO*** and Haruki OBARA****

The plastic deformation of circular membranes of a strain-rate dependent, work-hardening metal is analyzed based on the strain increment theory. Numerical solutions for aluminum membranes are obtained under various types of loading as well as impulsive pressure pulses caused by underwater wire explosions. When the duration of a loading pulse is less than one-third of the time required to finish the deformation of the membrane, the deflection of a deformed membrane depends almost only on the impulse of the loading pulse and the instantaneous profile of the membrane is found to be trapezoidal during deformation. The relation between impulse of loading and deflection is calculated and illustrated. The forming limit due to peripheral necking is discussed. The neglect of strain-rate dependency of material leads to a notable underestimation of the forming limit.

1. Introduction

The authors have made a systematic study on the electric discharge metal forming and reported already that the energy consumed by wire explosion is maximized under a certain relation of charging condition and wire dimensions^(1,7) and the relationships between the energy consumed during wire explosion and both peak value and impulse of the pressure pulse in water are clarified^(1,8). This paper deals with an analysis on the forming of a circular metal membrane due to the pressure pulse by underwater wire explosion.

Hudson⁽¹⁾ made an earlier study of the analysis of dynamic plastic deformation of a clamped circular metal membrane, where the membrane with initial velocity was deformed to a shape of cone by the characteristic wave propagating from edge to center in the membrane. It was reported by Kiyota⁽²⁾ that the deflection profile of a membrane during deformation obtained by Hudson gave a good approximation of the corresponding experiments. The complete equations of motion of a circular membrane were introduced by Kiyota⁽²⁾ and Munday et al.⁽³⁾ and the characteristics of the waves propagating in the membrane were detailed by Cristescu⁽¹⁰⁾. Some attempts to develop a numerical procedure for it were made by Witmer et al.⁽⁴⁾ and Sano⁽¹⁶⁾.

The dependency of the flow rule of materials on strain-rate, in general, is considerably significant in high strain-rate deformation of metals. However, no exact analysis of dynamic deformation of the membranes considering the strain-rate dependency has yet been reported as far as the authors know.

In this paper, an analysis of plastic deformation of circular aluminum membranes under impulsive loading is made based on the strain increment theory using the strain-rate dependent work-hardening rule. And also the influence of both time history of impulsive load and strain-rate dependency of materials on the deformation are discussed.

2. Flow Stress and Strain Relation Including Strain-Rate Dependency

With the aluminum JIS-3 (purity is more than 99%) which is mainly used in the experiments of electric discharge metal forming by the authors, the flow stress and strain relation including strain-rate dependency is estimated from the results which were already reported by various investigators.

Strain-rate dependencies of aluminum and its alloys obtained by various methods are shown at effective plastic strain of $\epsilon_{ef}^p = 0.1$ in Fig.1, where the ordinate is equivalent true stress σ_{ef} and the abscissa is effective plastic strain-rate $\dot{\epsilon}_{ef}^p$. In the figure, some materials have a considerably large strain-rate dependency and others have no appreciable dependency.

In the figure, the lines Nos. 1⁽⁵⁾, 2⁽⁶⁾, 8⁽¹²⁾, 9⁽¹²⁾, 11⁽¹³⁾ and 13⁽¹⁵⁾, which are the data on the similar materials to aluminum JIS-3, show a similar dependency of strain-rate to one another in spite of

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** Professor, Faculty of Engineering, Tohoku University, Sendai.

*** Associate Professor, Faculty of Engineering, Tohoku University.

**** Fujitsu FANUC Ltd..

Fig. 1 Relations between stress and strain-rate at $\epsilon_{ef}^p = 0.1$ in aluminum and its alloys (elastic strain is ignored)

the difference in experimental methods and in stress condition. It may be concluded that strain-rate dependency of a material is mainly affected by the purity of it. The same tendency is recognized in other materials such as steel, copper, lead and titanium. Generally speaking, the higher purity a material has, the larger dependency it shows.

In this paper, the flow stress and strain relation of aluminum JIS-3 including strain-rate dependency is determined as formula (1) on the basis of Lindholm's experiments which offered a plenty of data covering the widest range of strain-rate ($\dot{\epsilon}_{ef}^p = 10^{-4} - 2 \times 10^3$ 1/s).

$$\sigma_{ef} = \begin{cases} \sigma_0^* (\epsilon_{ef}^p)^n & : 0 < \dot{\epsilon}_{ef}^p \leq \dot{\epsilon}_{ef0}^p \\ \sigma_0^* (\epsilon_{ef}^p)^n + \sigma_1^* (\epsilon_{ef}^p)^m \log(\dot{\epsilon}_{ef}^p / \dot{\epsilon}_{ef0}^p) & : \dot{\epsilon}_{ef}^p > \dot{\epsilon}_{ef0}^p \end{cases} \quad (1)$$

$\sigma_0^* = 1050 \text{ kg/cm}^2$, $n = 0.24$, $\sigma_1^* = 160 \text{ kg/cm}^2$,
 $m = 0.54$, $\dot{\epsilon}_{ef0}^p = 10^{-3} \text{ 1/s}$

In equation (1), σ_0^* and n are the material constants concerning the quasistatic flow rule which were determined by the ordinary tensile test and σ_1^* , m , $\dot{\epsilon}_{ef0}^p$ are the constants concerning strain-rate which are determined by Lindholm's data.

3. Fundamental Equations

Deformation of a circular metal sheet is shown schematically in Fig. 2. Let us introduce a Lagrangian coordinate system r , z and fix its origin O at the center of the undeformed circular sheet. It is assumed that the bending rigidity of the sheet can be ignored because its thickness is very small compared with its diameter $2R_0$ and it is also assumed that a uniformly distributed pressure pulse acts on its under surface.

The equations of motion in horizontal r and vertical z directions are^{(2),(3)}

$$\rho_0 h_0 r_0 \frac{\partial^2 u}{\partial t^2} = -pr \frac{\partial w}{\partial r_0} + \frac{\partial}{\partial r_0} (\sigma_r h r \frac{\partial r}{\partial s}) - \sigma_\theta h \frac{\partial s}{\partial r_0} \quad (2)$$

$$\rho_0 h_0 r_0 \frac{\partial^2 w}{\partial t^2} = pr \frac{\partial r}{\partial r_0} + \frac{\partial}{\partial r_0} (\sigma_r h r \frac{\partial w}{\partial s}) \quad (3)$$

Fig. 2 Schematic diagram of the deformation of a circular membrane

where s is meridian distance measured from the center of the membrane; u , w are displacements in r and z directions; h is the thickness of the membrane; σ is true stress; ρ is density of the material; t is time. Suffixes s , θ , h denote the meridian, circumferential and thickness directions respectively and suffix o denotes the initial value. Logarithmic strain ϵ and nominal strain e are defined by the displacements u , w as

$$\epsilon_s = \ln(1 + e_s) = \ln \frac{\partial s}{\partial r_0} = \ln \left\{ \left(\frac{\partial r}{\partial r_0} \right)^2 + \left(\frac{\partial w}{\partial r_0} \right)^2 \right\}^{1/2} \quad (4)$$

$$\epsilon_\theta = \ln(1 + e_\theta) = \ln \frac{r}{r_0}, \quad \epsilon_h = \ln(1 + e_h) = \ln \frac{h}{h_0}$$

Considering plane stress condition, it is assumed that the material obeys Prandtl-Reuss's relation

$$\left. \begin{aligned} d\sigma_s &= \frac{E}{1-\nu^2} (d\epsilon_s^* + \nu d\epsilon_\theta^*) \\ d\sigma_\theta &= \frac{E}{1-\nu^2} (d\epsilon_\theta^* + \nu d\epsilon_s^*) \\ d\epsilon_s^* &= d\epsilon_s - \frac{3}{2} \frac{d\epsilon_{ef}^p}{\sigma_{ef}} \frac{2\sigma_s - \sigma_\theta}{3} \\ d\epsilon_\theta^* &= d\epsilon_\theta - \frac{3}{2} \frac{d\epsilon_{ef}^p}{\sigma_{ef}} \frac{2\sigma_\theta - \sigma_s}{3} \\ \sigma_{ef}^2 &= \sigma_s^2 + \sigma_\theta^2 - \sigma_s \sigma_\theta \\ d\epsilon_h &= \frac{1-2\nu}{E} (d\sigma_s + d\sigma_\theta) - d\epsilon_s - d\epsilon_\theta \end{aligned} \right\} \quad (5)$$

in plastic loading and obeys Hooke's law ($d\epsilon_{ef}^p = 0$ in eq.(5)) in both elastic loading and unloading, where E , ν are Young's modulus and Poisson's ratio, respectively. In calculation, the state of stresses is classified as

$$\left. \begin{aligned} \text{plastic: } d\epsilon_{ef}^p > 0 \text{ or } \sigma_{ef} \geq \sigma_0^* (\epsilon_{ef}^p)^n \\ \text{elastic: } d\epsilon_{ef}^p \leq 0 \text{ or } \sigma_{ef} < \sigma_0^* (\epsilon_{ef}^p)^n \end{aligned} \right\} \quad (6)$$

The boundary conditions of the membrane are

$$\left. \begin{aligned} r_0 = R_0 : u = w = 0 \\ r_0 = 0 : \begin{cases} u = 0 \\ \rho_0 h_0 \frac{\partial^2 w}{\partial t^2} = p \left(\frac{\partial r}{\partial r_0} \right)^2 + 2 \left(\frac{\partial r}{\partial r_0} \right) \frac{h \sigma_s}{1 + e_s} \frac{\partial^2 w}{\partial r_0^2} \\ \epsilon_s = \epsilon_\theta = \ln \frac{\partial r}{\partial r_0} \end{cases} \end{aligned} \right\} \quad (7)$$

and the initial conditions are

$$t=0 \quad \left\{ \begin{array}{l} u(r_0)=w(r_0)=0 \\ \varepsilon_r(r_0)=\varepsilon_\theta(r_0)=\varepsilon_h(r_0)=0 \\ \sigma_r(r_0)=\sigma_\theta(r_0)=0 \\ 0 \leq r_0 \leq R_0 \end{array} \right\} \dots\dots\dots (8)$$

Now the nondimensional variables are defined as follows:

$$\left. \begin{array}{l} \bar{r}_0 = \frac{r_0}{R_0}, \quad \bar{r} = \frac{r}{R_0}, \quad \bar{u} = \frac{u}{R_0}, \quad \bar{w} = \frac{w}{R_0}, \quad \bar{s} = \frac{s}{R_0}, \quad \bar{h} = \frac{h}{h_0}, \quad \bar{p} = \frac{R_0}{h_0} \frac{p}{\sigma_0^*}, \\ \bar{p}_m = \frac{R_0}{h_0} \frac{p_m}{\sigma_0^*}, \quad \bar{\sigma}_r = \frac{\sigma_r}{\sigma_0^*}, \quad \bar{\sigma}_\theta = \frac{\sigma_\theta}{\sigma_0^*}, \quad \bar{\sigma}_{ef} = \frac{\sigma_{ef}}{\sigma_0^*}, \quad \bar{\sigma}_1^* = \frac{\sigma_1^*}{\sigma_0^*}, \quad \bar{E} = \frac{E}{\sigma_0^*}, \\ \bar{t} = \frac{t}{T}, \quad \bar{\varepsilon}_{ef}^p = T \dot{\varepsilon}_{ef}^p, \quad \bar{\varepsilon}_{ef}^p = T \dot{\varepsilon}_{ef}^p, \quad T = R_0 \sqrt{\frac{\rho_0}{\sigma_0^*}}, \quad \bar{F}_t = \int_0^\infty p dt / h_0 \sqrt{\rho_0 \sigma_0^*} \end{array} \right\} \dots\dots\dots (9)$$

4. Numerical Procedures

Numerical integrations of finite-difference approximations of the equations of motion have been carried out. Initial locations of the mesh intersections are specified at time $t=0$ in the coordinate r_0 . The calculation sequence necessary to determine the transient response from the known variables at any time t is as follows:

1. Finite difference expressions of accelerations $\ddot{u}_t, \ddot{w}_t$ at time t are obtained by the equations of motion (2), (3). The first approximations of accelerations at time $t+\Delta t$, $\ddot{u}_{t+\Delta t}, \ddot{w}_{t+\Delta t}$, are assumed to be equal to $\ddot{u}_t, \ddot{w}_t$ respectively.
2. Velocities $\dot{u}_{t+\Delta t}, \dot{w}_{t+\Delta t}$ and displacements $u_{t+\Delta t}, w_{t+\Delta t}$ at time $t+\Delta t$ are calculated by

$$\left. \begin{array}{l} \dot{u}_{t+\Delta t} = \dot{u}_t + \Delta t / 2 (\ddot{u}_t + \ddot{u}_{t+\Delta t}), \\ \dot{w}_{t+\Delta t} = \dot{w}_t + \Delta t / 2 (\ddot{w}_t + \ddot{w}_{t+\Delta t}), \\ u_{t+\Delta t} = u_t + \dot{u}_t \Delta t + \Delta t^2 / 6 (2\ddot{u}_t + \ddot{u}_{t+\Delta t}), \\ w_{t+\Delta t} = w_t + \dot{w}_t \Delta t + \Delta t^2 / 6 (2\ddot{w}_t + \ddot{w}_{t+\Delta t}) \end{array} \right\} \dots\dots\dots (10)$$

3. Strains $\varepsilon_{st+\Delta t}, \varepsilon_{\theta t+\Delta t}$ at time $t+\Delta t$ are obtained by substituting displacements $u_{t+\Delta t}, w_{t+\Delta t}$ into the finite difference expressions of Eqs. (4). Strain increments during time interval Δt are

$$\begin{aligned} \Delta \varepsilon_s &= \varepsilon_{st+\Delta t} - \varepsilon_{st} \\ \Delta \varepsilon_\theta &= \varepsilon_{\theta t+\Delta t} - \varepsilon_{\theta t} \end{aligned}$$

4. Substituting the strain increments during time interval Δt and stresses at time t into the constitutive equations of the material (1), (5) and (6), stress increments $\Delta \sigma_s, \Delta \sigma_\theta$ during Δt and stresses $\sigma_{st+\Delta t}, \sigma_{\theta t+\Delta t}$ at time $t+\Delta t$ are obtained. When the stresses are in elastic state, $d\varepsilon_{ef}^p$ must be zero in Eqs. (5).
5. The second approximations of accelerations $\ddot{u}_{t+\Delta t}, \ddot{w}_{t+\Delta t}$ are calculated from the equations of motion (2), (3) in a manner similar to step 1.
6. Using new approximations of accelerations $\ddot{u}_{t+\Delta t}, \ddot{w}_{t+\Delta t}$ obtained in step 5, calculations from step 2 to step 5 are repeated until the new values of $\ddot{u}_{t+\Delta t}, \ddot{w}_{t+\Delta t}$ become equal to the previous values within a given error.

In step 4, the stress increments $\Delta \sigma_s, \Delta \sigma_\theta$ during time interval Δt are obtained by the numerical computation of an algebraical equation on $\Delta \lambda = (3/2) \Delta \varepsilon_{ef}^p / \sigma_{ef}^p$ which is derived from both flow stress-strain

relation (1) and Prandtl-Reuss's Equation (5), where the values of stresses are the mean values of those at time t and $t+\Delta t$.

The stress states which appear in computation are shown schematically in Fig.3. Point (a) shows the plastic state in which both stress and plastic strain increase by loading. Point (b) shows the plastic state in which plastic strain increases in spite of the decrease in stress. This state is characteristic of the strain-rate dependent flow rule. Point (c) shows the unloading elastic state.

As a system of partial differential equations of motion (2), (3) is hyperbolic, the time interval Δt must be determined in connection with the position interval Δr_0 in numerical procedure. In this study, Δt was determined as

Fig.3 Stress states in calculation
(a), (b): plastic
(c) : elastic

Table 1 Conditions and dimensions in calculation

	physical quantities	non-dimensional quantities
material	aluminum JIS-3	
material constants	$\rho_0 = 2.75 \times 10^{-6} \text{ kg s}^2/\text{cm}^4$	$\bar{E} = 753$
	$E = 7.91 \times 10^5 \text{ kg/cm}^2$	$\bar{\sigma}_0^* = 1$
	$\sigma_0^* = 1050 \text{ kg/cm}^2$	$\bar{\sigma}_1^* = 0.152$
	$\sigma_1^* = 160 \text{ kg/cm}^2$	$\bar{\varepsilon}_{ef}^p = 2.56 \times 10^{-7}$
	$\dot{\varepsilon}_{ef}^p = 10^{-3} \text{ 1/s}$	
	$\nu = 0.3$	
	$n = 0.24$ $m = 0.54$	
dimensions	$R_0 = 5.0 \text{ cm}, h_0 = 0.1 \text{ cm}$	

$$\Delta t = \Delta r_0 / C_{\max}$$

where $C_{\max}$ is the maximum inclination of the families of characteristics among all positions of mesh at the time.

5. Examinations of Errors Involved in Numerical Procedures

To examine the influence of the position interval Δr_0 on the calculation, preliminary calculations of deformation of the membrane by a pressure pulse of exponential decay

$$p = p_0 \exp(-t/t_0), \quad p_0 = 100 \text{ kg/cm}^2, \quad t_0 = 20 \mu\text{s}$$

were made under the conditions and dimensions shown in Table 1. The position interval Δr_0 affects strongly both the final deflection and the final strains at the center, as shown in Figs. 4 and 5. The final deflection w_{cf} of membrane increases with a decrease in Δr_0 and the difference between the estimated deflection corresponding to $\Delta r_0 = 0$ and the deflection at $\Delta r_0 = 0.05$, which corresponds to twenty divisions of the radius of the membrane R_0 , is about 5%. On the other hand, the final strains at the center increase with a decreasing $\log \Delta r_0$.

Deformation of an ideally plastic membrane were calculated and deflections and thickness strains are compared with the corresponding result by Hudson⁽¹⁾ in Fig. 6. In the calculation, the flow rule of an ideally plastic material is approximated as follows:

$$\sigma_{ef} = \sigma_0 * (\epsilon_{ef})^n, \quad \sigma_0 = 1050 \text{ kg/mm}^2, \quad n = 0.01$$

where n must be zero essentially. ... (11)

Calculated profiles and thickness strains during deformation are in good agreement with Hudson's results except final ones in the vicinity of center. One of the reasons for the differences may be considered to be that the position interval in calculation is not sufficiently fine, because the use of a finer position interval makes the difference smaller.

The time required in the computation increases in proportion to a square of number of divisions. In the following computations, the position interval is chosen as $\Delta r_0 = 0.05$, provided that the errors involved were within 5% except the final strains in the vicinity of the center.

6. Dynamic Deformation of Membrane

In this section, the deformation process of the membrane due to the impulsive pressure which is caused by wire explosion and has been measured by the pressure-bar in the previous paper⁽¹⁸⁾ is detailed. Unless otherwise specified, conditions and dimensions in calculation are as shown in Table 1.

Fig. 7 shows the pressure pulse employed in the calculation. The peak value of the pressure p_m is 470 kgf/cm² ($\bar{p}_m = 22.4$) and the impulse which is defined by the time integral of the pressure is

$$F_t = \int_0^\infty p dt = 1.96 \times 10^{-3} \text{ kg s/cm}^2 (\bar{F}_t = 0.365).$$

Fig. 4 Relation between final deflection at center and position interval

Fig. 5 Relation between final thickness strain at center and number of divisions

(a)

(b)

Fig. 6 Comparison with Hudson's result (a) profile (b) thickness strain

Calculated results of deformation process of the membrane are shown in Fig.8. The circles in Fig.8(a) indicate the instantaneous positions of the specified mesh points at each time $\bar{t}$ after the initiation

Fig.7 Impulsive pressure caused by wire explosion employed as loading pulse in calculation

solid line : measured
broken line : used in calculation
 $C_0 = 5.8 \mu\text{F}$: capacity of condenser
 $V_0 = 17.2 \text{ kV}$: charging voltage
 $l_w = 80 \text{ mm}$: copper wire length
 $s_w = 0.067 \text{ mm}^2$: wire sectional area
 $L_D = 100 \text{ mm}$: standoff distance

(b)

of loading. In Fig.8(d), the oblique line denotes the inclination of a family of the characteristics

$$C_I = \frac{d\bar{r}_0}{d\bar{t}} = - \left\{ \frac{1+e_\theta}{1+e_r} \bar{h} \bar{\sigma}_r \right\}^{1/2} \dots \dots \dots (12)$$

which mean the velocity of the transverse wave propagating in the membrane. The broken line indicates the elastic-plastic boundary. Total time required in deformation is about $\bar{t} = 1.1$ ($t = 0.27 \text{ ms}$) and the final deflection reaches $\bar{w}_{cf} = 0.38$ ($w_{cf} = 19 \text{ mm}$). The maximum strain-rate amounts to about

$$\bar{\epsilon}_{ef}^p = 0.72.$$

It is recognized that the transient profile of the membrane is trapezoidal and the central flat portion is in a balanced state of stress, while the outer conical portion is almost in a final state of deformation. These characters of deformation are similar to Hudson's results as a whole. The reasons may be as follows:

1. As the duration of pressure pulse is significantly short compared with the time required in deformation, the deformation

(c)

(d)

Fig.8 Dynamic deformation of membrane
($\bar{t}=1$ corresponds to $256 \mu\text{s}$, $\bar{r}_0$, $\bar{r}$ or $\bar{w}=1$ corresponds to 5 cm)
(a) profiles (b) strains (c) stresses
(d) inclinations of characteristic C_I

can be approximated as the one initiated by the initial velocity corresponding to the impulse of the pressure pulse.

2. The balanced biaxial state of stress in the central flat portion is secured by the fact that the velocity of longitudinal wave C_{II} is sufficiently fast compared with the velocity of transverse wave C_I .

7. Effects of Shape and Duration of Pressure Pulse

Final deflections of the membrane for triangular pressure pulse with various durations and peaks are shown in Fig.9 with respect to impulse $\bar{F}_t$ of the pulse, where circles denote the cases of variable peak

Fig.9 Final deflections for triangular pressure pulse with various durations and peaks as a function of $\bar{F}_t$

Fig.10 Final deflections due to both linear decay and rectangular pressures

Fig.11 An example of deformation profiles by the pressure pulse with long duration (triangular pressure)

pressure under the constant duration $t_d = 0.0313$ and triangles denote the case of variable duration under the constant peak value $\bar{P}_m = 23.8$. The rise time $\bar{t}_r$ equals to zero in these cases. Final deflections depend only on the impulse $\bar{F}_t$ of the pressure pulse and do not depend on its shape in these cases; it may be due to the fact that the durations of the pressure are less than one-fifth of the time required in deformation $\bar{t}_s$ ($0.8 < \bar{t}_s < 1.3$). It is expected that the deformation by longer pressure pulses may be affected by their shapes. Fig.10 shows the final deflections by both linear decay and rectangular pressure pulses with various durations but with constant impulses ($\bar{F}_t = 0.744$ and 0.372). When the duration of pulse $\bar{t}_d$ is less than one-third of $\bar{t}_s$, the final deflection can be estimated only from the amount of impulse $\bar{F}_t$ within 2% error.

An example of deformation process by the pressure pulse of longer duration is shown in Fig.11, where the pressure increases in proportion to time until $\bar{t} = 1.56$. The profiles in the beginning of deformation are dynamic, but the final profile at $\bar{t}_s = 1.8$ is similar to a profile by static pressure.

Fig.12 Effect of strain-rate dependency of material on deforming processes of the central deflection

Fig.13 Comparison of profiles between strain-rate dependent and independent materials

8. Effects of Strain-Rate Dependency of Material

Deforming processes of the central deflection of a membrane with the initial velocity corresponding to $\bar{F}_t=0.37$ and its instantaneous profiles are shown in Figs.12 and 13 respectively, showing the effect of strain-rate dependency of the material. Deforming rates of deflection in both materials are the same, but the final deflection of strain-rate independent material is about 15% larger than that of strain-rate dependent material; this may be due to the fact that the propagating velocity of transverse wave C_T (Eq.(12)) of the former is smaller than that of the latter.

9. Relation Between Final Deflection of Membrane and Impulse of Pressure

In this section, the deformation of a membrane with initial velocity corresponding to the impulse $\bar{F}_t$ is discussed. Deforming processes of central deflection $\bar{w}_c$ due to various impulses $\bar{F}_t$ are shown in Fig.14. The time required in deformation decreases with an increasing impulse. The relation between final deflection $\bar{w}_{cf}$ and

Fig.14 Deforming processes of central deflection due to various amount of impulse

Fig.15 Relationship between final deflection and impulse

impulse $\bar{F}_t$ is shown in Fig.15. The arrows show succeeding deformations due to peripheral necking which indicate one of the forming limits in dynamic formings. It is notable that the forming limit due to the peripheral necking of the strain-rate independent material is about 35% less than that of the strain-rate dependent material. Fig.16 shows the strain distributions at time $\bar{t}=0.7$ for various impulses, where the arrows denote the boundary positions between the central flat portion and the conical portion estimated from the deforming profiles. In the case of impulse $\bar{F}_t=0.37$, the central portion is in a balanced biaxial state of stresses as mentioned in Fig.8. In stronger impulses, $\bar{F}_t \geq 0.74$, the strain state is $\epsilon_s > \epsilon_\theta$ in the central portion. At $\bar{F}_t=1.48$, an extreme increase of meridian strain ϵ_s is found in the vicinity of periphery $r_0=R_0$ corresponding to the peripheral necking above mentioned.

Let us consider these phenomena in a simplified case of rigid-plastic, strain-rate independent material. When the loading impulse is weak and consequently stresses in the membrane are small, the velocity C_{II} of the longitudinal wave, which causes an extension of the membrane, is sufficiently fast in comparison with the velocity C_I of the transverse wave which changes the shape of the membrane. However, as an increase of the impulse accompanies a decrease of C_{II} , the velocity C_{II} is not so faster than the velocity C_I in a stronger impulse such as $\bar{F}_t \geq 0.74$. The velocity C_{II} is expressed by

$$C_{II} = \frac{dr_0}{dt} = \frac{1}{1+e_s} \sqrt{\frac{d\bar{\sigma}_s}{d\epsilon_s}} \bar{\sigma}_s \quad \dots\dots\dots (13)$$

$$\bar{\sigma}_s = \left(\frac{2}{\sqrt{3}} \right)^{n+1} \epsilon_s^n$$

at the peripheral boundary $r_0=R_0$, where $\epsilon_\theta=0$. According to Eq. (13), $C_{II}=0$ at $\epsilon_s=n$; this is also the plastic instability condition for diffuse contraction as well as localized contraction at the periphery.

Fig.16 Strain distributions at time $\bar{t}=0.7$ for various impulses

10. Conclusions

The dynamic deformation of a circular membrane of strain-rate dependent, strain-hardening metal was analyzed based on the strain increment theory. Numerical solutions for aluminum membranes were obtained under various types of loading as well as impulsive pressure pulse caused by the underwater wire explosion.

Following are concluded:

1. When the duration of loading pressure is less than one-third of the time required in deformation of a membrane, the final deflection of the membrane depends on the impulse of the pressure pulse, which is defined by time integral of pressure, and does not depend upon both duration and shape of the pressure pulse.
2. Under the condition mentioned at (1), the deforming profile is trapezoidal.
3. Under the condition mentioned at (1), the relation between the final deflection of the membrane and the impulse of the pressure pulse is obtained.
4. The forming limit due to peripheral necking in no strain-rate dependent material is only about 65% of one in strain-rate dependent material.

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