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Performance of Rotating Cascades under the Inlet-Distortions

(1st Report, Performance under the Radial Distortions)

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Some experimental and theoretical studies of performances of rotating cascades operating under the radial-distortions are shown.

The transmission of the distorted velocity profiles through the cascades, changes in characteristic curves and aerodynamic loading on the blade elements are obtained experimentally, and some discussions of the criterion of the onset of stall in the rotating cascades are made. The flow fields near the rotating cascades are analyzed by stream-line curvature method and the results are compared with the experimental ones.

The rotating cascades have effects to attenuate the distorted velocity profile both at the design point and off-design points. The rotating cascades operate stably even when the aerodynamic loading near the root exceeds the stall limit of the two-dimensional cascades. The stall limit of the rotating cascade is determined by the equivalent diffusion ratio at a specific radius near the tip.

The analytical results of transmission of the distorted velocity profiles through the rotating cascades, changes in the aerodynamic loading and stall limits agree with the experimental ones with sufficient accuracy.

1. Introduction

In the design of the rotors of axial-flow blowers and compressors, the two-dimensional cascade data with some corrections of three-dimensional effects(1) have been used extensively, and the validity of the procedure has been verified in the operation at the design point.

In the off-design operations, however, the performances of rotors(rotating cascades) are not yet clarified but only a few studies of performance of rotating cascades operating near the stall limit have been made(2).

Recently, the advent of high-speed digital computers allows us to calculate easily the flow in the turbomachinery at the design and off-design points by the use of 'blade element method'(3), but the agreement of calculated stall limit and experimental one is relatively poor in many cases(4). One of the most important cause of the discrepancy seems to originate from the criterion of the onset of a stall in the rotating cascades. In most of the studies, it is considered that the rotating cascades stall when the aerodynamic loading at any radius exceeds the stall limit of the two-dimensional cascades at the same radius. The work done by the rotating cascades operating near the stall limit is quite unequal in the spanwise direction and the

interference between stream surfaces at different radii becomes remarkable, then the three-dimensional effects appear more extensively than in the operation at the design point. In such a process, the flow fields of the rotating cascades near the stall limit are different from ones of the two-dimensional cascades.

On the other hand, most of the rotating cascades of the axial-flow turbomachinery do not operate under the uniform inlet flow but usually under the radial velocity distortions due to boundary layers on the hub and casing surfaces, and the distortions influence the characteristic curve and stall limit of the cascades. In such a case, the operations of the rotating cascades become complicated and the three-dimensional effects appear severely. Therefore, the performance of rotating cascades operating near the stall limit under the various radial inlet-distortions must be known to clarify the stall limit of the actual axial-flow turbomachinery.

In the present paper, the difference in characteristic curves, transmission of the distorted velocity profiles through the cascades and the changes in the aerodynamic loading are obtained experimentally using three different types of rotating cascades, and some discussions of the factor which plays an important role in determining the stall limit of the rotating cascades are presented. The flow fields near the rotating cascades operating under the inlet-distortions are analyzed by stream-line curvature method(5) and the results are compared with the experimental ones.

Symbols

ϕ ; flow rate $= \bar{v}_z / u_t$

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ϕ_t ; total pressure rise coefficient
 $= \Delta \bar{p}_t / (\rho u_1^2 / 2)$
 ϕ^* ; flow rate at design point
 v_z ; axial velocity
 $\bar{v}_z$; axial velocity averaged over the annulus
 v_θ ; circumferential velocity
 p_t ; total pressure
 $\bar{p}_t$; mass weighted averaged total pressure
 $= \frac{1}{n} \sum_{i=1}^n (\bar{p}_t)_i \frac{(v_z)_i}{\bar{v}_z}$
 $J\bar{p}_t$; mean total pressure rise through the rotor
 u_t ; rotor-tip velocity $= r\omega$
 ρ ; air density
 r ; radius
 r_o ; outer radius of the annulus
 r_c ; radius of curvature of the projected stream-line, Fig.9
 λ ; slope of the projected stream-line
 ω ; angular velocity of cascades
 Deq ; equivalent diffusion ratio
 β ; relative flow angle
 i ; incidence angle
 i^* ; incidence angle at reference point
 σ ; solidity

Subscripts

- 1 ; before the cascade
 2 ; after the cascade

2. Experimental apparatus and procedure

2.1 Test installation The scheme of the test rig is illustrated in Fig.1. The test rig consists of an annulus of 400 mm in outer diameter (hub-tip ratio of 0.5) with three cascades, i.e., test rotating cascade ③, stator ④ and rotor for drafting ⑤. In the case of small distance between rotating cascade and stator, it has been shown that the stall characteristics of the rotating cascade are severely affected by the stator position (6), therefore the stator is located far downstream of the cascade to prevent such an interaction. The rotor ⑤ was used to make it possible to test the cascades in the wide flow region by combining with the throttle ⑥.

In the test, three different types of the test cascades were used, which are designed in the types of guidevaneless and free vortex. The design conditions and the essential dimensions are listed in Tables 1 and 2.

2.2 Distortion screens The inlet-flow distortions were made by the distortion screens illustrated in Fig.2, which were installed at the station ② indicated in Fig.1.

Fig.1 Test rig

Fig.2 Distortion screens

2.3 Measurement of flow fields The flow fields near the test cascades were obtained with an arrowhead type yawmeter inserted just before and after the cascades as indicated in Fig.1. The flow rate was measured with a Prandtl type Pitot tube ① in Fig.1, and the mean total pressure rise through the cascades was so evaluated that $\Delta \bar{p}_t = (\bar{p}_t)_2 - (\bar{p}_t)_1$ from the measurement of the flow fields.

Generally, the onset of a stall in the rotating cascades of the low-pressure compressors is designated by the rotating stall appearing at a part of the span. In the cascades tested, a rotating stall occurs in the tip as reported in the previous paper (6), so that the stall limit is determined with a hot-wire anemometer inserted just before the cascades.

Table 1 Design conditions of the cascades

cascade	Ro.1	Ro.2	Ro.3
capacity m ³ /min	120	100	80
pressure rise mmAq	45	40	35
flow rate ϕ^*	0.487	0.405	0.324
pressure rise coeff. ϕ_t	0.419	0.372	0.326
rotational speed rpm	2000	2000	2000
number of blades	30	16	10
chord length mm	40	50	60
cross-sectional profile	NACA65	NACA65	NACA65
tip clearance mm	1.0	1.0	1.0
blockage factor	0.96	0.96	0.96

Table 2 Configurations of cascades

cascade	Ro.1			Ro.2			Ro.3		
radius r mm	100	158	199	100	158	199	100	158	199
relative inlet-flow angle β_1 deg	44.6	57.3	63.1	49.8	61.9	67.0	56.0	67.1	71.3
relative exit-flow angle β_2 deg	9.3	46.1	57.4	16.8	52.7	62.4	27.3	60.5	67.9
stagger angle deg	24.8	48.2	57.0	29.5	54.0	61.5	37.6	60.0	67.1
solidity	1.91	1.21	0.96	1.27	0.80	0.64	0.96	0.60	0.48

3. Experimental results and discussions

3.1 Inlet-distortion profiles The inlet-flow velocity profiles measured just before the cascade are shown in Fig.3. The velocity distribution of Screen No.0, which was used as the supporting screen for the fine mesh, is almost uniform in the center region (in the present paper, 'center region' means the region where the influences of the hub and casing surfaces can be neglected). The velocity profiles of Screens No.1~4 have stepped distributions in the center region, and velocity difference between high-speed region and low-speed one is about (30~35)% of the mean velocity. The inlet-distortion profiles at any other flow rates maintain similar to ones in Fig. 3 so long as the cascades are unstalled.

3.2 Transmission of the distorted flow profiles through the cascades The velocity distributions just after the cascade are given in Fig.4. Fig.4(a) indicates the results near the design point ($\phi/\phi^*=0.925$), which shows that the distorted velocity profile remains even after the cascade, but the profiles become more uniform. The velocity difference in the center region is about (17~20)% of the mean flow, so that the attenuation ratio is about 0.6.

In general, velocity distribution after the rotating cascade is determined in a form that satisfies both the equation of motion and the cascade characteristics such as total pressure loss coefficient and

incidence angle-deviation angle relation. Since the cascade characteristics are affected by the flow conditions before the cascades, the transmission of the distorted velocity profiles in the off-design condition may be changed. The flow fields near the stall point are illustrated in Fig. 4(b), which indicates that the attenuation is almost same as that at the design point.

Further experiments using other cascades indicate similar results to ones described above, therefore it can be concluded that the inlet-distortions are attenuated by the rotating cascades at both design and off-design points.

Fig.5 Characteristic curves (Ro.1)

Fig.3 Inlet-velocity profiles (Ro.1, $\phi=0.45$)

Fig.4 Velocity profiles after the cascade (Ro.1).

3.3 Changes in the characteristic curves The characteristic curves are given in Fig. 5. The pressure rise at the relatively large flow rate is not affected by the inlet-velocity profiles but the stall limit is affected severely, i.e., the stall limit shifts toward the high flow rate region when the flow velocity near the tip is small (such as Screens No.1 and 2).

Almost the same results are obtained from the experiments using the other cascades, but the amount of the changes in the stall limit are not equal in all cases. Therefore, it is not enough to discuss the influence of the inlet-distortions on the stall limit using the characteristic curve, and the changes in the local aerodynamic loading on the blade elements will be discussed in the next section.

3.4 Aerodynamic loading The streamlines through the cascade can be calculated from the velocity profiles before and after the cascade, Fig.6(a), and the velocity diagram on the stream surface can be represented as Fig.6(b). Since the axial velocities before and after the rotating cascade are not equal in many cases, the velocity diagram must be modified using the mean axial velocity as indicated in Fig.6(b).

The amount of the radial shift of the stream-line is small enough to consider that the stream surface is cylindrical, so that the local aerodynamic loading on the

(a) Stream-line through the cascade

(b) Velocity vector diagram

Fig.6 Stream-line and velocity diagram

blade elements at every radius is calculated in similar manner to the two-dimensional cascade flow with the same velocity diagram as the modified one in Fig.6(b).

The aerodynamic loading of the two-dimensional cascade can be represented by the equivalent diffusion ratio(7), which is defined by

$$D_{eq} = \frac{\cos \beta_2}{\cos \beta_1} \left[1.12 + a(i - i^*)^{1.43} + 0.61 \frac{\cos^2 \beta_1}{\sigma} (\tan \beta_1 - \tan \beta_2) \right]$$

where the coefficient a is 0.0117 for the NACA65-series profiles used in the cascades tested. Distribution of Deq of the cascades operating under the inlet-distortions is presented in Figs.7(a)~(c). Fig.7(a) gives the case of Screen No.0, which shows that Deq near the hub and casing surfaces becomes extremely large and the fact suggests that the blade elements near the walls do not operate normally even at the design point. In the center region, however, Deq increases gradually in every radius with a decrease in flow rate. The dotted line in the figure indicates the distribution of Deq at the stall point obtained from the data in the unstalled region by extrapolation. Fig.7(b) illustrates the case in which the velocities near the tip are larger than those near the root, and almost the same tendency as Fig.7(a) can be seen.

In the cases described above, Deq in the center region does not exceed (2.0~2.2) till a stall occurs in the cascade.

On the other hand, when the velocities near the hub are smaller, Fig.7(c), Deq near the root increases rapidly as the flow rate decreases, and Deq in the center region grows as large as (2.5~3.0). In the case of the two-dimensional cascades, the total pressure loss increases rapidly when Deq exceeds (2.0~2.2) and the cascades become stalled condition(7).

The results described above indicate that the rotating cascades can operate stably even when the aerodynamic loading exceeds locally the stall limit of the two-dimensional case at the radius near the root. The fact may be explained as follows. The flow fields of the two-dimensional cascades are uniform in the spanwise direction, and it is reasonable to consider that the stall occurs at the same time along the span. On the contrary, in the case of the rotating cascades, however, even when the aerodynamic loading reaches the two-dimensional stall limit at any radius, the flow fields are restricted by the flow on the neighboring stream surfaces and the onset of a stall is delayed. In such a process, a noticeable difference in stall limit appears in the rotating cascades.

3.5 Stall limit of rotating cascades

As described above, the difference in stall limit of rotating cascades from that of two-dimensional ones is evident, so that the most important factor that determines the stall limit of the rotating cascades will be discussed.

Fig.8 shows the distribution of Deq at the stall limit under the inlet-distortions. Although the test cascades have different design parameters and blade configurations, a stall occurs when the distribution of Deq assumes the same pattern for every inlet-distortion, which suggests that Deq is a valid parameter to represent the stall limit of the rotating cascades. It must be noted that in the cases of Screens No.3 and 4 Deq near the root at the stall limit reaches (3.5~4.0), moreover a stall does occur not near the root but near the tip.

From the experimental results described above, it can be concluded that a stall of the rotating cascades occurs when Deq at $r/r_0 = (0.925 \sim 0.95)$ reaches (1.9~2.1) in all cases.

Fig.7 Equivalent diffusion ratio (Ro.1)

Fig.8 Equivalent diffusion ratio at stall limit

4. Analysis of distorted flow through the cascades

4.1 Governing equations and calculating procedure The Crocco's equation for a steady, inviscid and incompressible flow is given by

$$\vec{v} \times (\nabla \times \vec{v}) = \frac{1}{\rho} \nabla p_t \quad \dots\dots\dots (1)$$

In the case of an axisymmetric flow, eq.(1) can be rewritten as eq.(2) using the cylindrical coordinate system (r, θ, z) .

$$\begin{aligned} \frac{\partial v_z^2}{\partial r} + 2 \left(\frac{\sin^2 \lambda}{r} + \frac{1}{r_c \cos \lambda} + 2 \tan \lambda \frac{\partial \lambda}{\partial r} \right) v_z^2 \\ = 2 \cos^2 \lambda \left(\frac{1}{\rho} \frac{\partial p_t}{\partial r} - \frac{v_\theta}{r} \frac{\partial r v_\theta}{\partial r} \right) \quad \dots\dots\dots (2) \end{aligned}$$

where λ and r_c are the slope and the radius of curvature of the projected stream-line in the meridional plane as shown in Fig.9. Solving eq.(2) by the iterative process, we can rewrite eq.(2) in the following integral form(5).

$$\begin{aligned} v_z^2 = \exp \left\{ - \int P(r) dr \right\} \\ \times \left[\int \exp \left\{ \int P(r) dr \right\} T(r) dr + v_i^2 \right] \quad \dots\dots\dots (3) \end{aligned}$$

where v_i is a constant of integral determined from the equation of continuity in every calculating plane. The functions $P(r)$ and $T(r)$ are given by eqs.(4) and (5).

$$P(r) = 2 \left(\frac{\sin^2 \lambda}{r} + \frac{1}{r_c \cos \lambda} + 2 \tan \lambda \frac{\partial \lambda}{\partial r} \right) \quad \dots\dots\dots (4)$$

$$T(r) = 2 \cos^2 \lambda \left(\frac{1}{\rho} \frac{\partial p_t}{\partial r} - \frac{v_\theta}{r} \frac{\partial r v_\theta}{\partial r} \right) \quad \dots\dots\dots (5)$$

Fig.9 Coordinate system

In the calculation by stream-line curvature method, (10~15) calculating planes in axial direction with (10~20) stream-lines are used. At first, λ and r_c are obtained from the assumed velocity profiles at every plane by a suitable curve fitting procedure, and the new velocity profiles are calculated from eq.(3), the same process being repeated until the velocity profiles at all planes converge. The key point to obtain a suitable convergency is the method for curve fitting in the calculation of λ and r_c . In the present study a quartic method of 5 points accompanied by the smoothing process using weighted moving average with optimum frequency response, which shows quite favorable accuracy and convergency as demonstrated in the analysis of the transonic compressors(9), is adopted.

The values of P_t and v_θ appearing in eq.(5) can be evaluated from the cascade characteristic. The total pressure loss coefficient is so estimated that the value corresponding to the incidence angle at which Deq reaches 2.0 is twice that in the minimum loss condition and a parabolic change vs. incidence angle is assumed. In addition, incidence angle-deviation angle relation is evaluated from Lieblein's formula(10).

4.2 Transmission of distortions

Fig.10 presents the analytical results of changes in velocity profiles through the

(a) Before the cascade (b) After the cascade
Fig.10 Transmission of the distortion through the cascade (Ro.1, $\phi/\phi^*=0.925$)

Fig.11 Calculated Deq (Ro.1)

cascade, which correspond to the experiment shown in Fig.3 and Fig.4(a). The flow fields near the walls are extremely different from the experiment because of the assumption of an inviscid flow model, but the velocity profiles in the center region agree with the experiment relatively well.

A further analysis at the flow rate corresponding to Fig.4(b) indicates that the attenuation is similar to that in Fig. 10, and agreement with the experiment is favorable.

4.3 Equivalent diffusion ratio The analytical changes in Deq are illustrated in Fig.11. For the uniform inlet flow corresponding to Fig.7(a), the changes in Deq due to a decrease in the flow rate agree with the experiment relatively well except in the wall region, Fig.11(a). It must be noted, however, that Deq at the mid-span or near the root reaches 2.0 at the flow rate as large as $\phi/\phi^*=0.9$, therefore if we adopt the criterion of the onset of a stall in the rotating cascades as paper(8) that a stall occurs when Deq at any radius reaches (2.0~2.2), the cascade may stall at flow rate of 90% of the design condition. Such a result is not consistent with the experimental one at all, which may suggest a requirement for the new criterion of the onset of a stall in the rotating cascades. Figs.11(b) and (c) indicate the results corresponding to Figs.7(b) and (c), which may lead to the conclusion that the

changes in the aerodynamic loading can be evaluated analytically from the method presented here with sufficient accuracy. The distributions of Deq at the stall limit are given in Fig.12. In the analysis, we cannot discuss the flow near the wall, but the blade elements near the hub and casing surfaces do not operate normally at any flow rate. The stall limit of the rotating cascades is determined from the aerodynamic loading of the blade elements near the tip in the center region, and the operations of the blade elements near the hub and casing surfaces do not affect the stall limit of the rotating cascades.

From the above discussion, it can be concluded that the stall limit of the rotating cascades can be evaluated from the procedure presented here.

4.4 Stall limit The calculated stall limit $(\phi_s/\phi^*)_c$ is compared with the experimental one $(\phi_s/\phi^*)_E$ in Fig.13. In the calculation, the following criterion of the onset of a stall is adopted, that is a stall in the rotating cascades occurs when Deq at $r/r_0=(0.925\sim 0.95)$ reaches 2.0. The figure shows that the agreement is relatively good and the discrepancy is less than 5% of the designed flow rate.

Fig.12 Comparison of Deq at stall limit (Ro.1)

Fig.13 Comparison of analytical and calculated stall limit

The criterion used in the calculation is obtained from the experiments using the cascades with hub-tip ratio of 0.5 and all of them are 'tip critical'. The stall in the rotating cascades does not always appear near the tip but sometimes near the root(11). In the present cases, Deq near the root grows as large as (3.5~4.0) without experiencing a stall in the root region, but it may be reasonable to consider that the blade elements near the root have a finite aerodynamic loading limit. As shown in many papers(12)(13), however, a stall in the rotating cascades with low hub-tip ratios tends to occur near the tip, in addition the present results are obtained from the experiments using three cascades with different design parameters and blade geometries operating under various inlet-distortions. Therefore the criterion proposed above can be expected to be applicable to other rotating cascades with low hub-tip ratios.

5. Conclusions

A series of experiments on the operating performance of the rotating cascades under the radial inlet-distortions are made, and flow fields and aerodynamic loading are analyzed by stream-line curvature method. The results obtained are summarized as follows.

(1) The radial inlet-distortions are attenuated by the rotating cascades at both design and off-design points.

(2) The rotating cascades can operate without experiencing a stall under a larger aerodynamic loading near the root than the loading limit of the two-dimensional cascades.

(3) On the basis of the experiments using three test cascades with hub-tip ratio of 0.5, a criterion of the onset of a stall in the rotating cascades is proposed. That is 'a stall occurs when Deq at $r/r_0 = (0.925 \sim 0.95)$ reaches (1.9~2.1)':

(4) The attenuation of the inlet-distortion and the changes in aerodynamic loading calculated by stream-line curvature method can represent the experimental ones relatively well, and the stall limit agrees with the experimental one with sufficient accuracy.

References

- (1) Lieblein, S., et al., NACA RM, E56 B03a(1956).
- (2) Nagano, S., et al., Trans. Japan Soc. Mech. Engrs. (in Japanese), Vol. 39, No. 328(1973-12), p. 3653.
- (3) Serovy, G.K. and Lysen, J.C., Trans. ASME, Ser. A, Vol. 85, No. 1(1963-1), p. 1.
- (4) Shirato, T., et al., Trans. Japan Soc. Mech. Engrs. (in Japanese), Vol. 32, No. 234(1966-2), p. 288.
- (5) Novack, R.A., Trans. ASME, Ser. A, Vol. 89, No. 4(1967-10), p. 478.
- (6) Tanaka, S. and Murata, S., Bull. JSME, Vol. 18, No. 117(1975-3), p. 256.

- (7) Lieblein, S., Trans. ASME, Ser. D, Vol. 81, No. 3(1959-9), p. 387.
- (8) Swan, W.C., Trans. ASME, Ser. A, Vol. 83, No. 3(1961-7), p. 322.
- (9) Maekawa, A. and Tanaka, S., Preprint of Japan Soc. Mech. Engrs. (in Japanese), No. 764-4(1976-3), p. 46.
- (10) Lieblein, S., Trans. ASME, Ser. D, Vol. 82, No. 3(1960-9), p. 575.
- (11) Iura, T. and Rannie, W., Trans. ASME, Vol. 76, No. 3(1954-4), p. 463.
- (12) Hatta, K., et al., Bull. of the Aeron. Res. Inst. University of Tokyo (in Japanese), Vol. 1, No. 1(1959-3), p. 190.
- (13) Valensi, J., J. Aeron. Sci., Vol. 25, No. 1(1958-1), p. 1.