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Reattachment Flow Issuing from a Finite Width Nozzle*

By Tsutomu NOZAKI**, Keiji HATTA***,

Masahiro NAKASHIMA**** and Hirohisa MATSUMURA*****

It was found for reattachment jets having large offset ratios that an agreement is good between the experimental results and the results of the approximate calculations obtained by using the solutions of a two-dimensional jet. But flows having rather small offset ratios show different values from the results of calculation. Existence of similarity in the reattachment jet flows having large offset ratios can be well expressed by introducing an appropriate parameter which represents a bubble pressure. Experiments also show that the behaviours of a two-dimensional reattachment jet flow are independent of Reynolds number, but the behaviours of a reattachment jet flow are dependent on Reynolds number in the case of three-dimensional flows having small aspect ratios.

1. Introduction

Recently, many investigations concerning the effects of a reattachment jet flow have been done actively, which appear in flows in combustion chambers and reattachment type fluidic devices. The case of a flow having rather large offset ratios has been reported by Bourque et al.⁽¹⁾, Sawyer^{(2),(3)}, McRee et al.⁽⁴⁾ and Kumada et al.⁽⁵⁾ In most these investigations, the flows are independent of the nozzle width because of large offset ratios. But in the case of small offset ratios as a flow in the fluidic devices, the flow depends on the offset ratios. The experiments on flows having rather small offset ratios have scarcely been done, so far as the authors are aware. In this paper, the results of approximate calculations of reattachment streamline and reattachment distance and the experimental results of a flow having rather small offset ratios are reported.

The approximate calculations of a two-dimensional reattachment flow are done by using the solutions of a two-dimensional free jet flow issuing from a finite width nozzle⁽⁶⁾. The experiments of a two-dimensional reattachment flow (nozzle aspect ratio $AR = 8$) are done for comparison with calculated results. Experiments are also done in the case of flows having aspect ratios 3, 2 and 1 to compare the effect of

the nozzle aspect ratio.

A jet issuing from a finite width nozzle consists of two regions, one being the zone of flow establishment where the center line velocity is kept constant and the other the zone of established flow where the center line velocity decays. Therefore, the behaviour of the reattachment flow is divided into two types, one being a flow having small offset ratios and influenced fairly by the existence of the zone of flow establishment, and the other a flow having large offset ratios and not influenced by the existence of the potential core. For the discussions of the experimental results, a new coefficient of pressure is introduced.

2. Approximate calculation of a two-dimensional reattachment flow

A schema of a reattachment flow model and symbols are shown in Fig.1. Let the origin O be taken at the center of the nozzle.

Fig.1 Schema of a reattachment flow model

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** Lecturer, Faculty of Engineering, Kagoshima University, Koorimoto, Kagoshima.

*** Professor, Kyoto Institute of Technology.

**** Assistant, Kagoshima Technical College.

***** Professor, Faculty of Engineering, Kagoshima University.

zle exit, the axis of x in the direction of the flow at the nozzle exit, the axis of y perpendicular to the reattachment wall and the axis of z perpendicular to them. In this paper, the offset ratio is defined as the ratio of the distance between the x -axis and the reattachment wall to half width of the nozzle b_0 .

2.1 Center line of a curved jet

It is assumed that a constant pressure difference acts across the flow which is considered two-dimensional, steady and incompressible. Thus a jet center line is deflected by acceleration perpendicular to the streamwise direction. Let the axis x' be taken along the curved jet center line and the axis of y' perpendicular to x' -axis toward the center of curvature of the jet center line, as shown in Fig. 1. The velocity components in both directions are u and v , respectively. It is assumed that the velocity distribution of the free jet is still valid for the curved jet with this new coordinates system.

Let p denote the pressure, ρ the density of the fluid, ϵ the eddy kinematic viscosity and R the radius of curvature of the jet center line. The equation of motion considering the curvature of the x' -axis is given by

$$\frac{R}{R-y'} u \frac{\partial v}{\partial x'} + v \frac{\partial v}{\partial y'} + \frac{u^2}{R-y'} = -\frac{1}{\rho} \frac{\partial p}{\partial y'} + \epsilon \frac{\partial^2 v}{\partial y'^2} \quad \dots (1)$$

When the radius of curvature is large compared to the jet width, Eq.(1) is reduced to

$$u \frac{\partial v}{\partial x'} + v \frac{\partial v}{\partial y'} + \frac{u^2}{R} = -\frac{1}{\rho} \frac{\partial p}{\partial y'} + \epsilon \frac{\partial^2 v}{\partial y'^2} \quad \dots (2)$$

By integrating Eq.(2) with respect to y' from $-\infty$ to $+\infty$ considering the above assumptions, we obtain

$$\frac{R}{b_0} = \frac{2\rho U_0^2}{\Delta p}, \quad \dots (3)$$

where Δp denotes the pressure difference in both sides of the jet and U_0 the uniform velocity at the nozzle exit. It is assumed that the pressure difference Δp is constant regardless of x' , then Eq.(3) signifies that the radius of curvature of the curved jet center line is constant, that is, the center line is a circular arc, and it is determined by the momentum of the flow issuing from the nozzle per unit time, no matter how the velocity distribution of the jet is.

2.2 Reattachment streamline

In the schema of the flow model shown in Fig.1, it is assumed that there is no flow across the curved jet center line, thus there is no increase in flow in the reattachment wall side, since there is no entrainment flow from the surroundings. There is a streamline connecting the edge of the nozzle to a reattachment point, which is called a reattachment streamline. In order to calculate the equation of this streamline, the velocity distribution in the whole region of a non-reattachment side

of the flow and the region between the jet center line and the reattachment streamline is assumed to be the same as that of a free jet with respect to x' , y' -axes, as shown in Fig.1.

In the zone of flow establishment, let $2b_c$ denote the width of the invicid core having the uniform velocity U_0 , and $2b$ the width of the jet. By using a parameter

$$\eta' = \frac{y' - b_c}{b - b_c}, \quad \dots (4)$$

the velocity distribution in $b_c < y' < b$ is assumed as

$$u = U_0 f(\eta') \quad \dots (5)$$

In the zone of established flow, let U denote the velocity on the jet center line, and by using a parameter

$$\eta' = \frac{y'}{b}, \quad \dots (6)$$

the velocity distribution is assumed as

$$u = U f(\eta') \quad \dots (7)$$

The function $f(\eta')$ contained in Eqs.(5) and (7) is a polynomial of η' and is given as follows:

$$f(\eta') = 1 - 6\eta'^2 + 8\eta'^3 - 3\eta'^4 \quad \dots (8)$$

Let y_a' denote the distance between the jet center line and a reattachment streamline, then y_a' has to satisfy the following equation by conservation of mass:

$$\int_0^{y_a'} u dy' = U_0 b_0 \quad \dots (9)$$

In the zone of flow establishment, by putting $\eta' = \eta_a'$ for $y' = y_a'$ in Eq.(4) and substituting Eqs.(5) and (8) into Eq.(9), we obtain

$$\frac{b_c}{b_0} + \frac{b-b_c}{b_0} \int_0^{\eta_a'} (1 - 6\eta'^2 + 8\eta'^3 - 3\eta'^4) d\eta' = 1 \quad \dots (10)$$

By substituting the results of the calculation of b and b_c of a two-dimensional free jet issuing from a finite width nozzle⁽⁶⁾ into Eq.(10), $\eta_a' = 0.341$ is obtained regardless of x' . Then a reattachment streamline in the zone of flow establishment is given by

$$\frac{y_a'}{b_0} = 1 + 0.0196 \frac{x'}{b_0} \quad \dots (11)$$

In the zone of established flow, substituting Eqs.(7) and (8) into Eq.(9), we obtain

$$\frac{U}{U_0} \frac{b}{b_0} \int_0^{\eta_a'} (1 - 6\eta'^2 + 8\eta'^3 - 3\eta'^4) d\eta' = 1 \quad \dots (12)$$

By using the values of U and b of a two-dimensional free jet, Eq.(12) becomes

$$\left(0.0721 \frac{x'}{b_0} + 0.286\right)^{\frac{1}{2}} \left(\eta_a'^2 - 2\eta_a'^3 + 2\eta_a'^4 - \frac{3}{5}\eta_a'^5\right) = \frac{2}{7} \quad \dots (13)$$

Therefore, a reattachment streamline in the zone of established flow is given as follows by using η_a' which satisfies Eq.(13):

$$\frac{y_a'}{b_0} = \left(0.252 \frac{x'}{b_0} + 1.002\right) n_a'. \quad \dots (14)$$

2.3 Reattachment distance

A reattachment point is defined as a point of intersection of a reattachment streamline and a reattachment wall which is given by $y=h$. The distance between the nozzle exit plane and the reattachment point is called a reattachment distance x_a . The reattachment streamline is at the distance y_a' from the curved jet center line (x' -axis). In the zone of flow establishment, y_a' is calculated by Eq.(11), and in the zone of established flow by substituting the corresponding n_a' given by Eq.(13) into Eq.(14).

3. Experimental equipment and procedure

An outline of the experimental equipment and the detail of the test section of the two-dimensional reattachment flow are shown in Figs. 2 and 3, respectively. The aspect ratio of the nozzle 8 is used. Reynolds number is defined as $Re = 2 U_0 b_0 / \nu$ for convenience, where ν denotes the kinematic viscosity of air. The air flow from the centrifugal blower passes through three stainless steel screens in the plenum chamber to suppress the flow turbulence and then issues from the nozzle with a uniform

Fig.2 Experimental apparatus

Fig.3 Detail of the test section

velocity profile. In order to investigate the reattachment flow, the pressure distributions along the reattachment wall are measured. The pressure taps are bored on the wall in staggered arrangement near the nozzle exit as shown in Fig.3 to avoid interaction among them. A reattachment point where a flow divides is determined as the position where the velocity near the wall detected by a hot-wire probe shows the lowest value.

Similar equipment is also used in the case of a flow having the aspect ratios 3, 2 and 1. In order to keep the thickness of the boundary layer developing on the side walls equal, the nozzle width is kept constant at 25mm for every aspect ratio. Then the nozzle depths in Fig.3 are 75 mm, 50 mm and 25 mm according to the aspect ratios 3, 2 and 1 respectively.

4. Experimental results and discussions

4.1 Mean bubble pressure

Mean bubble pressure p_{mean} was proposed by Sawyer⁽²⁾, as a mean value of the pressure on the reattachment wall over the range of 95% distance from the nozzle plane to the position where the pressure along the reattachment wall recovers to the atmospheric pressure p_a . The coefficient of the mean bubble pressure is defined as

$$Cp_{mean} = \frac{p_{mean} - p_a}{\rho U_0^2 / 2}.$$

The relation between the offset ratio and the coefficient of the mean bubble pressure is shown in Fig.4 and compared with the results of other investigators. This relation shows somewhat different behaviour according as the offset ratios are smaller or larger than 12, as Kumada et al.⁽⁵⁾ pointed out. But $|Cp_{mean}|$ does not show a constant value for the flow having the offset ratio smaller than 12.

In the following calculations, the mean bubble pressure by Sawyer's proposal is used for Δp of Eq.(3). The relation between $|Cp_{mean}|$ and h/b_0 is represented experimentally by Kumada et al.⁽⁵⁾ as follows:

$$|Cp_{mean}| = 0.824 \left(\frac{h}{b_0}\right)^{-0.75} \quad \dots (15)$$

For the offset ratio smaller than 12, $|Cp_{mean}|$

Fig.4 Mean bubble pressure

is represented by the equation

$$|C_{p\text{mean}}| = 0.193 \left(\frac{h}{b_0} \right)^{-0.164}, \dots (16)$$

from the authors' experimental results for $AR=8$.

4.2 Pressure distributions along the reattachment wall

The pressure distributions along the reattachment wall are shown in Fig.5 in the case of aspect ratio 8, by using a pressure coefficient defined by

$$C_p = \frac{p_s - p_a}{\rho U_0^2 / 2}, \dots (17)$$

where p_s is a pressure on the reattachment wall and p_a the atmospheric pressure. It is found from this figure that the pressure distributions are not much influenced by Reynolds number in its range of the present experiments. In the case of $AR=2$ also, the pressure distributions are almost independent of Reynolds number, as shown in Fig.6, except that the position of the maximum pressure moves slightly in the up-

Fig.5 Pressure distributions along the wall ($AR=8$)

Fig.6 Pressure distributions along the wall ($AR=2$)

stream direction for Reynolds number 1.0×10^4 . But the pressure distributions are influenced by Reynolds number in the case of $AR=1$, as shown in Fig.7. As Reynolds number increases, the maximum pressure points move gradually in the upstream direction and the bubble pressures tend to decrease. This is due to the effects of side walls. As it is evident from the isotach patterns of the bounded jet of aspect ratio unity which has no reattachment wall (Fig.8), the effect of side walls on the flow pattern already spreads all over the jet at $x/b_0=4.0$ and this effect changes with Reynolds numbers.

In representing the pressure distribution along the reattachment wall, a pressure coefficient normalized by the dynamic pressure at the nozzle exit, Eq.(17), has been used. This is, however, not adequate to investigate the similarity of a reattachment flow having different offset ratios, because the pressure along the wall may be dependent upon the jet velocity just before reattachment. Therefore, an imaginary velocity on the jet center line just before reattachment is used instead of U_0 .

By substituting $p_{\text{mean}} - p_a$ into Δp of Eq.(3) and using a mean pressure coefficient, the radius of curvature of the jet center line is given as follows:

$$\frac{R}{b_0} = \frac{4}{|C_{p\text{mean}}|}, \dots (18)$$

Also by substituting Eq.(15) or (16) into $|C_{p\text{mean}}|$, a circular arc expressing the jet center line is given. Let x_c' denote the distance between a nozzle exit and the reattachment wall along the circular arc and U the velocity on the jet center line at $x' = x_c'$ supposing that there is no reattachment wall. For the value of U the velocity at $x = x_c'$ in a free jet is used. In order to express the pressure along the reattachment wall, a new pressure coefficient

$$C_{p^*} = \frac{p_s - p_a}{\rho U^2 / 2}, \dots (19)$$

Fig.7 Pressure distributions along the wall ($AR=1$)

is introduced instead of C_p in Eq.(17). In Eq.(19) $U=U_0$ for reattachment in the zone of flow establishment and

$$\frac{U}{U_0} = \left(0.0721 \frac{x'}{b_0} + 0.286\right)^{-\frac{1}{2}} \dots\dots (20)$$

for the zone of established flow.

Non-dimensional parameter x/b_0 has been used as a value which represents a distance along the reattachment wall. But from the experimental results by Bourque et al.⁽¹⁾, Sawyer^{(2),(3)} and Kumada et al.⁽⁵⁾, as shown in Fig.9, the ratio x_a to h is considered to be as follows:

$$x_a/h = 1.4 (\text{constant}) \dots\dots (21)$$

Fig.8 Isotach patterns of the bounded jet ($AR=1$, $x/b_0=4.0$)

Fig.9 Relations between reattachment distance and offset ratio

Fig.10 Pressure distributions along the wall ($AR=8$)

for large offset ratios. Therefore it is considered reasonable to use x/h instead of x/b_0 . Although the above mentioned investigators showed distances between a nozzle plane and a maximum pressure point on the reattachment wall, these values are not much different from the reattachment distance x_a defined in the present paper in the case of large offset ratios, so they are shown together in Fig.9 and compared with the authors' experimental results.

The relations between a pressure coefficient C_p^* and x/h for the flow $AR=8$ are shown in Fig.10. This result may suggest that pressure distributions tend to coincide as the offset ratio becomes large.

But for small offset ratios they do not coincide. This is obvious from the fact that x_a/h is not kept constant as offset ratio changes, as shown in Fig.9. This tendency is almost the same for different aspect ratios as shown in Figs.11 to 13.

The relations between offset ratio and the maximum pressure coefficient $C_{p_{max}}^*$ or the minimum pressure coefficient $C_{p_{min}}^*$ are shown in Fig.14 together with the results of other investigators in the case of large offset ratios. These results suggest that both $C_{p_{max}}^*$ and $C_{p_{min}}^*$ tend to approach constant values respectively as the offset ratio becomes large. In the case of small offset ratios the flow reattaches near the nozzle exit, thus phenomenon of reattachment is scarcely influenced by the side walls even if the aspect ratio is small. Therefore the values of $C_{p_{max}}^*$ and $C_{p_{min}}^*$ for different aspect ratios of the nozzle

Fig.11 Pressure distributions along the wall ($AR=3$)

agree respectively.

Assuming a force to deflect the jet, a mean bubble pressure should be taken into consideration. The mean bubble pressure determined by Sawyer's proposal is shown in Fig.15 by using a new pressure coefficient. It appears that $C_{p\text{mean}}^*$ tends to approach a constant value as the offset ratio becomes large.

4.3 Reattachment distance

The reattachment distance obtained by the approximate calculation is compared with the experimental results. The radius of curvature of the jet center line is calculated from Eq.(18) by using $C_{p\text{mean}}$ of Eq.(15) in the case of a flow having large offset ratios and of Eq.(16) for h/b_0 smaller than 12, respectively. The equation of

Fig.12 Pressure distributions along the wall ($AR=2$)

Fig.13 Pressure distributions along the wall ($AR=1$)

the reattachment streamline referred to this jet center line is given by Eq.(14) with n_a' given by Eq.(11) in the zone of flow establishment and by Eq.(13) in the zone of established flow. Some of them are shown in Fig.16. A reattachment distance x_a is determined as a point of intersection of the reattachment wall and the reattachment streamline. As the offset ratio becomes small, the calculated reattachment distance becomes considerably larger than the experimental results. Milne-Thomson⁽⁷⁾ proved that the streamlines must intersect each other perpendicularly at a stagnation point for a potential flow. This condition seems to hold approximately for a reattachment flow as seen from the experimental results shown in Fig.17. The angle between

Fig.14 Maximum and minimum pressures on the wall

Fig.15 Mean bubble pressure

Fig.16 Examples of calculated reattachment streamline

a reattachment streamline and a reattachment wall shown in Fig.16 becomes considerably smaller than right angle, as the offset ratio decreases. This may be a reason for disagreement of the calculated and experimental results, comparison of which is made in Fig.9, where the calculated one is shown by broken lines. Reattachment distance x_a obtained by the experiments is smaller than the calculated value especially for small offset ratio.

Here a formula expressing the relation between x_a and h will be derived. Assump-

Fig.17 Reattachment streamline

Fig.18 Relation between reattachment distance and step ratio

Fig.19 Relations between reattachment distance and Reynolds number

tion is made that in the case of a flow having large offset ratio the reattachment streamline is approximately a circular arc which has the center on y -axis and radius R' , then its equation is

$$\left(\frac{R'}{b_0}\right)^2 = \left(\frac{x_a}{b_0}\right)^2 + \left\{\frac{R'}{b_0} - \left(\frac{h}{b_0} - 1\right)\right\}^2 \quad \dots\dots(22)$$

In this equation unity can be neglected compared with $(R' - h)/b_0$ when R' is large, and since R'/h is constant from the existence of the similarity of the flow having large offset ratios, we obtain from Eq.(22)

$$\frac{x_a}{h} = \sqrt{2C - 1}, \quad \dots\dots(23)$$

where $C = R'/h$. This equation has to agree with Eq.(21). When the offset ratio is sufficiently large, R' is nearly equal to R which is the radius of curvature of a jet center line. As shown in Fig.4 and Eq.(15), $|C_{p_{mean}}|$ is inversely proportional to $h^{3/4}$, thus R is proportional to $h^{3/4}$ from Eq.(18) in the range of offset ratios 12 to 70. Therefore, R' is also proportional to $h^{3/4}$, and Eq.(23) based on the assumption of $R' \propto h$ is slightly modified as follows:

$$\frac{x_a}{b_0} = 1.4 \frac{h}{b_0} + 4.0 \quad \dots\dots(24)$$

This equation agrees well with experimental results in the range of $h/b_0 \geq 6$ as shown in Fig.9.

Notwithstanding this modification, Eq.(24) does not hold in the case of a flow having small offset ratios, because the relation between R and h is different from that in a flow having large offset ratios and the existence of the zone of flow establishment affects the reattachment. In this case, the reattachment distance is given by

$$\frac{x_a}{b_0} = 3.51 \left(\frac{h}{b_0} - 1\right)^{0.754} \quad \dots\dots(25)$$

approximately, as shown in Fig.18, using the distance between a nozzle exit wall and a reattachment wall or a step ratio instead of an offset ratio...

The relation between a reattachment distance and Reynolds number is shown in Fig.19, using offset ratio as a parameter. The reattachment distance is not influenced by Reynolds number in the range from 2.0×10^4 to 7.0×10^4 in the case of $AR = 8$ shown in Fig.19(a). But in the case of $AR = 3, 2$ and 1 shown in the same figure (b), (c) and (d), respectively, the effect of Reynolds number increases as the aspect ratio decreases in the above mentioned range of Reynolds numbers.

5 Conclusions

The approximate solutions of a two-dimensional free jet issuing from a finite width nozzle are applied to a model of a reattachment jet, and a reattachment streamline is calculated. A reattachment distance for each offset ratio is calculated from a reattachment point assumed as the point where a reattachment streamline intersects with the reattachment wall. These results of the approximate calculations agree well with the experimental results for large offset ratios, but it is difficult to express the relation between the reattachment distance and the offset ratio in one equation. Therefore a reattachment streamline is considered a circular arc approximately, and a simple formula expressing the relation between reattachment distance and offset is derived for relatively large offset ratios. But this formula does not agree with the results of experiment for offset ratios smaller than 6. It is shown experimentally that a step height $h-b_0$ is a suitable parameter for expressing a flow having a small offset ratio. For this flow an experimental formula expressing the relation between step height and reattachment distance is also obtained.

Since the usual method to normalize a pressure on the reattachment wall by a dynamic pressure at the nozzle exit is not suitable to consider the similarity of a

reattachment flow, a new pressure coefficient is introduced here on the basis of the concept that the pressure depends on the velocity just before reattachment. Similarity of a reattachment flow for large offset ratio is confirmed by introducing a new parameter.

It is found from the experimental results of a two-dimensional reattachment jet having $AR=8$ that the reattachment distance is independent of Reynolds number in its range from 2.0×10^4 to 7.0×10^4 , whereas the effect of Reynolds number in the same range increases as an aspect ratio decreases.

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