A Method of Absorption for Surge Pressure in Conduits

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Surge pressure in a liquid conduit is often amplified extremely in the presence of trapped air or gases. In this paper the hydraulic transients in such cases were investigated, and on the basis of the results a method of absorption for surge pressure by means of an air chamber at the downstream end of conduit having a nozzle was studied. An appropriate arrangement of values of parameters in the presence of trapped air or gases absorbs the surge pressure most effectively.

1. Introduction

In many problems of long liquid conduit, it is important to predict and to absorb a surge pressure associated with a hydraulic transient. Hydraulic transients in liquid conduits such as waterhammer or oilhammer have been extensively studied. But most of the previous studies were restricted to the case where conduits were filled up with liquid. In practical long conduit, air can enter the liquid line and some gases can be evolved from liquid. These gaseous substances often form cavities in conduit. Although it is necessary to study the hydraulic transients in the above mentioned condition, only a few authors treated this problem.

When a gas such as air forms a cavity, a surge pressure is usually damped by the air cavity since the gas behaves as a cushion. But the air cavity does not always absorb the surge pressure. The author has already revealed that under certain conditions the surge pressure is enhanced by such an air existence. Consequently it becomes necessary to make a study on absorption of surge pressure under such conditions. Although Ichiru proposed in his paper an accumulator as a means for absorption of a surge pressure, many problems relating to the pressure absorption are not yet solved.

The author studied the hydraulic transient in a liquid conduit containing air cavities by precise experiments and analysis. The results of the study are used to develop a method for pressure surge reduction, where an air chamber with inlet restriction is attached at the end of the conduit. The surge reducing effect of parameters connected to the air chamber is studied in detail.

Nomenclature

\[ A_n = \frac{L_{eq}}{\alpha} \gamma p, \]
\[ a = \text{speed of pressure pulse} \]
\[ c, c' = \text{characteristic line} \]

\[ c : \text{discharge coefficient} \]
\[ D : \text{internal diameter of conduit} \]
\[ d_n : \text{diameter of nozzle} \]
\[ g : \text{gravitational acceleration} \]
\[ H : \text{pressure head} \]
\[ H' = H/(\rho_0 / \rho) \]
\[ h_i : \text{pressure head loss per unit length} \]
\[ h_{ij} = \gamma g L A_i / \rho \]
\[ K_i : \text{coefficient defined by Eq.(5)} \]
\[ K : \text{coefficient defined by Eq.(9)} \]
\[ k_i = K_i (p_i / \rho_i) \]
\[ k_i = K_i (p_i / \rho_i) \]
\[ L : \text{length of conduit} \]
\[ L_{eq} : \text{equivalent conduit length of air volume at initial condition} \]
\[ N : \text{number into which the conduit is subdivided for computations} \]
\[ p : \text{absolute pressure} \]
\[ p_i : \text{gauge pressure} \]
\[ p_i = p_0 \]
\[ p_i : \text{supplied pressure} \]
\[ p_i : \text{air volume} \]
\[ W : \text{air volume} \]
\[ W' = W / W_i \]
\[ \mu : \text{weighting function} \]
\[ x : \text{distance along conduit from valve} \]
\[ x' = x / L \]
\[ x_i : \text{distance from valve to the first air cavity} \]
\[ x_i = x / L \]
\[ z : \text{distance from valve to the second air cavity} \]
\[ z_i = x / L \]
\[ \alpha : \text{hydraulic impedance} \]
\[ \alpha : \text{rate of reduction of surge pressure} \]
\[ \alpha : \text{ratio of specific heat} \]
\[ \alpha : \text{kinematic viscosity} \]
\[ \varrho : \text{density} \]
\[ \alpha : \text{angular velocity} \]

Subscripts

\[ 0 : \text{initial value} \]
\[ A, B, C, P : \text{grid point} \]
\[ d : \text{air chamber} \]
\[ m : \text{air cavity at the middle of conduit} \]

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2. The method of numerical calculation

The boundary condition to calculate the hydraulic transients becomes complicated when air forms a cavity in conduit. In this paper the characteristics method is employed to analyze such hydraulic transients.

Schematic model of the system to be analyzed is shown in Fig.1. In Fig.1(a), one end of the conduit is connected to a reservoir with a constant head through a valve, and the other end is closed. After the pressure in conduit is equalized with the atmospheric pressure, the valve at conduit end is suddenly opened. Thus a step input of pressure equivalent to the pressure head of the reservoir is given to the conduit end. Pressure variation at the closed end of the conduit would be a well-known rectangular pulse, if the effect of viscosity and other minor effects were ignored for simplicity. However, if there exists an air cavity or chamber shown in Fig.1(b), (c), (d), the pressure variations will become more complicated.

(a) conduit without an air cavity

(b) conduit with an air cavity

(c) conduit with an air chamber

(d) conduit with an air cavity and an air chamber

Fig.1 Schematic models of conduit

The equations of motion and continuity for one-dimensional flow in horizontal conduit are given as follows.

\[ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} \frac{\partial H}{\partial x} + \frac{\partial h}{\partial t} = 0 \]  \hspace{1cm} (1)

\[ \frac{\partial H}{\partial t} + \frac{\partial H}{\partial x} \frac{\partial V}{\partial x} + \frac{\partial h}{\partial x} = 0 \]  \hspace{1cm} (2)

These equations can be transformed into a pair of total differential equations by the characteristics method. These total differential equations can be transformed into the following difference equations if the difference is taken on the characteristic lines C and C’ in Fig.2, provided that the second terms in Eq.(1) and (2) are omitted since \( V < a \).

\[ C - V_{P} = C_{0} - C_{D} \frac{t}{H} \]  \hspace{1cm} (3)

\[ C' - V_{P} = C_{0} - C_{D} \frac{t'}{H} \]  \hspace{1cm} (4)

where

\[ C - V_{P} = C_{0} - C_{D} \frac{t}{H} \]

\[ C' = \gamma \]

\[ C_{0} = V_{P} \]

\[ \alpha = \frac{1}{L} \]

\[ \alpha' = \frac{1}{L'} \]

2.1 Boundary condition at the valve

The flow through the valve can be represented as follows.

\[ Q = \frac{k}{V} \frac{V}{H_{A}} \frac{V}{H_{B}} \]  \hspace{1cm} (5)

Although \( K \) varies with time during valve stroking, it can be regarded as a constant at \( t > 0 \), since the valve opening time is short enough compared with the wave return-travel time \( 2L/a \). Unknown variables at the valve in Fig.3 are determined using Eqs.(4) and (5).

2.2 Boundary condition at the air cavity

If the change of the state of the air in the cavity is assumed as adiabatic, we have \( P_{W_{0}} = p_{W_{0}} \) \hspace{1cm} (6)

The relations between the air volume and the velocity at the air cavity in Fig.4 are as follows.

- case of Fig.1(a)

\[ \frac{dV_{P}}{dt} = -SV \]  \hspace{1cm} (7)

- case of Fig.1(b)

\[ \frac{dV_{P}}{dt} = -S(V_{P} - V_{A}) \]  \hspace{1cm} (8)

These two equations will be easily solved if the rectangular approximation is used. But the method can give satisfactory result only when the step size is very small. Therefore these equations are transformed into a linear finite-difference form by the trapezoidal approximation. The equations giving this boundary condition cannot be solved analytically, hence the unknown variables are determined using the Newton-Raphson method.

2.3 Boundary condition at the air chamber

The equation of inlet nozzle of an air chamber at the closed end of conduit in Fig.5 is as follows.

\[ V_{P} = \frac{k}{V_{P}} \frac{V_{P}}{H_{A}} \frac{V_{P}}{H_{B}} \]  \hspace{1cm} (9)

where \( k = \sqrt{2gH_{0}} \)

In the same way as with an air cavity, we have equations (10) and (11).

\[ P_{W} = P_{W_{0}} \]  \hspace{1cm} (10)
Finally, for the expression of $W_{re}$, we have

$$W_{re} = W_{re} + \frac{3}{2} \left( \frac{V_p + V_e}{C_0} \right)^{1/2} \left( \frac{V_p + V_e}{C_0} \right) \frac{V_e}{C_0}$$

where $H_r = 1/2 \left( 2A + B^r \right) - \left( B^r + 1 \right)^2 \left( C - A \right)$

$$A = \frac{C_0}{C_1}, \quad B = \frac{K_e}{C_1}, \quad C = \frac{P_t \left( W_{re} \right)}{r \left( W_{re} \right)} - 1$$

$W_{re}$ is determined using the Newton-Raphson method, because it is also unsolvable analytically.

3. Experiment

3.1 Experimental setup

Schematic representation of the experimental setup is shown in Fig. 6. It is constructed with an oil hydraulic power source and a liquid conduit. The hydraulic pump together with a relief valve and an accumulator of the oil hydraulic power source generates an oil flow with constant pressure head. The volume of the accumulator is $25.5 \times 10^{-3}$ m$^3$. One end of the liquid conduit is connected to the power source with a directional control valve. The conduit is approximately $35.7$ mm inside diameter and $106$ m long. The conduit consists of commercial pipes for gas line connected to other with flanges. Any protuberances of flanges into the inside of conduit are completely rejected. To compose the air chamber or air cavity, containers shown in Fig. 7 are set at selected flanges. The containers are made of plexiglass pipes, so that we can settle air volumes easily. When the container is used as an air chamber, a nozzle is attached at its inlet as a restriction. Pressure transducers of piezo-resistance type are set at the power source, at the air cavity and at the closed end of the conduit. Outputs of the transducers are recorded on an electro-magnetic oscillograph.

3.2 Experimental procedure

Before the experiment is carried out, the oil in conduit is sufficiently circulated to obtain uniform temperature distribution and to remove all the extra trapped air in the conduit. Then the pressure in conduit is set to the atmospheric pressure, and the volumes of the air cavity and air chamber are settled.

The speed of pressure pulse in conduit was measured to be $1210$ m/s.

The valve opening time was set at about $0.014$ s, which was short enough com-
pared with the wave return-travel time.

4. Experimental and calculated result

4.1 Hydraulic transient in the conduit containing an air cavity

The surge pressure observed with the conduit setup shown in Fig.1(b) is greater than that in Fig.1(a). Fig.8 shows an experimental trace of the pressure variation at the closed end of the conduit containing an air cavity at the end. Where the supplied pressure \( p \) from the power source is equal to 9.1 bar, and kinematic viscosity of oil is \( 3.3 \times 10^{-6} \text{ m}^2/\text{s} \). It is well-known that the surge pressure is about two times the supplied pressure, when no air cavity exists in the conduit. Fig.8 shows that the surge pressure becomes about three times the supplied pressure in the existence of the air whose volume is only about 0.1% of that of the oil in conduit. It is clear that the existence of air in conduit causes a remarkable increase of surge pressure.

![Fig.8 Experimental trace of pressure surge \((L_e=1.0 \text{ m})\)](image)

The relation between surge pressure and volume of air cavity at the midpoint \( x=52.8 \text{ m} \) of the conduit is given in Fig.9. Since the air cavity is not settled exactly at the equally subdivided point of the conduit, for the numerical calculation the air is assumed to exist at the nearest subdivided point to the actual settled point. The supplied pressure is slightly lowered due to the insufficiency of volume of accumulator in the power source when volume of the air cavity is extremely large. To compare the experimental result with the corresponding theoretical value, \( p \) measured pressure at the power source \( p_e \) is used for numerical calculation. In Fig.9 the surge pressure increases stepwise with a smooth increase of air volume. The phenomena are caused by the repeated wave transmissions and reflexions.

Surge pressure takes a maximum value at a certain value of air volume. Fig.10 shows the relation between this maximum value and the position of the air cavity. The maximum value of the surge pressure at the closed end of conduit maintains almost a constant value, whereas that at the air cavity decreases as the position of the air cavity \( x \) is located farther from the closed end. This shows that the air cavity affects the surge pressure at the nearer point to the closed end of conduit more strongly.

![Fig.10 Maximum value of surge pressure versus location of air cavity](image)

4.2 Influence of the air chamber with an inlet restriction on the surge pressure

An accumulator or an air chamber is often used to reduce the surge pressure in a conduit. This is a utilization of a gas as a buffer. However the existence of gas
in liquid conduit does not necessarily reduce the surge pressure, as was explained in the preceding section. To reduce the surge pressure, an inlet restriction at an air chamber plays an important role.

A pressure surge generated at any point in a circuit propagates along the conduit, and at a point where the homogeneity of the conduit is lost, e.g., a transition to a hydraulic element or change of conduit configuration. Consequently, the surge pressure changes. The surge pressure changes extremely, if the conduit is closed. Although a circuit with a closed end is practically rare, the similar circumstances can be produced through the valve operation. Hence the air chamber with inlet restriction to reduce the surge pressure should be set at the closed end of conduit.

4.2.1 Conduit without air cavity

For the inlet restriction in conduit model shown in Fig.1(c), one of the three semicircular edged nozzles in Fig.11 is used. Supplied pressure from the power source is 10.2 bars. Fig.12 reproduces the traces of the pressure variation at the closed end with an air chamber of \( L_a = 0.4 \text{ m} \). Though the experimental and calculated results do not agree well during transient time, the surge pressures show a good agreement. Fig.13 shows relation between the surge pressure and volume of the air chamber where the parameter is diameter of the nozzle. Under the assumption of sufficient pressure recovery in the air chamber, discharge coefficient of the nozzle is taken as 0.95. It is seen in Fig.12 that the surge pressure is amplified at \( d_n = 10.2 \text{ mm} \) and it begins to be reduced at \( d_n = 6.4 \text{ mm} \). During experiments a cavitation was observed at the inlet restriction of the air chamber, but it did not affect the surge pressure at all.

4.2.2 Conduit with an air cavity

It was shown in section 4.1 that the air cavity causes an extreme amplification of the surge pressure. Therefore, in this case the effect of surge reduction under use of an air chamber will be different from that in the case without an air cavity.

Using the conduit configuration shown in Fig.1(d), experiments and numerical calculation similar to the previous section were performed under various combinations of volumes of the air cavity and its position in the conduit.

Fig.14(a), (b) show the surge pressures at the closed end versus the volume of air cavity, where the parameter is the volume of the air chamber. In both of Fig.14(a) and (b), experimental and calculated results show a good agreement.

Experimental values of the surge pressure at the inlet of the air chamber are given in Fig.15. Variation of surge pressure is small when position of the air cavity is varied from \( n' = 0 \), namely just behind the directional control valve, to \( n' = 1 \), namely at the closed end of the conduit.

5. Discussions

5.1 Nondimensional parameters affecting hydraulic transients

Following nondimensional variables are introduced for the basic equations and boundary conditions to describe the hydraulic transients.
Fig. 14 Surge pressure

\[
\frac{\Delta p}{p_i} = \frac{\Delta p'}{p_i'} = \frac{V - V'}{V} \quad \frac{H}{H_i} = \frac{H'}{H_i'} = \frac{W - W'}{W} \quad \frac{F_i}{F_i} = \frac{F_i'}{F_i'} = \frac{Q_i}{Q_i'}
\]

Then the equations can be nondimensionalized and the following nondimensional parameters are induced:

\[
A_1 = \frac{\Delta p}{p_i}, \quad A_2 = \frac{\Delta p'}{p_i'}, \quad A_3 = \frac{V}{V'}, \quad A_4 = \frac{W}{W'}
\]

\[
k_1 = \frac{K_i}{(g/\alpha)^{1/2}p_i/p_i'}, \quad k_2 = \frac{K_i}{(g/\alpha)^{1/2}p_i/p_i'}
\]

5.2 Surge reduction by an air chamber with an inlet restriction (conduit without air cavity)

Surge pressure at the closed end of the conduit is varied effectively by the nondimensional nozzle area \( S' \), and it takes a minimum value at a certain value of \( S' \). Fig. 16 shows the calculated results of surge pressure with 10.2 bars of supplied pressure \( p \), where the volume of the air chamber is taken as parameter. We define \( \alpha \) as the rate of reduction of the surge pressure:

\[
\alpha = \frac{\text{surge pressure with the air chamber}}{\text{surge pressure without the air chamber}}
\]

Conditions for the numerator and denominator are the same except presence of an air chamber or none. The surge pressure reduction as an effect of the air chamber corresponds to \( \alpha < 1 \). \( \alpha \) becomes greater than unity when \( S' \) is increased, that is to say the pressure is enhanced by the air chamber. Consequently \( S' \) should be selected such as to make \( \alpha \) less than unity. The upper limit of \( S' \) for \( \alpha \) smaller than unity varies with the volume of the air chamber. Fig. 17 shows this upper limit of \( S' \) satisfying the condition \( \alpha < 1 \). The upper limit of \( S' \) tends to decrease with an increase of the supplied pressure. In Fig. 17 the upper limits of \( S' \) in cases where the restriction of the valve or the viscous loss is ignored are shown also. The upper limits of \( S' \) suffer the same effect as if the supplied pressure were increased, when these losses are neglected.

5.3 Estimation of the optimal diameter of the nozzle using an acoustical approximation

To reduce the surge pressure at the end of the conduit, conditions of pressure wave reflection at the end should be investigated. The condition of wave reflection at the conduit end can be changed by setting an air chamber. The surge pressure at that position can be reduced when the condition of reflection is simulated to the condition of no-reflection.
Although the boundary condition at the conduit ends with the air chamber is nonlinear, we can suppose through linear approximation that there exists a boundary condition where the surge pressure is reduced most effectively with the aid of acoustic theory. If the equations are linearized in the neighbourhood of the initial value, the hydraulic impedance at the end of conduct is expressed as follows.

\[ Z = R + \frac{P_s}{\frac{1}{W_s} Jw} \]

where \( R \) is the coefficient of the nozzle restriction. The characteristic impedance in a frictionless system is as follows.

\[ Z = \frac{\rho u}{S} \]

Substituting \( Z \) of Eq.(18) into \( Z \) of Eq.(17), we have as the condition of no reflection the following expressions.

\[ \frac{P_s}{W_s} = 0 \]

\[ \frac{\rho u S}{S} = R \]

Eq.(19) means that the volume of air chamber \( W_s \) should be sufficiently large. Since discharge characteristic of the nozzle is expressed by Eq.(9), \( R \) in Eq.(20) is not constant but varies with the differential pressure across the nozzle. When we take a supplied pressure as the reference of the differential pressure, we have from Eq.(16) and Eq.(20).

\[ R = \frac{1}{\rho u} \int_0^1 \frac{dp}{dt} = \frac{S}{S} \]

\[ R = \frac{A_t}{A_t} \]

If \( A_t \) is used instead of \( R \) in Eq.(20), \( k \), becomes \( 2/3 \). Fig.18 shows the minimum value of \( a \) and the corresponding value of \( k \). With an increase of \( A_t \), \( k \) approaches the value obtained from the acoustic approximation. At the same time \( a \) approaches about 0.5, where the surge pressure is reduced effectively.

5.4 Surge reduction by an air chamber (conduit with air cavities)

Since the surge pressure can be amplified by the existence of air cavities, the procedure to handle the inlet nozzle should be altered from the method shown in the preceding section. Under any combinations of the volume of the air chamber, diameter of the nozzle and position of the air chamber, the surge pressure at the conduit end takes a maximum value at a certain volume of air cavity as shown in Fig.14. The maximum value of surge pressure is shown in Fig.19 where the volume of the air chamber and the position of the air cavity are taken as parameters. The maximum surge pressure takes the smallest value when \( S_r \) takes a certain value. When \( S_r \) is less than the value which makes the maximum value of surge pressure minimum, the maximum value of surge pressure is determined only by the position of air cavity and is not affected by the volume of the air chamber. Conversely, when \( S_r \) is greater than the critical value, the result is inverted. At the same time the maximum value of surge pressure becomes identical with the value when no air cavity exists in the conduit, which is represented by a broken line in Fig.19. This shows that the maximum value of surge pressure is determined by air cavity when the nozzle area is small and by air chamber when the nozzle area is large.

Although the transient phenomena in conduit containing two or more air cavities become more complicated, they have characters similar to that of a single air cavity. Namely, the maximum surge pressure is determined by air cavities or air chamber owing to the value of nondimensionalized nozzle opening. Fig.20 shows the maximum value of surge pressure in conduit contain-
When the conduit is filled up with liquid, surge pressure can be reduced most effectively by an appropriately designed air chamber. The optimal dimension of inlet restriction at the air chamber for surge reduction can be estimated using the acoustical approximation.

For a conduit with air cavities there exists an optimal combination of parameters relating to air chamber for the most effective surge reduction. Since the optimal dimensions vary with the number of air cavities, it is necessary to determine the system parameters appropriately according to the system configuration. The optimal dimensions for surge reduction in each case can be determined by numerical calculations.

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References


Fig.20 Maximum value of surge pressure

6. Conclusions

A method for absorption of surge pressure by means of an air chamber having an inlet restriction at the end of conduit was studied, and its validity was verified.