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A Transmissibility for Single-Loop Spatial Mechanisms*

By Hiroshi SHIMOJIMA**, Kiyoshi OGAWA***

and Toru KAWANO****

Transmissibilities for single-loop spatial mechanisms are expressed quantitatively, and a guide for designing mechanisms with sufficient movability is suggested by considering the flow of power.

One of the former transmission indices is improved with respect to its application. Then two methods for designing the mechanisms with sufficient movability are suggested: one is to synthesize the mechanism considering the above transmission index, and the other is controlling the flow of power by attaching supplementary mechanisms. Such methods are applied to a spatial path-generating mechanism, and the relationships among the transmission index, power and the movability are investigated theoretically and experimentally.

1. Introduction

As a general case, not only the output accuracy but also the transmissibility of a mechanism should be taken into consideration in synthesis of a non-uniform motion mechanism. The methods for expressing the transmissibility quantitatively have been considered, and now, a pressure angle or a transmission angle^{(1)~(3)} is generally used for a cam or a simple planar link mechanism. And further some expressions of transmissibility applicable to certain spatial mechanisms have been suggested^{(4), (5)}.

In this paper, the range of applications of the above transmissibility is magnified by determining the output loads, and the transmissibility of a general spatial mechanism is expressed by a transmission index. Then two methods are suggested to obtain a mechanism with sufficient movability; one is to consider the transmission index, and the other is to regulate the power by attaching another mechanism. Such methods are applied to a spatial path-generator⁽⁶⁾, and are investigated theoretically and experimentally.

2. Transmission index of single loop mechanisms

2.1 Definition of transmission index

The pressure angle applicable to planar simple link mechanisms has been defined as a ratio of the magnitude of the force acting on the output link to its component along the velocity of the link^{(1)~(3)}. The transmission index of a spatial mechanism has been defined, similarly, as follows. The force and moment acting on the output link

can be represented by use of the motor⁽⁷⁾, as shown in Fig.1.

$$\hat{F} = (F + \epsilon M)a = F(1 + \epsilon p_f)(a + \epsilon a_0) \quad \dots\dots(1)$$

where F, M ; magnitudes of the force and moment

$p_f (= M/F)$; pitch of the force

a ; unit motor

a, a_0 ; unit vectors

And the velocity screw of the output link is represented by

$$\hat{V} = (Q + \epsilon V)b = Q(1 + \epsilon p_v)(b + \epsilon b_0) \quad \dots\dots(2)$$

where V, Q ; magnitudes of the velocity and angular velocity

$p_v (= V/Q)$; pitch of the displacement

b ; unit motor on the instantaneous screw axis

Then the power transmitted to the output link is represented by

$$P = \text{Dual}(\hat{F}, \hat{V}) = FQ((p_f + p_v)\cos\beta - r\sin\beta) \quad \dots\dots(3)$$

where r, β ; distance and angle between a , and b . The maximum power when the force changes only its direction is given by

$$P_{\max} = FQ\sqrt{(p_f + p_v)^2 + r^2} \quad \dots\dots(4)$$

Then the ratio TI of P to $P_{\max}$ is shown as follows.

Fig.1

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** Associate Professor, Tokyo Institute of Technology, Ookayama, Meguro-ku, Tokyo.

*** Professor, Tokyo Institute of Technology.

**** Staff, Japan Air Line Co., LTD.

$$TI = \frac{P}{P_{\max}} = \frac{(p_f + p_e) \cos \beta - r \sin \beta}{\sqrt{(p_f + p_e)^2 + r^2_{\max}}} \quad (5)$$

TI is called the transmission index⁽⁵⁾, and may be used for representing the transmissibility. From Eq.(5), $0 \leq TI \leq 1$, and $TI=0$ means that the power cannot be transmitted from the input link to output link, and $TI=1$ means that the maximum power is transmitted. In the case of the planar 4-link mechanism with four revolute pairs, $TI = r/r_{\max}$, which coincides with cosine of the pressure angle. Eq.(5) is applicable even when the magnitude of the pair and the friction force are taken into consideration. The joint force and its acting point of the above mechanism are, for instance, shown by $\hat{F}_2$, R_2 in Fig.2 when the magnitude of the joint is considered, and further by $\hat{F}_3$, R_3 when the friction force of the joint is considered.

2.2 Application

In order to obtain the transmission index by applying Eq.(5) to an actual mechanism, the joint force and moment should be known beforehand. Therefore the applicable mechanisms have been restricted to those whose output link is an arm and the degree of freedom of the output fixed joint is 1.

Now, let F and M be the outer force and moment acting along the motor a on the output link, as shown in Fig.3, and F' , M' be the load balancing with them. The component of the load to the screw axis is

$$\left. \begin{aligned} F_n &= -F \cos \beta \\ M_n &= -M \cos \beta + Fr \sin \beta \end{aligned} \right\} \quad (6)$$

and the pitch is

$$p_t = \frac{M_n}{F_n} = p_f - r \tan \beta \quad (7)$$

which has a relationship with the pitch p_f of the outer force. This means that the force and moment acting from the coupler to the output link are varied by the pitch of the load, and then the transmission index is not determined. Now, put

$$p_t p_e = 1 \quad (8)$$

then the acting forces are determined. About the input link, let the pitch of the input force coincide with that of the input

displacement, similarly to Eq.(8), then the transmission index is uniquely determined for a mechanism.

3. Movability of mechanism

3.1 Conditions for sufficient movability

The movability of a spatial mechanism depends on the magnitude of the fluctuation of the input force or torque, which is affected not only by the geometrical configurations but also by the dynamical conditions. The former involves the transmission index, and the smaller the transmission index is, the less the input becomes provided that the dynamical conditions are almost constant. The latter determines the power transmitted from the input to the output link, and if we choose those conditions such that the power may be little when the transmission index is small, a movable mechanism will be obtained. The dynamical conditions are easily changeable by attaching a mass or another mechanism to a link or combining two links by a spring. Such a mechanism part used for the purpose of controlling the power flow is here called the supplementary mechanism.

Table 1 Classification of the positions of supplementary mechanisms

Flow of power	Examples
Supplementary mechanism → Input link → Output link	
Input link → Supplementary mechanism → Output link	
Input link → Output link → Supplementary mechanism

Fig.2

Fig.3

3.2 Classification and characteristics of supplementary mechanisms

The supplementary mechanisms can be classified into the inertia force type, the elastic force type, and the gravity type according to the energies applied. The positions of these mechanisms are fundamentally classified as shown in Table 1 with examples.

Now let x be the input displacement, and P_s be the output power of a supplementary mechanism. Moreover, for the inertia force type mechanism, let M_s be the mass (or moment of inertia) of the moving link, and $y=f(x)$ be its displacement (or angular displacement), then we have

$$\left. \begin{aligned} P_s &= M_s \ddot{y} \dot{y} \\ &= M_s G \end{aligned} \right\} \dots\dots\dots (9)$$

where

$$G = (f''\dot{x}^2 + f'\ddot{x})f'\dot{x}, \quad f' = df(x)/dx, \quad \dot{x} = dx/dt$$

Therefore the power fluctuation is determined by the function G .

On the other hand, for the elastic force type mechanism, let $y=f(x)$ be the displacement of the elastic body, and $F_s(y)$ be the force or moment generated, then we have

$$\left. \begin{aligned} P_s &= \frac{d}{dt} \int F_s(y) dy \\ &= F_s(y) \dot{x} \end{aligned} \right\} \dots\dots\dots (10)$$

Therefore the power fluctuation is determined by the force function F_s .

4. Synthesis of mechanism considering transmissibility and power

Table 2 Spatial mechanism synthesized

Triad	Parameters	Former values	Corrected values
R-R triad (input)	Position of J_{ab}	(0.457, 0.684, 0.250)	(0.604, 0.858, 0.250)
	Direction of J_{ab}	(-0.992, -0.119, 0.0)	(-0.964, 0.262, 0.0)
	b	0.418	0.270
	c	0.561	0.786
	β_{12}	90.0°	61.4°
R-P triad (output)	Swing angle $\Delta\theta$	100°	120°
	Position of J_{ga}	(0.5, 0.5, 0.5)	
	Direction of J_{ga}	(0.577, 0.577, -0.577)	
	g	0.707	
Planar 4-link mechanism	β_{56}	54.8°	
	Swing angle $\Delta\phi$	120°	
	b_0	0.375	0.375
	c_0	0.9	0.9
	d_0	0.493	0.428
	λ	40.5°	57.0°

Fig. 4

Fig. 5

4.1 Procedure for synthesis

In the beginning, as shown in Fig.4, obtain the mechanisms satisfying the required output statically, and calculate the transmission indices of the output links with use of Eq.(5). Then obtain the minimum values $TI_{\min}$ of those mechanisms and choose a mechanism among them which has the maximum value of $TI_{\min}$. Moreover, if the transmission indices of the other links of such a mechanism are smaller than the allowable value, a supplementary mechanism should be attached such as to minimize P/TI through a period.

Fig.6

Fig.7

Fig.8

4.2 Examples

4.2.1 When transmissibility is considered In the former paper, the authors have synthesized an **RRSPR** spatial 5-link mechanism⁽⁶⁾ as shown in Fig.5 and the left column of Table 2 which can trace the path along the edges of a cube. Now let the generating surface of the **R-R** triad be an oblique torus, and increase a parameter of the mechanism, which may be used for considering the transmissibility. The mechanism synthesized is shown in the right column of Table 2. The transmission indices of the former and the present mechanisms are

Table 3 Dynamical parameters of spatial mechanism

Links	Masses gs ² /mm	Moment of inertias about center of gravities gmm ²		
b	$m_b=0.0357$	208.0		
c	$m_c=0.0276$	200.0	4.6	181.6
d	$m_d=0.0088$	1.4	0.83	1.34
e	$m_e=0.0087$	3.12	0.60	3.12
f	$m_f=0.0422$	426.0		
g	$m_g=0.0720$	373.0

Fig.9

Fig.10

as shown in Fig.6, and it is evident that the minimum value has been increased to about three times of the former value.

4.2.2 When power transmission is considered The transmissibility of the mechanism shown in the left column of table 2 may be improved by attaching a supplementary mechanism between the fixed and output link g . The transmission index is as shown in Fig.6 by the one dot chain line, which has an extreme value at about $\theta=47^\circ$ similarly to TI_{br} . Now let T_0 be the moment component to the fixed joint axis of the dual force acting on the output link, W_g be the gravity of the link g , I_0 be the moment of inertia around the fixed joint axis, and g' be the distance from the fixed joint to the center of gravity, then the power transmitted to the output link is

$$P = T_0 \dot{\phi} = (I_0 \ddot{\phi} + W_g g' \cos \phi) \dot{\phi} \dots (11)$$

The power obtained from Eq.(11) under the condition that a 4-link mechanism shown in the lower section of Table 2 should be attached and rotated uniformly (at 5 rpm) has the maximum value near $\theta=47^\circ$ ($\phi=49^\circ$) at which the transmission index becomes small. When a supplementary mechanism is attached directly to the link g , the output power is represented by

$$P_s = F_s(\phi) \dot{\phi} \dots (12)$$

Then the force function $F(\phi)$ $= P/\dot{\phi}$ obtained from P is as shown in Fig.7 by the broken line. Now we suggest a nonlinear leaf spring device as shown in Fig.8 which will satisfy $F_s(\phi) = F(\phi)$. When the displacement is little, the generating torque is given by

$$T_s = 3EI \frac{l_2}{(l_2 - l_1)^2} \phi \dots (13)$$

where E and I are Young's modulus and second moment of area of the leaf spring, and when the displacement increases as in Fig.(b), the torque decreases. Put $l_1=9$ mm, $l_2=17$ mm, then $F_s(\phi)$ is shown in Fig.7 by the solid line, which coincides with $F(\phi)$ at where the transmission index is small. The ratio of the total power to the transmission index is shown in Fig.9, the maximum value of which has been decreased to 1/3 of that of the original mechanism.

5. Experiments

Now we express the movability of a mechanism by the measurable input torque and joint forces, and prove the relationships among the transmission index, power regulation, and the movability of a mechanism experimentally.

5.1 Trial making

The unit length of Table 2 is 200 mm. The spherical pair is replaced by three revolute pairs, and then two links d, e

Fig.11

Fig.12 (a)

Fig.12 (b)

Fig.13 (a)

are added. The dynamical constants are as shown in Table 3, and the mechanism synthesized in this paper and the supplementary mechanism with nonlinear leaf spring are as shown in Fig. 10.

5.2 Experimental results

About three types of mechanisms; the former mechanism of the left column of Table 2, the synthesized mechanism of the right column, and the former mechanism with the supplementary mechanism, the errors of static displacements, the input torques, and the joint forces are obtained and compared with each other.

The error of static displacement of the synthesized mechanism approximates to that of the former mechanism, as shown in Fig. 11, even though the transmission index is considered. The errors of the coupler displacement and output angular displacement under the condition for the torque of $\pm 20 \text{ Kg cm}$ applied on the link g are given in Fig. 12, which shows that the larger the transmission index is, the smaller becomes the output error influenced by the joint clearances and the link flexibilities.

The input torques and joint forces of both mechanism measured under the condition that the planar 4-link mechanism should be driven uniformly are as shown in Fig. 13 and 14. Although the torques and forces have large values at about $\theta = 90^\circ$ and 300° where the transmission indices become minimum, those of the synthesized mechanism have reduced to 1/4 of the former values. On the other hand, the characteristics of the former mechanism have been improved also by attaching the supplementary mechanism as shown in Fig. 15, and it has been proved that the movability can be improved by regulating the flow of power even though the transmission index is small. Fig. 16 represents the input torque when the input angular velocity is increased until the maximum

Fig.13 (b)

Fig.14 (a)

Fig.14 (b)

Fig.15 (a)

value of the torque coincides with that of Fig. 13(a), which shows that the driving velocity can be increased by making the transmission index large.

6. Conclusions

The results obtained in this paper are summarized as follows.

(1) When the output link makes a screw motion, as a general case, it has been proved that the transmission index should be a function of the pitch of the load, and that a transmission index could be matched to a mechanism by coinciding the pitch of the load to that of the displacement.

(2) As a result of synthesis and experiment of a spatial path-generator, it has been confirmed that the larger the transmission index, the smaller the error of displacement, input torque, and the joint force are.

(3) It has been pointed out that the ratio of the power to the transmission index should be considered in order to estimate the movability of a mechanism, and use of a supplementary mechanism has been suggested to regulate the flow of power. As a result of attaching such a supplementary mechanism to a mechanism with small transmission index, the movability has been improved.

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Fig.15 (b)

Fig.16