Self-Excited Oscillations of Gas-Liqud Two-Phase Systems in a V-Tube Connected with Reservoirs

By Hikaru TASHIRO**, Takamoto SAITO*** and Hideo UCHIDA***

In this report are discussed the self-excited oscillations due to air slugs in a V-tube connected with two reservoirs. The experimental results and calculated results are summarized as follows:
(1) The parameters which control the periods and the amplitudes of self-excited oscillations are the number of air slugs supplied in a unit time and the flow rate of supplied air.
(2) The periods of self-excited oscillations are from one to five or six times as large as those of free oscillations without any loss.
(3) The waveforms of self-excited oscillations, including the change of the water level in reservoirs, are similar to those of relaxation oscillations.

1. Introduction

In a cryostat system, it is required that the level of the liquid of low temperature in a reservoir be kept at a desired height, therefore in some cases the liquid of low temperature is continuously supplied corresponding to the thermal load in the reservoir. When the liquid velocity is comparatively small, the heat input at the pipeline or at the valve of the supplying system will cause an evaporation of the liquid, which results in the formation of a gas-liquid two-phase flow in the tube, sometimes growing into the so-called slug flow. When the supplying system to a number of cryostat systems has a number of rising pipe branches connected with cryostats, the variation of the liquid level in the cryostat is caused depending upon which of the pipelines is chosen for the rising of the gas slug, which sometimes leads to a periodical phenomenon. Of course, such a phenomenon should be avoided. This report is intended to clarify the characteristics of the oscillating phenomenon in a system where the saturated gas-liquid two-phase flow mentioned above is replaced with an air-water two-phase flow, and where the water flows to and fro but the average water level is constant. The similar phenomena will be encountered in the field of chemical engineering that deals with the gas-liquid two-phase flow.

In the past, related to this phenomenon, Beck and Davidson reported on the phenomenon that an oscillating having a period close to the free oscillating period of a single liquid phase is produced by the air bubbles supplied to the liquid in a reservoir system or in a V-tube system. The analysis is based on the balance between the supply of the oscillating energy by the air bubbles and the energy dispersion from the fluid. In this report, the equation of motion related to a system in which pipelines of a V-tube are connected with the bottoms of the reservoirs is analyzed and compared with the experimental results, and the confirmation is made about the existence of the self-excited oscillating phenomenon, the period of which is greater than the free oscillating period of a single liquid phase.

Nomenclature

- \( a \) : sectional area of a V-tube cm²
- \( A \) : sectional area of a reservoir cm²
- \( c \) : elbow loss coefficient
- \( d \) : inner diameter of a V-tube cm
- \( D \) : inner diameter of a reservoir cm
- \( g \) : acceleration due to gravity cm/s²
- \( h \) : liquid level in a reservoir cm
- \( 2a h \) : amplitude=maximum liquid level minus minimum liquid level cm
- \( l \) : length of a V-tube cm
- \( n \) : number of air slugs supplied in a unit time \( 1/s \), or number of air slugs in a V-tube
- \( \rho \) : pressure g/cm²
- \( \Delta \rho \) : pressure loss g/cm²
- \( Q \) : flow rate cm³/s
- \( Re \) : Reynolds number \( u_d \mu \)
- \( T \) : time s
- \( T \) : period s
- \( \nu \) : water velocity in a V-tube cm/s
- \( U_a \) : air slug velocity in a V-tube cm/s
- \( U \) : water velocity in a reservoir cm/s
- \( U_b \) : air bubble velocity in a reservoir cm/s
- \( \gamma \) : volume cm³
- \( \chi \) : location of an air slug or an air bubble cm
- \( \Gamma \) : relative velocity coefficient of an air slug \((U_a-U)\)/\( U \)
- \( \theta \) : angle of an inclined tube measured from the horizontal direction deg
- \( \lambda \) : tube friction coefficient
- \( \mu \) : viscosity coefficient g/cm²
- \( \nu \) : dynamic viscosity coefficient cm²/s
- \( \rho \) : density g/cm³
- \( \Delta \rho \) : difference of density g/cm³
- \( \sigma \) : surface tension g/cm

Subscripts

- \( 0 \) : still water, or initial value, or bottom of a V-tube
- \( 1 \) : tube 1, or reservoir 1
- \( 2 \) : tube 2, or reservoir 2
- \( 1 \) : bottom of a V-tube (plus for tube \( 1 \) → tube 2)
- \( 2 \) : bottom of a V-tube (plus for tube 2 → tube 1)
- \( a \) : air
- \( e \) : elbow loss
- \( f \) : friction loss
- \( m \) : average value in the air-water two-phase flow

* Received 28th March, 1977.
** Graduate Student, University of Tokyo, Hongo.
*** Professor, University of Tokyo.
2. Oscillating phenomenon

An example of pictures of the oscillating phenomenon is shown in Fig.1. Depending upon the direction of the water flow in the V-tube, the choice of the entry of the supplied air slugs to either side of the V-tube is determined. In such a way, the air slugs are supplied to one side of the V-tube one after another, causing an increase of the difference in the water head between the two reservoirs with the lapse of time. When the restoring force caused by the head difference becomes larger than the driving force by the air slugs in the V-tube, the water velocity is decreased until the flowing direction will be reversed. Then the air slugs successively supplied will enter the other side of the V-tube, thus keeping up the oscillation.

3. Analysis

3.1 Oscillation model

An oscillation model is shown in Fig. 2, in which the following conditions from (1) to (10) are assumed:

(1) The choice to which side of the V-tube the air slugs are supplied depends upon the sign of the water velocity in the V-tube \( \psi_i \) (i=1,2).

(2) The volume \( \psi_s \) of an air slug and the number \( n \) of air slugs supplied in a unit time are fixed. Further, the volume \( \psi_s \) of an air slug or an air bubble is fixed regardless of the location, and the disruption or the coalescence of air slugs or air bubbles does not take place.

(3) Air slugs and air bubbles are treated as particles.

(4) When air slugs are supplied, a water level difference between two reservoirs is caused according to the sign of the water velocity in the V-tube \( \psi_i \) (i=1,2), i.e., when \( \psi_i > 0 \), only the water level in the reservoir i rises by \( \Delta h_i = \psi_i A \), and the water level in the same reservoir falls by \( \Delta h \) when the air bubbles are released to the atmosphere.

(5) From the experimental results shown in Fig.3, when \( \theta < 90^{\circ} \), the rising velocity \( \psi_{gas} \) of the air slugs in the standing water in a vertical tube and \( \psi_{gas} \) in an inclined tube with an angle of \( 90^{\circ} \) to the horizontal direction are given by the following equations:

\[
\psi_{gas} = \frac{2\psi}{\sqrt{3} \rho} \quad \cdots \cdots \quad (1)
\]

\[
\psi_{gas} = 1.2 \psi_{gas} = 0.20 \sqrt{3} \rho \quad \cdots \cdots \quad (2)
\]

The velocity \( \psi_s \) of the air bubbles rising in the water in a large vessel is

\[
\psi_s = \frac{1}{1,8} \left( \frac{\sigma}{\rho} \right)^{1/2} \cdots \cdots \cdots \cdots \quad (3)
\]

(6) The relation among the velocity \( \psi_{as} \) of an air slug in a tube, the rising velocity \( \psi_s \) of an air slug in the standing water and the water velocity \( u \) is shown in Fig.4, where \( \gamma_m \) (\( \psi_{as} - \psi_s \)) is the relative velocity coefficient. Consequently,

\[
\psi_{as} = \psi + \nu_s \quad \cdots \cdots \cdots \cdots \quad (4)
\]

is almost valid. Similarly,

\[
\psi_{as} = \psi + \nu_s \quad \cdots \cdots \cdots \cdots \quad (5)
\]

is valid for an air bubble in a reservoir.

(7) The density and the momentum of air are neglected in comparison with those of water.

(8) The friction loss \( \Delta \rho_i \) and the elbow loss \( \Delta \rho_i \rho \) are taken into consideration.

Fig.1 Oscillating phenomenon

**Fig.2 Oscillation model**

![Oscillation model](image)

**Fig.3 Rising velocity of air slugs in an inclined tube**

![Rising velocity graph](image)
As to single water phase flow, the friction loss coefficient of tubes $\lambda_1$, $\lambda_2$, and the pressure loss $\Delta P_{\text{m}}^1$, $\Delta P_{\text{m}}^2$ (1, j = 1, 2) are considered as follows:

$$\lambda_1 = 0.64/R_{\text{m}} \text{ for the laminar flow} \left( R_{\text{m}} \leq 2500 \right)$$

$$\lambda_2 = 0.3164/R_{\text{m}}^{1/4} \text{ for the turbulent flow} \left( R_{\text{m}} > 2500 \right)$$

$$\Delta P_{\text{m}} = \lambda_1 \frac{D}{R_0} \rho \frac{u^2}{2}$$

$$\Delta P_{\text{m}} = \lambda_2 \frac{D}{R_0} \rho \frac{u^2}{2}$$

The elbow loss coefficient $C_{\text{el}}$ and the pressure loss $\Delta P_{\text{el}}$ are

$$\Delta P_{\text{el}} = \frac{C_{\text{el}}}{2} \frac{D}{R_0} \rho \frac{u^2}{2}$$

Various losses of the air-water slug flow and the air-water bubble flow are regarded as follows. Referencing the study of Griffith and Stenning, when $i, j = 1, 2$, the friction loss of the air-water slug flow is shown below. When $u = u_0$, $\beta = \beta_0$, $\rho = \rho_0$, the friction coefficient of the tube $\lambda$, and the pressure loss $\Delta P_{\text{m}}^i$, are respectively,

$$\lambda = 0.64/R_{\text{m}} \text{ for the laminar flow} \left( R_{\text{m}} \leq 2500 \right)$$

$$\lambda = 0.3164/R_{\text{m}}^{1/4} \text{ for the turbulent flow} \left( R_{\text{m}} > 2500 \right)$$

$$\Delta P_{\text{m}}^i = \lambda \frac{D}{R_0} \rho \frac{u^2}{2} \left( 1 + \frac{Q_1}{A} \right)$$

That is equal to finding the solution of Eq. (11) using Eq. (12) as to the average density $\rho$, and the average velocity $u$ of the air-water slug flow.

$$\Delta P_{\text{m}}^i = \lambda \frac{D}{R_0} \rho \frac{u^2}{2} \left( 1 + \frac{Q_1}{A} \right)$$

Here, using the average density and the average velocity, the loss in the air-water slug flow and in the air-water bubble flow is found in the following way. When the friction coefficient is $\lambda_1 = 1$, $u_{\text{m}} = u_0$, $Q_1 = 0$, the pressure loss $\Delta P_{\text{m}}^i$ is as follows.

$$\lambda \left( \frac{d}{A} \right)^{1/2} + 1 - 2 \left( \frac{Q_1}{A} \right) \sin \theta$$

The relation of the flow rate at the connecting point of the reservoir $i$ with the V-tube $i$ is

$$\Delta t \left\{ P_{\text{m}} \left( A_i - n_{\text{v}} \rho \right) \right\}$$

where $i, j = 1, 2$. The relation between the water velocity and the water level in the reservoir $i$ is

$$\Delta t \left\{ P_{\text{m}} \left( A_i - n_{\text{v}} \rho \right) \right\}$$

The location $x_i$ of the air slugs and air bubbles is determined by Eq. (22).

$$x_i = \frac{1}{2} u_0 \Delta t$$

The number of the air slugs $n_i$ or the number of the air bubbles $N_i$ is expressed in the following way.

$$n_i = \left[ \frac{\text{the number of the air slugs}}{\text{the number of the air bubbles}} \right]$$

$$N_i = \left[ \frac{\text{the number of the air bubbles}}{\text{the number of the air slugs}} \right]$$

where $t_0$ is the time when the air slugs are supplied at the bottom, and $t_i$ is the time when the air slugs transfer from the V-tube to the reservoir. On the other hand, the initial condition at $t_0$ is given by

$$h_{t_0} = h_0, u_{t_0} = u_0, 0 < \frac{\Delta x}{\Delta t}$$

$$h_{i, j} = h_i, u_{i, j} = u_j, 0 < \frac{\Delta x}{\Delta t}$$

and by the condition that the water veloc-
ility is \( u_i \). In this case, since the term indicating the acceleration of the fluid in the reservoir included in the left side of Eq. (16) is much smaller than the terms in the right side, the acceleration term is neglected.

\[
\frac{d}{dt}\left[p_u(a_2-i(nu\text{ Va}))u_i\right]
\]

\( \approx \) each term in the right side \( \cdots (25) \)

Further, in Eq. (19), the variation of the void fraction in the V-tube \( d/dt(nu\text{ Va})/dL \) is neglected in comparison with the variation rate of water velocity \( (d/dt)u_i \).

\[
\frac{d}{dt}\left[p_u(a_2-i(nu\text{ Va}))u_i\right] = \frac{d}{dt}\left[p_u(a_2-i(nu\text{ Va}))\frac{d}{dt}u_i\right]
\]

\[
\frac{d}{dt}u_i = \left[p_u\frac{d}{dt}(a_2-i(nu\text{ Va}))\right]
\]

\[
\frac{d}{dt}u_i = \left[p_u\frac{d}{dt}(a_2-i(nu\text{ Va}))\right] \cdots (27)
\]

By carrying out the two approximations mentioned above and also the arrangement by the pressure \( P_i \), \( P_0 \) from Eqs. (16)-(21), the following Eqs. (27)-(29) are obtained.

\[
\frac{d}{dt}u_i = \left[p_u\frac{d}{dt}a_2-i(nu\text{ Va})\frac{d}{dt}u_i\right]
\]

\[
\frac{d}{dt}u_i = \left[p_u\frac{d}{dt}(a_2-i(nu\text{ Va}))\right] \cdots (28)
\]

4. Experiment

The experimental apparatus shown in Fig. 5 is composed of a compressor, a flow rate, a visigraph, and two reservoirs connected with a V-tube. In the experiment, the initial water level in the reservoir is set to 20 cm, and air is supplied from the bottom of the V-tube. At the supplying portion, the air forms intermediate air slugs which enter the V-tube system.

On the other hand, the representation parameter for the oscillation, two parameters, i.e., the supplied quantity of air \( Q_a \) and the number \( n \) of air slugs supplied in a unit time are selected.

Further, the supplied air quantity is measured by a float type flow rate meter and the alternation of the supply of the air slugs is measured by the visigraph that utilizes the variation of the electric resistance between two electrodes. The oscillation of the water level is measured by the water level meter that utilizes the variation of the electric resistance between two leading wires. In the experiment, the supplied air quantity \( Q_a \) is varied in five stages including 2.4, 6.8, and 10 cm/s, and the number \( n \) of air slugs supplied in a unit time is varied in eight or nine stages in the range of 0.2-20 slugs/s.

5. Result and discussion

5.1 Waveform of the oscillation

Examples of waveforms are shown in Fig. 6 and Fig. 7. When the number of slugs supplied in a unit time is small, the waveform of water level is shown in Fig. 6, in which the waveform of the water velocity in the V-tube indicates that the
water velocity abruptly increases when the air slugs are supplied one by one into the V-tube, and abruptly decreases when the air-slug moves from the V-tube into the reservoir. On the other hand, when the number of air slugs supplied in a unit time is large, the waveform of water level is of the relaxation oscillation type as is shown in Fig.7, in which the waveform of the water velocity in the V-tube shows that the water velocity attains the maximum or the minimum value almost at the same time as the flowing direction of water is reversed, and it decreases or increases almost steadily accompanied with small oscillations until the flowing direction of water is reversed in the next turn.

In Fig.6 and Fig.7, the waveform of oscillation in the experiment corresponds to the steady condition, while the waveform obtained by the calculation indicates the one that has started just after the supply of the first air slug. The result of the calculation seems to indicate that the condition of oscillation becomes almost steady from the beginning without passing the transient condition.

5.2 Period and amplitude of the oscillation

The relation of the number of air slugs supplied in a unit time with the period of oscillation is shown in Fig.8, and the relation with the amplitude of oscillation of water level is shown in Fig.9.

In Fig.8, the period $T$ of oscillation is compared with the period $T_0$ of free oscillation of the system. The latter system is composed of a single water phase without any loss, therefore $T_0 = \frac{2 \pi \sqrt{l}}{\sqrt{g}} \sqrt{\frac{A}{h_m}}$ (when $l = 50 \text{ cm}$, $A/h_m = 100\text{ cm}$, $g = 10 \text{ cm/s}^2$). The period $T$ of self-excited oscillation is almost as large as the period $T_0$ of free oscillation when the number of air slugs supplied in a unit time is small, while $T$ becomes five or six times as large as $T_0$ when the number of air slugs is large. Also, in Fig.8 and Fig.9, both the period $T$ and the amplitude $2\alpha h$ of self-excited oscillation greatly vary before and behind the point where the number of air slugs is $0.4$ to $0.6$ slugs/sec, and they tend to saturate when the number of air slugs becomes large.

5.3 Difference between experimental values and calculated values

About the difference between the experimental and calculated values as to the period $T$ and the amplitude $2\alpha h$ of oscillation, the reason is considered as follows by comparing the assumption of the modelling of oscillation with the actual phenomena.

(1) A switching condition is set such that the choice of the supply of an air slug to either tube of the V-tube would depend upon the direction of the water flow in the V-tube. However, the practical condition of supplying air slugs to the V-tube is complicated. With due consideration of the supplying condition of air slugs, the reason why the oscillation does not necessarily become steady is pointed out.

(a) When the water velocity is small just before the reversing of the flow direction in the V-tube, it sometimes occurs as is shown in Fig.10 that the air slug splits and they are supplied to both sides of the V-tube. Thereby, it is not rare that the oscillation is suppressed or the direction of the water flow is reversed.

(b) Similarly, when the water velocity in the V-tube is small, it sometimes occurs as is shown in Fig.11 that the air slug is supplied against the flow direction of the water. This is because the air slug can rise in the V-tube against the flow direction of water when the water velocity is small compared with the rising velocity of the air slug in the standing water. Further, it is not rare that the reversing of the flow direction of the water is triggered by the reverse supply of air slugs.

(c) When the supplied quantity of air is large, the air slug in one side of the V-tube sometimes branches into the other side of the V-tube as is shown in Fig.12. Thereby, the oscillation is suppressed.

As is described above, the assumed switch-
ing condition depending upon the water velocity in the V-tube is not always correct.

(2) Both the air slug and the air bubble are regarded as particles. However, unlike in the case of the air bubble, regarding an air slug as a particle is questionable. That is, when the ratio of the air slug length $L_a$ to the inner diameter of the V-tube $d$ is large, the void fraction in the V-tube greatly varies with the entry of the air slug in the V-tube or its discharge from the V-tube into the reservoir, so that the approximating formula (26) becomes improper.

(3) Various losses of the single water phase flow, the slug flow and the bubble flow are calculated under the steady flow condition. However, the flowing velocity greatly varies in one period of actual phenomenon. Therefore, it should be investigated further whether the loss evaluation formula for the steady flow is generally applied in practice.

6. Conclusions

The self-excited oscillation phenomenon of air slugs in the V-tube was modelled and numerical analysis was carried out. From the result of the experiment and analysis, the following characteristics were clarified.

(1) By the assumption that the choice of the supply of the air slug to either side of the V-tube depends upon the direction of the water velocity in the V-tube, the oscillating mechanism was clarified. (however, because of such phenomena as the splitting, branching and the reverse supply of the air slug as shown in Fig. 10-12, the oscillation sometimes becomes irregular.)

(2) In a given oscillation system, the period of the self-excited oscillation $T$ and the amplitude of the oscillation $2\theta b$ vary with the number $n$ of air slugs supplied in a unit time and the supplied quantity of air $Q_a$. The influence of the number $n$ of air slugs is greater.

(3) In comparison of the period $T$ of self-excited oscillation with the period $T_b$ of free oscillation of the single water phase, the former varies from one to five or six times as widely as the latter depending upon the number $n$ of air slugs. Furthermore, when the number $n$ of air slugs becomes large, the period $T$ tends to be saturated.

(4) The waveform caused on the water level in the reservoir presents a similarity to a sine wave in a range of small numbers $n$ of air slugs. In the range of large numbers $n$, the waveform is a relaxation oscillation type. Further, when the number $n$ of air slugs becomes large, the amplitude of oscillation $2\theta b$ tends to be saturated.

(5) The relation between the period of self-excited oscillation and the amplitude of oscillation $2\theta b$ can be roughly evaluated by the formula $T=(4\pi^2/\omega^2)\sqrt{m/a}$, using the average velocity of one-directional flow $\omega$ in the V-tube. This evaluating formula should be especially used in the range in which the number $n$ of air slugs is large, in other words, in the range in which the period $T$ of self-excited oscillation is fairly greater than the period $T_b$ of free oscillation of the single water phase.

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References

[(1) Beek, W.J., Symp. on Two Phase Flow, Exter, Ser. F (1965-6), p. 401.]
[(3) Garland, C.J. and Davidson, J.P., VDI-Ber., Bd. 30 (1975), S. 177.]