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The Method of Calculating the Mechanical

Coefficients of Fluid Mixtures*

(11. The Coefficient of Internal Resistance
in Two-Phase Flows)

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The coefficient of internal resistance in dispersive two-phase flows is considered. The method of self-consistent cell model is applied to determine the coefficient as a function of the volume fraction of particle phase. A certain spherical cell is formed around a randomly chosen particle. It is assumed that inside the cell the fluid is Stokesian while outside the fluid is Stokes-Darcian. The solutions in both sides are connected on the cell surface by suitable conditions. The condition of coincidence is considered, which is indispensable to determine the coefficient considered and supplements the boundary conditions. The numerical calculation is carried out and the prediction is reasonable.

1. Introduction

The basic equations in the dynamics of fluid-mixtures are expressed with the aid of various mechanical coefficients. The present author considered the inertia coefficients, one of the referred mechanical coefficients, in the preceding paper (1). The flow field in the fluid mixtures is tremendously complicated if observed in minute scale. The cell model method employed in the preceding paper offers a simple and powerful means to estimate average properties of the flow. By making use of the cell model method again in the present paper, we shall estimate the coefficient of internal resistance in two-phase flows.

The resistance coefficient was briefly studied in the preceding paper, where there was some ambiguity about the boundary conditions on the cell surface. To eliminate this ambiguity, we shall employ the method of improved cell model, i.e., the so-called self-consistent cell model (2). For that purpose, the flow outside the cell is replaced by a smoothed-out average flow. It is found that this outside field is the flow of a Stokes-Darcian fluid, which will be treated generally in the present paper. The motion of a Stokes-Darcian fluid is affected by the effective viscosity as well as the effective resistance. It is, though, the ultimate aim of the present paper to estimate the coefficients of these transport effects by the method of self-consistent cell model. To avoid complication, we use the simple cell model method to estimate the effective viscosity. The self-consistent cell model method requires a

little more complicated procedure than the simple method.

2. Statement of the problem

We shall consider flows of dispersive two-phase mixtures composed of usual fluid and solid particles. When the mean velocity of a cluster of solid particles is different in magnitude and direction from that of a continuous fluid, there arises an internal resistance. It is reasonable to assume that the internal resistance is proportional to the relative velocity between the particle phase and the continuum, unless the relative velocity exceeds a certain limit. The aim of the present paper is to estimate the coefficient of the internal resistance. Since the resistance depends simply on the relative velocity, it is assumed that the particles are fixed in space. Then, the continuum flows through the space among the fixed particles (Fig. 1).

The particles are assumed to be small spheres of uniform size, the radius of which is denoted by a . It is also assumed that n particles are uniformly dispersed in the space of a unit volume. Let the viscosity of the continuous phase be μ_c , where the continuum is taken to be Stokesian. The assumption of Stokesian fluid becomes adequate when the particles are densely dispersed.

The flow around the fixed particles will have great complexity because of infinitely many-body interaction, hence, an approximating method must be devised. Choose a particle randomly, and observe the flow from the center of the particle. Near the particle the flow is governed by the usual fluid dynamic equations. Far from the origin, the flow becomes smooth and simple, where the detailed flow is assumed to be averaged spatially and statistically. The smoothed out flow experiences resistance from the background. In addition, since the flow is not

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uniform when viewed from the origin, the effect of viscosity must be taken into consideration. Consequently, the fluid far from the origin must be assumed to be Stokes-Darcian.

Fig.1 The flow pattern

We shall simplify the situation as much as possible. Let us form a cell with an appropriate radius $\frac{1}{2}$ around a particle chosen randomly. Fill the cell region $a \leq r \leq \frac{1}{2}$ with Stokesian fluid and outside the cell $r \geq \frac{1}{2}$ with Stokes-Darcian fluid where r is the distance from the origin. Solve the flow problem in which the velocity approaches U as $r \rightarrow \infty$. Refer to Fig. 2. This is the self-consistent cell model in the present problem. The space allotted to a particle is, on the average, of volume $1/n$ including the particle itself. By assuming this space to be spherical, the radius $\frac{1}{2}$ already used above is given by

$$\frac{1}{2} \equiv (3/4\pi n)^{1/3}. \quad (1)$$

Fig.2 The self-consistent cell model

3. Stokes-Darcian fluid and the dissipation function

The flow exerts a force on the fixed particles. The force is proportional to the velocity of the flow. Let the proportionality constant be β . Denote the viscosity by μ and the velocity by v_i . The coordinate system is orthonormal and rectangular, where the component is specified by the index i ($= 1, 2, 3$). The dissipation

function in region V is expressed as

$$\Phi_{(v)} = (1/2) \int_V \beta v^2 dx + (1/2) \int_V \mu \partial_i \partial_i v_j \partial_i v_j dx, \quad (2)$$

where $\partial_i \equiv \partial/\partial x_i$ and $(\)$ indicates the mixing operation: $\partial_i \partial_i v_j \equiv (\partial_i v_j + \partial_j v_i)/2$. The summation convention is applied for repeated indices as usual.

The volume fraction of the continuous phase (fluid) is denoted by z_c , and that of the dispersed phase (particles) by z_d . The flow rate of the continuous phase is $z_c v_i$ and the equation of continuity is given by

$$\partial_i (z_c v_i) = 0. \quad (3)$$

Now, minimize the dissipation function (2) under the auxiliary condition (3). Take the Lagrange multiplier P , and transform the integral by making use of the divergence theorem, then we get

$$\delta \Phi_{(v)} = \int_V [\beta v_i - z_c \partial_i \{ \mu \partial_i v_i \} + z_c \partial_i P] \delta v_i dx + \int_S \{ 2\mu \partial_i v_i - z_c P \delta n_i \} \delta v_i dS = 0, \quad (4)$$

where S is the boundary surface of V , dS the surface element, and the index n means a component normal to the surface. From Eq. (4) it follows first that

$$z_c \partial_i P = z_c \partial_i (\mu \partial_i v_i) - \beta v_i \quad \text{in } V. \quad (5)$$

Define the pressure tensor P_{ij} by

$$P_{ij} \equiv z_c P \delta_{ij} - z_c \mu \partial_i v_j. \quad (6)$$

Next, let us consider boundary conditions on discontinuous surfaces. First Eq. (3) gives

$$(z v_n)_+ = (z v_n)_-, \quad (7)$$

where the index n means a component normal to the discontinuous surface. We put the condition in the tangential direction as

$$(v_{i'})_+ = (v_{i'})_-, \quad (8)$$

where the index i' means the tangential component. If the above condition is not satisfied, there arises infinite shear rate on the discontinuous surface, and as the result the dissipation function diverges. The normal component of velocity can have a jump as seen from the condition (7), for the pressure P may jump to compensate the discontinuity of the normal component of velocity. The surface integral in Eq. (4) suggests the conditions for the pressure tensor on the discontinuous surface, that is,

$$(P_{nn}/z)_+ = (P_{nn}/z)_-, \quad (9)$$

$$(P_{ni'})_+ = (P_{ni'})_-, \quad (10)$$

where P_{nn} is the normal pressure and $P_{ni'}$ the tangential force on the surface. When v_i and P satisfy Eq. (3) and Eq. (5) in V and Eqs. (7) through (10) on the discontinuous surfaces in V , the dissipation function (2) can be transformed as

$$\Phi(r) = -\frac{1}{2} \int_S p_{ni} v_i dS. \quad (11)$$

4. Elementary solutions of Stokes-Darcian flow

Consider a case where the coefficients β , μ and the volume fraction α_c can be assumed to be constant. In this case, Eq. (3) and Eq. (5) reduce to

$$\nabla \cdot \mathbf{v} = 0, \quad (12)$$

$$\nabla p' = \mu \Delta \mathbf{v} - \beta \mathbf{v}, \quad (13)$$

respectively, where $p' = \alpha_c p$ is the partial pressure of the continuous phase. An elementary solution of p' is given by

$$p' = \alpha_c \phi_n, \quad (14)$$

where ϕ_n is a solid spherical harmonics of degree n . A particular solution of $\mathbf{v}$ corresponding to the above p' is easily found to be

$$\bar{\mathbf{v}} = -\beta^{-1} \nabla \phi_n, \quad (15)$$

where $\beta \neq 0$ is assumed. The complementary solution $\hat{\mathbf{v}}$ must satisfy the following equations:

$$\nabla \cdot \hat{\mathbf{v}} = 0, \quad (16)$$

$$\Delta \hat{\mathbf{v}} - k^2 \hat{\mathbf{v}} = 0, \quad k \equiv (\beta/\mu)^{1/2}. \quad (17)$$

Let us consider a similar but scalar equation $\Delta \psi - k^2 \psi = 0$. As easily seen, there is an elementary solution defined by

$$\psi = e^{-kr}/r. \quad (18)$$

The partial derivatives of any order are also the solutions. Consider a tensorial function defined by

$$\psi_{ij} \equiv \partial_i \partial_j \psi - k^2 \psi \delta_{ij}. \quad (19)$$

This function satisfies the equations

$$\partial_i \psi_{ij} = 0, \quad \Delta \psi_{ij} - k^2 \psi_{ij} = 0.$$

Namely, ψ_{ij} can be a solution to Eqs. (16) and (17). The derivatives of ψ_{ij} are also the solutions. By substituting (18), we have specifically

$$\psi_{ij} = \left\{ -(1+kr+k^2 r^2) \delta_{ij} + (3+3kr+k^2 r^2)(x_i x_j / r^2) \right\} (\psi / r^2), \quad (20)$$

$$\begin{aligned} \partial_k \psi_{ij} &= \left\{ (3+3kr+k^2 r^2)(\delta_{ki} x_j / r) + (6+6kr+3k^2 r^2+k^2 r^3)(\delta_{ij} x_k + \delta_{kj} x_i) / 2r \right. \\ &\quad \left. - (15+15kr+6k^2 r^2+k^2 r^3)(x_k x_i x_j / r^3) \right\} \\ &\quad \times (\psi / r^3). \end{aligned} \quad (21)$$

In the present problem, the flow field is governed by the externally imposed velocity $\mathbf{U}$. Because of linearity and isotropy of the problem the solution is easily obtained by using the elementary solutions stated above. First, the partial pressure is given by

$$p' = -\beta (C_1 + C_2 / r^3) x_j U_j, \quad (22)$$

where C_1 and C_2 are arbitrary constants. Correspondingly, from the formula (15) the particular solution becomes

$$\bar{v}_i = \left\{ C_1 \delta_{ij} + (C_2 / r^3)(\delta_{ij} - 3x_i x_j / r^2) \right\} U_j, \quad (23)$$

whence

$$\begin{aligned} \partial_k \bar{v}_i &= - (C_2 / r^4) \left\{ 3(\Sigma x_h \delta_{ij} / r) \right. \\ &\quad \left. - 15(x_h x_i x_j / r^3) \right\} U_j, \end{aligned} \quad (24)$$

where we have used the abbreviation defined by

$$\Sigma x_h \delta_{ij} = x_h \delta_{ij} + x_i \delta_{hj} + x_j \delta_{hi}.$$

The complementary solution is obtained from the expression (19), namely,

$$\hat{v}_i = C_3 \psi_{ij} U_j, \quad (25)$$

where C_3 is an arbitrary constant. As for $\partial_k \hat{v}_i$, the expression (21) is applicable. A complete solution of $\mathbf{v}$ is given by

$$\mathbf{v} = \bar{\mathbf{v}} + \hat{\mathbf{v}}. \quad (26)$$

The pressure tensor is obtained from the definition (6), i.e.,

$$-p_{hi} / \mu = - (p' / \mu) \delta_{hi} - 2 \partial_i v_j. \quad (27)$$

We shall need later the expression of the component of the pressure tensor $p_{ri} = p_{hi}(x/r)$ acting on the surface $r = \text{const}$. The expressions of v_i and p_{ri} are arranged in the following form:

$$v_i = \left\{ V_r(x_i x_j / r^2) + V_t(\delta_{ij} - x_i x_j / r^2) \right\} U_j, \quad (28)$$

$$p_{ri} = \left\{ P_r(x_i x_j / r^2) + P_t(\delta_{ij} - x_i x_j / r^2) \right\} U_j. \quad (29)$$

The suffixes r and t imply the radial and the transversal directions, respectively. By making use of the expressions (18), (20), (21) and (22), we obtain

$$V_r = C_1 - 2C_2 / r^3 + 2(1+kr)e^{-kr} C_3 / r^3, \quad (30a)$$

$$V_t = C_1 + C_2 / r^3 - (1+kr+k^2 r^2)e^{-kr} C_3 / r^3, \quad (30b)$$

$$\begin{aligned} -r P_r / \mu &= k^2 r^2 C_1 + (12+k^2 r^2) C_2 / r^3 \\ &\quad - 4(3+3kr+k^2 r^2)e^{-kr} C_3 / r^3, \end{aligned} \quad (31a)$$

$$\begin{aligned} -r P_t / \mu &= -6 C_2 / r^3 \\ &\quad + (6+6kr+3k^2 r^2+k^2 r^3)e^{-kr} C_3 / r^3. \end{aligned} \quad (31b)$$

5. The coefficient of internal resistance

The region $a \leq r \leq \frac{1}{2}$ is occupied by the Stokesian fluid with viscosity μ_c . The solution for this case is given in the preceding paper, " which is rewritten as follows:

$$p = -\frac{3}{2} \frac{\mu_c}{a^2} \left(\frac{a^3}{r^3} C + 5 \frac{a^3}{r^3} C' \right) x_i U_i, \quad (32-a)$$

$$v_i = \left\{ \tilde{V}_r \frac{x_i x_i}{r^2} + \tilde{V}_t (\delta_{ij} - \frac{x_i x_j}{r^2}) \right\} U_j, \quad (32-b)$$

$$P_{ri} = \left\{ \tilde{P}_r \frac{x_i x_i}{r^2} + \tilde{P}_t (\delta_{ij} - \frac{x_i x_j}{r^2}) \right\} U_j, \quad (32-c)$$

where C, C' are arbitrary constants and

$$\begin{aligned} \tilde{V}_r &\equiv \frac{1}{2} (2 - 3 \frac{a}{r} + \frac{a^3}{r^3}) C \\ &- \frac{1}{4} \frac{a^3}{r^3} (3 \frac{r^2}{a^2} - 5 + 2 \frac{a^3}{r^3}) C', \quad (33-a) \end{aligned}$$

$$\begin{aligned} \tilde{V}_t &\equiv \frac{1}{4} (4 - 3 \frac{a}{r} - \frac{a^3}{r^3}) C \\ &- \frac{1}{4} \frac{a^3}{r^3} (6 \frac{r^2}{a^2} - 5 - \frac{a^3}{r^3}) C', \quad (33-b) \end{aligned}$$

$$\begin{aligned} -\frac{r}{\mu_c} \tilde{P}_r &\equiv \frac{3}{2} \left\{ (3 \frac{a}{r} - 2 \frac{a^3}{r^3}) C \right. \\ &\left. + \frac{a^3}{r^3} (3 \frac{r^2}{a^2} + 2 \frac{a^3}{r^3}) C' \right\}, \quad (33-c) \end{aligned}$$

$$\begin{aligned} -\frac{r}{\mu_c} \tilde{P}_t &\equiv \frac{3}{2} \left\{ \frac{a^3}{r^3} C \right. \\ &\left. - \frac{1}{2} \frac{a^3}{r^3} (3 \frac{r^2}{a^2} + 2 \frac{a^3}{r^3}) C' \right\}. \quad (33-d) \end{aligned}$$

The region $r \geq \frac{1}{2}$ is occupied by the Stokes-Darcian fluid, and the solution is obtained in the preceding section. It is noted, however, that the drag coefficient β as well as the viscosity μ used therein is not known yet. These coefficients are rather the objects to be solved by the method of the present cell model. To avoid complication, μ will be determined by using the simple cell model in the next section.

The condition at infinity is put naturally as $v_i \rightarrow U_i$ as $r \rightarrow \infty$ which gives

$$C_1 = 1. \quad (34)$$

The remaining four constants C, C', C_2 and C_3 are determined by conditions (7) through (10). Hence, the flow is solved for any β . To determine β , one more condition is required. We will seek for the condition. Once the flow field has been known, one can calculate the dissipation function by Eq. (2) or Eq. (11):

$$\Phi_R = \frac{1}{2} \mu k^2 \frac{4\pi}{3} R^3 U^2 (1 - \frac{C_2}{R^2} + \dots),$$

as $R \rightarrow \infty$, where Φ_R is the dissipation function in the region $r \leq R$. On the other hand, the smoothed-out field is uniform everywhere with the velocity U , hence the dissipation function is given by

$$\Phi_R = \frac{1}{2} \beta U^2 \frac{4\pi}{3} R^3, \quad (35)$$

which is rather the definition of β . The former expression of Φ_R must coincide with the above expression, where the difference should be within the dissipated energy per particle. Thus we demand

$$C_2 = 0. \quad (36)$$

We will call (35) the condition of coincidence.

The outside solutions (28) and (29) are connected with the inside solution (32) at the cell surface $r = \frac{1}{2}$. The conditions of connection are (7) through (10). By eliminating $C, C', C_2 e^{-\beta/b^2}$ after laborious calculation, we get the following equation which serves to determine β as a function of Z_d :

$$\frac{A_N}{A_D} = \frac{B_N}{B_D}, \quad (37)$$

where

$$A_N \equiv 3\delta(3 + 9\delta + 9\delta^2 + 6\delta^3 - 2\delta^5), \quad (38-a)$$

$$\begin{aligned} A_D &\equiv 3(1 + \delta + \delta^2)(4 + 5\delta + 6\delta^2 + 2\delta^3 - 2\delta^4) M_1 \\ &- 3(2 + 3\delta + 3\delta^2 + 2\delta^3) M_2 \\ &+ (1 - \delta)(4 + 7\delta + 4\delta^2) \tilde{\mu} M_4, \quad (38-b) \end{aligned}$$

$$\begin{aligned} B_N &\equiv 19\delta(1 + \delta + \delta^2)(2 + 4\delta + 3\delta^2 + \delta^3 - \delta^4 \\ &+ \delta^5) - (1 - \delta)(4 + 7\delta + 4\delta^2) \tilde{\mu} M_2, \quad (38-c) \end{aligned}$$

$$\begin{aligned} B_D &\equiv 6(1 + \delta + \delta^2)\{ (4 + 7\delta + 7\delta^2 - 3\delta^3 - 3\delta^4 \\ &+ 3\delta^5) M_1 - (4 + 5\delta + 6\delta^2 + 2\delta^3 - 2\delta^4) M_2 \} \\ &- (1 - \delta)(4 + 7\delta + 4\delta^2) \tilde{\mu} M_3, \quad (38-d) \end{aligned}$$

and

$$\tilde{\mu} \equiv \mu/\mu_c. \quad (39-a)$$

$$M_1 \equiv 2(1 + m), \quad (39-b)$$

$$M_2 \equiv -(1 + m + m^2), \quad (39-c)$$

$$M_3 \equiv -4(3 + 3m + m^2), \quad (39-d)$$

$$M_4 \equiv 6 + 6m + 3m^2 + m^3, \quad (39-e)$$

$$\delta = a/\lambda, \quad m = kl = \lambda(\beta/\mu)^{1/2}. \quad (40)$$

The equation (37) determines m as a function of $Z_d = \delta^2 : m(Z_d)$. The coefficient of internal resistance β is finally given by

$$\frac{\beta}{\beta_*} = \frac{2}{9} \frac{\mu}{\mu_c} \frac{m^2}{\delta}, \quad (41)$$

where β_* is defined by

$$\beta_* = 6\pi\mu_c a R = (9/2)(\mu_c a/\delta^2).$$

6. The effective viscosity

We shall determine the effective viscosity μ by the method of simple cell model. As before, the fluid is supposed to flow through the space among fixed particles. When the local mean velocity is not uniform, the flow around a particle can be resolved

into (A) and (B) in Fig. 3. The flow (A) produces the Darcy resistance and the flow (B) causes the energy dissipation by viscosity. In the case of suspension, particles move with the fluid. Therefore, the viscosity of suspension may be different to some extent from that of the present problem.

Fig.3 Fixed particles and a non-uniform flow

Take a flow with uniform strain rate at infinity:

$$U_i = E_{ij} x_j, \quad (E_{ij} = E_{ji}, E_{ii} = 0). \quad (42)$$

In the simple cell model, the flow at distance prevails up to the cell surface. Inside the cell the fluid is governed by the differential equations

$$\nabla \cdot \mathbf{v} = 0, \quad \nabla p = \mu_c \Delta \mathbf{v}, \quad (43)$$

and by the boundary conditions

$$\mathbf{v} = 0 \quad \text{at } r = a, \quad (44)$$

$$\mathbf{v}_r = z_c U_r, \quad \mathbf{v}_i = U_i \quad \text{at } r = l. \quad (45)$$

The latter condition correspond to (7) and (8). The volume fraction of the continuous phase is z_c , while that of the dispersed phase is z_d :

$$z_c = 1 - z_d, \quad z_d = (a/l)^3 = \delta^3. \quad (46)$$

If z_c is set formally at unity in (45), the problem reduces to one of suspension studied by R. Simha⁽³⁾.

Since the flow is governed by the external tensor of the second order E_{ij} , the appropriate solid harmonics for solution are as follows:

$$\phi_2 = (x_i x_j - \frac{1}{3} r^2 \delta_{ij}) E_{ij},$$

$$\phi_{-2} = \phi_2 / r^5.$$

The solution is constructed by using these harmonics⁽⁴⁾.

$$v_i = \{V_r \frac{x_i x_j}{r^2} + V_t (\delta_{ij} - \frac{x_i x_j}{r^2})\} x_k E_{jk}, \quad (47)$$

where

$$V_r = (2 - 5 \frac{a^3}{r^3} + 3 \frac{a^5}{r^5}) \bar{C}$$

$$+ \frac{r^2}{a^2} (15 - 21 \frac{a^2}{r^2} + 6 \frac{a^4}{r^4}) \bar{C}, \quad (48a)$$

$$V_t = 2 (1 - \frac{a^5}{r^5}) \bar{C}$$

$$+ \frac{r^2}{a^2} (25 - 21 \frac{a^2}{r^2} - 4 \frac{a^4}{r^4}) \bar{C}, \quad (48b)$$

$\bar{C}, \bar{C}$ being arbitrary constants.

The boundary conditions (45) are written as

$$(V_r)_{r=a} = 1 - z_d, \quad (V_t)_{r=a} = 1. \quad (49)$$

The solution is obtained as follows:

$$\bar{C} = \{2(1 - \delta^3) - \frac{1}{5} z_d (25 - 21 \delta^2 - 4 \delta^4)\} / D, \quad (50a)$$

$$\delta^2 \bar{C} = \{-\delta^3 (1 - \delta^2) + \frac{2}{5} z_d (1 - \delta^5)\} / D, \quad (50b)$$

where

$$D = 4 - 25 \delta^3 + 42 \delta^5 - 25 \delta^7 + 4 \delta^{10}.$$

The pressure tensor on the surface $r = \text{const.}$ is given by

$$P_{ik} = \{P_r \frac{x_i x_k}{r^2} + P_t (\delta_{ik} - \frac{x_i x_k}{r^2})\} x_n E_{jn}, \quad (51)$$

where

$$-\frac{r}{\mu_c} P_r = 2(2 + 15 \frac{a^3}{r^3} - 12 \frac{a^5}{r^5}) \bar{C} - 3 \frac{r^2}{a^2} (5 + 14 \frac{a^2}{r^2} + 16 \frac{a^4}{r^4}) \bar{C},$$

$$-\frac{r}{\mu_c} P_t = 2(2 - 5 \frac{a^3}{r^3} + 8 \frac{a^5}{r^5}) \bar{C} + 2 \frac{r^2}{a^2} (40 - 21 \frac{a^2}{r^2} + 16 \frac{a^4}{r^4}) \bar{C}.$$

The dissipation function per particle Φ , is calculated by the formula (11), where the following integral is useful

$$\int_{r=a} x_i x_k x_l x_m dS = (4\pi a^4 / 15) \times (\delta_{ij} \delta_{lm} + \delta_{jl} \delta_{km} + \delta_{jm} \delta_{kl}).$$

The formula (49) gives

$$\Phi = \frac{1}{10} \mu_c |\mathbf{E}|^2 \frac{4\pi a^4}{3} (2P_r^* + 3P_t^* - 2z_d P_r^*), \quad (52)$$

where

$$|\mathbf{E}|^2 = E_{jk} E_{jk},$$

$$P_r^* = 2(2 + 15 \delta^3 - 12 \delta^5) \bar{C} - 3(5 + 14 \delta^2 + 16 \delta^4) \delta^2 \bar{C}, \quad (53a)$$

$$P_t^* = 2(2 - 5 \delta^3 + 8 \delta^5) \bar{C} + 2(40 - 21 \delta^2 + 16 \delta^4) \delta^2 \bar{C}. \quad (53b)$$

The dissipation function per unit volume $\bar{\Phi}$ is equal to $n\Phi$, which must be equal to $(1/2) \mu_c |\mathbf{E}|^2$. Consequently, the effective viscosity is formulated as follows:

$$\frac{\mu}{\mu_c} = \frac{1}{10} (2P_r^* + 3P_r^* - z z_d P_r^*). \quad (54)$$

By substituting (50) and (53), the effective viscosity can be calculated as a function of $z_d = \phi^3$.

7. Numerical calculation and discussions

First, the result of numerical calculation of (54) is shown by a solid line in Fig. 4. A chain line in the figure shows the viscosity of suspension μ_s calculated by R. Simha, which is also obtained from (54) by setting $z_d = 0$ as stated before. It seems strange that the flow through fixed particles has lower viscosity than the flow of suspension. In the former flow the energy dissipation mainly takes place through Darcy resistance and the flow around a sphere is smooth compared with the suspension flow.

Next, the drag coefficient or the coefficient of internal resistance β is calculated from Eqs. (37) and (41), where the value determined by (54) is used as μ . The result is shown by the solid line in Fig. 5. The double dotted chain line shows the result obtained in the preceding paper by Method 11. The chain line does the curve obtained by using the suspension viscosity μ_s in (38), for comparison. The dotted line shows the viscous part of Ergun-Orning's empirical formula (5), which is obtained from the measurement of pressure drop in pipe flows. In the present notation, the formula becomes

$$\frac{\beta}{\beta_s} = \frac{25}{3} \frac{z_d}{1-z_d} + \frac{7}{12} \frac{z_d U}{\mu_c / P_c}.$$

The first term is the viscous part of resistance as shown in Fig. 5 by a dotted line. It is seen that the solid line is the best among three in the sense that it is the nearest to the empirical curve.

Fig.4 Effective viscosity vs. volume fraction

Fig.5 Drag coefficient vs. volume fraction

8. Conclusion

For two-phase flows composed of a fluid and solid particles, the coefficient of internal resistance is studied. When the average velocities of the two component fluids are different, there arises an internal resistance proportional to the relative velocity. The proportionality constant is the coefficient of internal resistance. This coefficient is determined as a function of the volume fraction of particle phase by the method of self-consistent cell model.

It is assumed that inside the cell the fluid is Stokesian while outside the cell the fluid is Stokes-Darcian. The elementary solutions to the motion of the latter fluid are obtained. The solutions in both sides are connected on the cell surface. The condition of connection is determined by considering the principle of variation. The condition of coincidence is required to determine the drag coefficient. The numerical calculation is carried out, and the prediction is reasonable. One of the viscosities in dispersive two-phase flow is obtained by the method of simple cell model.

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