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Temperature and Strain Rate Effects upon the Strength of Zinc Single Crystals*

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The temperature and the strain rate effects upon the strength of zinc single crystals were investigated experimentally in the following three cases; (1) when only the basal slip can be operative, (2) when both the basal slip and the mechanical twinning can be operative and (3) when both the basal slip and the pyramidal slip can be operative. Results are summarized as follows: The static strength in almost all cases decreases with an increase of temperature. On the other hand, the dynamic strength increases with a rising temperature in the case of (1) and also increases in some case of (2), when the basal slip is followed by the mechanical twinning. In the case of (3), the dynamic strength at small strain has similar dependency to that observed for the basal slip at high strain rate, while the strength at large strain decreases with an increase of temperature.

1. Introduction

Temperature and strain rate effects on the strength of metals have been studied by many investigators^{1,2}. An understanding of the strength of polycrystals may be enhanced when combined with a study of the strength for various kinds of deformation modes in single crystals³⁻⁷. In the ranges of the temperatures and the strain rates at which the sliding and the migration of grain boundaries are difficult to occur, various kinds of deformation modes have to operate simultaneously in grains of polycrystalline zinc in order that material continuity can be preserved from grain to grain. It is not always possible that any one of mechanical twinings of $(10\bar{1}2)$ operates under the action of a load in an arbitrary crystallographic direction⁸. Therefore, the operation of pyramidal slip of $(11\bar{2}2) \langle 11\bar{2}3 \rangle$ which can produce a slip in the c-axis direction will be induced in a grain in which a mechanical twinning can not operate. Though the slip in a direction perpendicular to the c-axis can be accomplished by the operation of the pyramidal slip, it is also accompanied with the slips in the basal plane (0001) and in the prismatic plane $(10\bar{1}0)$. Among these slips, the slip in the basal plane is preferred for its low Peierls force⁹ and it is always operative.

In the present paper the effects of the temperature and the strain rate on the strength of zinc single crystals are clarified in the following three cases; (1) when only the basal slip can be operative, (2) when both the basal slip and the mechanical twinning can be operative, and (3) when both the basal slip and the pyramidal slip can be operative.

2. Material and Experimental Procedure

Single crystals were grown by using a Bridgman furnace shown in Fig.1. Chemical compositions of the material used are tabulated in Table 1. The material in the form of rods (13mm $\times$ 13mm in section, 300mm in length) was set into a split graphite crucible pointed at one end. As shown in Fig.1, a tightly fitting Mullite tube was placed around the upper part of the split crucible in an attempt to obtain a unidirectional heat flow along the growth direction in most of the experiments.

The rods were melted in a vacuum and heated at 470°C for half an hour to insure a complete melt. The furnace was moved from the bottom of the crucible at 3cm/hr

Table 1 Chemical compositions of material

Zn	Cd	Pb	Fe	Sn
99.99	.0009	.0013	.0005	non

Fig.1 Details of furnace and crucible

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in most of the experiments and crystals were grown. The temperature was measured with thermocouples placed into the drilled holes (shown in Fig.1) to detect the temperature gradient along the zinc in the crucible. The temperature gradient along the zinc with a Mullite tube was about $10^{\circ}\text{C}/\text{cm}$ at solidification and single crystals without sub-boundaries were grown when the furnace was moved at a speed lower than $4\text{cm}/\text{hr}$. When the furnace was moved at a speed higher than $6\text{cm}/\text{hr}$, crystals with sub-boundaries were grown. The temperature gradient along the zinc without the Mullite tube was lower than about $5^{\circ}\text{C}/\text{cm}$ and small grains were accidentally yielded in crystals.

Crystallographic orientation of the growth direction was determined by back reflection x-ray diffraction. Crystals were cut with a chemical saw using a 50% HNO_3 and polished in 20% HNO_3 so as to be formed into cuboid test pieces of $4\text{mm}\times 4\text{mm}\times 3.5\text{mm}$ or of $12.5\text{mm}\times 12.5\text{mm}\times 12\text{mm}$. Smooth and parallel compression surfaces were accomplished by polishing with moist carborundum. Specimens were annealed at 200°C for 24 hours and the surfaces were chemically polished for 30~60sec in $16\text{grCr}_2\text{O}_3 + 2\text{grNa}_2\text{SO}_4 + 50\text{mlH}_2\text{O}^{10}$. Stereographic representation of the compression direction of the specimens is given in Fig.2. The values of χ_0 and λ_0 which are the angles between the compression direction and the normal to the basal plane and between the compression direction and the slip direction, respectively, are tabulated in Table 2.

An Instron testing machine was used for static compression tests. In the case of the slip in the basal plane, the temperature and the strain rate effects on the strength are not remarkable, and to make clear these effects, the differential tests in which the temperature and the strain rate are abruptly changed in the course of the deformation are carried out as follows: Specimens were deformed at 0°C with a cyclic change of the strain rate during the

Table 2 Values of χ_0 and λ_0

No.	1	2	3	4	5	6	7	8
χ_0 (degree)	36	51	51	60.5	55	59	88	89.5
λ_0 (degree)	54.5	39	40.5	31	40	38	12.5	6

Fig.2 Stereographic representation of compression axis

deformation up to the required strain. After the specimens were unloaded at the strain, the temperature was abruptly decreased to the required temperature. Then, the specimens were deformed again at a new temperature with a cyclic change of the strain rate in the course of the deformation. To bring about an abrupt change of temperature, the temperature of the specimen is at first controlled at 0°C and a new required temperature is attached by installing the cooling vessel to the testing machine. The temperature is sensed momentarily by Alumel-Chromel thermocouples attached to the specimen.

In the case of operations of non-basal slip and of mechanical twinning, the static compression tests at various constant temperatures with the cyclic change of strain rate were performed.

A split Hopkinson pressure bar apparatus was used for the dynamic compression tests. Two kinds of striker bars are used to determine the stress increment caused by an abrupt increase of the strain rate at various constant temperatures. One is a step-shaped striker bar¹¹. A thinner part of the bar is of Duralmin 10mm in diameter and 300mm in length, and the other thicker part is of stainless steel 20mm in diameter and 750mm in length. The other is a composite bar with an elastic and a plastic part. The elastic part is a stainless steel bar of 17mm in diameter and of 500mm in length, and has a $10\text{mm}\phi\times 10\text{mm}$ drilled hole at one end. The plastic part is a pure aluminium of 6mm in diameter and 12mm in length. The plastic part is put into the hole in alignment and in contact with the bottom surface of the hole. By using these two kinds of striker bars, a step-shaped loading stress wave is generated. The first step of the loading wave is produced by the plastic deformation of the aluminium. Due to the step shaped bar and the composite striker bar, the strain rate is abruptly increased two or four times during deformation, respectively. The specimens deformed were examined by an optical microscope to ascertain a slip system in operation.

3. Results and Consideration

3-1. In the case when only the basal slip can be operative

Relations between the shear stress and the shear strain on the basal plane at static strain rate ($\dot{\gamma}=10^{-4}/\text{s}$) are shown in Fig.3. In the relations specimens are deformed up to 20% shear strain at 0°C with a cyclic change of the strain rate from a strain rate $\dot{\epsilon}_1$ to a higher strain rate $10\times\dot{\epsilon}_1$ and from $10\times\dot{\epsilon}_1$ to $\dot{\epsilon}_1$. Specimens are unloaded at the required strain and the temperature is abruptly changed to the required temperatures. At the new temperatures the specimens are loaded again at a strain rate $\dot{\epsilon}_2$ with a cyclic change of the strain rate from $\dot{\epsilon}_2$ to $10\times\dot{\epsilon}_2$ and from $10\times\dot{\epsilon}_2$ to $\dot{\epsilon}_2$. In the figure the notation $L\rightarrow L$ indicates the case that the strain rate $\dot{\epsilon}_2$ at the new temperature is $\dot{\epsilon}_1$ and the notation $L\rightarrow H$ is used for the case when the strain rate $\dot{\epsilon}_2$ is 5

$\times \dot{\epsilon}_1$. It is seen that the shear stress is only slightly dependent on the strain rate $\dot{\epsilon}$ at every temperature. Denoting the shear stress increment as $\Delta\tau$ caused by a change of the strain rate, and the logarithmic increment of the shear strain rate as $\Delta \ln \dot{\gamma}$, the value of $(\Delta\tau/\tau)/\Delta \ln \dot{\gamma}$ is obtained as about 0.01 regardless of the temperature. At temperatures of -195°C , -150°C and -72°C when the strain rate is abruptly changed from $10 \times \dot{\epsilon}_2$ to $\dot{\epsilon}_2$, the stress-strain relation at $\dot{\epsilon}_2$ immediately returns to the relation obtained by an appropriate extrapolation of the stress-strain relation at $\dot{\epsilon}_2$ to the required strain. This fact suggests that the stress-strain relation is independent of the strain rate history. On the other hand, when the strain rate at strains beyond about 10% returns to $\dot{\epsilon}_1$ from $10 \times \dot{\epsilon}_1$ at 0°C , the stress does not change immediately but gradually changes as shown in Fig.3. Drouard et al.¹² investigated thermal recovery of the strength of zinc single crystals in the case of the basal slip, and detected the thermal recovery operating even at -30°C . The gradual stress response to the strain rate in the present experiment may be understood by the thermal recovery.

At temperatures below 0°C , weak yielding phenomena at large strains are accompanied with an abrupt increase of the strain rate as shown in the figure in detail. The increase of the stress due to the abrupt decrease of the temperature is understood as a thermally activated process.

An activation volume v^* in the thermally activated mechanism is defined as:

$$v^* = kT \partial \ln \dot{\gamma} / \partial \tau^* \quad (1)$$

where k , T and τ^* are Boltzman's constant, the absolute temperature and the effective shear stress, respectively. In the relation the effective stress τ^* is expressed as

$$\tau^* = \tau - \tau_u$$

where τ and τ_u are the applied shear stress and the athermal component of the stress. The values of v^* in the present experiment are $(4 \sim 8) \times 10^{-19} \text{ cm}^3$ at the static strain rate on condition that effective stress increment is evaluated by an instantaneous increment of the applied stress due to the abrupt increase of the strain rate. The present value agrees well with the value of $3 \times 10^{-19} \text{ cm}^3$ obtained by Lukáč.¹³

Figs.4 and 5 represent the stress-

Fig.3 Shear stress-shear strain relation for basal slip (static)

strain curves at high strain rates. The vertical arrows in the figure indicate the strain at which the strain rate is abruptly increased about two or four times. An example of the strain rate to the time relations with an abrupt change during the deformation is shown in Fig.6. Deformation commences at almost a constant lower strain rate and proceeds subsequently at almost a constant higher strain rate. In Fig.4 the stress-strain relation at the higher strain rate following the lower strain rate agrees fairly well with the relation at the constant higher strain rate. It is also found that the work hardening rate is independent of the temperature. The strain rate sensitivity at high strain rates is much more remarkable than that at low strain rates, and the value of $(\Delta\tau/\tau)/\Delta \ln \dot{\gamma}$ is obtained as about 0.20.

Microscopic observations give direct evidence of a homogeneous deformation accompanied only with a basal slip up to about 30% strain in static tests. In the dynamic tests, the deformation proceeds without twinning up to about 20% strain as shown in Fig.7, while at about 30% strain the deformation at temperatures above room temperature is accompanied with mechanical twinings and non-basal slips. The gradual increase of the work hardening rate starting at about 20% strain at temperatures above room temperature will be understood in connection with the commence of twinning and non-basal slip. Next it is relevant to discuss the control mechanism operating at high strain rates.

Fig.8 represents the relations be-

Fig.4 Shear stress-shear strain relation for basal slip (dynamic)

Fig.5 Shear stress-shear strain relation for basal slip (dynamic)

tween the shear stress at 15% strain and the shear strain rate. In the figure the arrows are drawn connecting the stresses at the lower and the higher strain rate in the dynamic differential tests. The respective straight lines through the experimental points at every temperature have the following relation:

$$\tau = \alpha \dot{\gamma} + \tau_b \quad (2),$$

where τ_b is the back stress.

The values of α and τ_b in the relation are given in Table 3 as a function of temperature. The shear stresses at 15% shear strain are plotted against the temperature in Fig. 9. The temperature dependency of the shear stress in the static tests will be combined with a thermally activated mechanism controlling the static strength, while the behaviour at the high strain rates can not be understood as a thermally activated one.

Various kinds of damping mechanisms of the dislocation motion have been proposed by a number of investigators. Dislocation interaction with electrical wave is only predominant at cryogenic temperatures. The thermoelastic effect does not produce a drag for screw dislocations and is small even for edge dislocations. Dislocation damping due to the scattering

Table 3 Values of α and τ_b

Temperature ($^{\circ}\text{C}$)	-195	-45	17	100	200
$\alpha \times 10^5$ ($\text{kg} \cdot \text{s} / \text{mm}^2$)	3.8	6.0	6.8	7.6	8.0
τ_b (kg / mm^2)	0.29	0.32	0.35	0.40	0.40

Fig. 6 Shear strain rate shear strain relation with an abrupt change of strain rate

Fig. 7 Basal slip traces on specimen No. 7 (dynamic)

of thermal phonons by moving dislocations¹⁶ and damping mechanism related to phonon viscosity¹⁷ are now considered. When the dislocation motion is damped in a viscous manner, the equation of motion is expressed by the following equation:

$$(\tau - \tau_b) b = Bv \quad (3),$$

where b , v and B are Burger's vector, the velocity of dislocation and the damping constant, respectively. In the equation the left hand side represents the force acting on the unit length of moving dislocation and the right hand side shows the damping force against the dislocation. The shear strain rate $\dot{\gamma}$ is related to the mobile dislocation density ρ as:

$$\dot{\gamma} = b\rho v \quad (4).$$

Combining the equations (2), (3) and (4), the damping constant B can be related to α . Then,

$$B = \alpha \rho b^2 \quad (5).$$

The values of the damping constant B s for phonon scattering and the damping constant Bv for phonon viscosity were calculated by Ferguson et al.⁴ in the case of zinc for the basal slip, as given in Table 4, as a function of temperature. When the values of $\alpha \rho b^2$ at 100°C in the present experiment are given by the values of B s and Bv in Table 4, the mobile dislocation density ρ s and ρv for the respective cases of phonon scattering and of phonon viscosity can be evaluated as $1.1 \times 10^8 / \text{cm}^2$ and $1.7 \times 10^7 / \text{cm}^2$, respectively. The evaluation is carried out by taking $b = 2.67 \times 10^{-8} \text{cm}$ for zinc and by using the values of α in Table 3. The values of $\alpha \rho b^2$ and $\alpha \rho v b^2$ are evaluated as a function of temperature and are plotted

Fig. 8 Shear stress-shear strain rate relation for basal slip (dynamic)

Fig. 9 Shear stress-temperature relation for basal slip

as solid and open circles in Fig.10, respectively. The full lines in the figure are obtained by connecting the values of B_s and B_v calculated by Ferguson et al. It can be seen in Figs.4 and 5 that the work hardening rate is almost independent of the temperature. Under the condition of the constant mobile dislocation density, the temperature dependency of damping constant evaluated in the present experiment agrees to the prediction of the phonon viscosity rather than that of the phonon scattering. For the basal slip in zinc single crystals, Ferguson et al. have shown almost a constant yield stress at high strain rates independently of the temperature from 27°C to 380°C. Yoshida et al. also investigated the deformation behaviour of zinc single crystals at high rates of strain and pointed out that the strength increases with a rising temperature from -195°C to 100°C. In the case of the phonon viscosity effect, Ferguson et al.⁴ predict that the strength increases remarkably with a rising temperature from -195°C to about 100°C and is almost independent of temperature above about 100°C. In the present experiment the shear stresses at high strain rates are remarkably dependent on temperature below about 100°C and this behaviour can be understood as a phonon viscosity effect. The mobile dislocation density estimated as $1.1 \times 10^8/\text{cm}^2$ is a reasonable amount for slightly strained metals.

3-2. In the case when both the basal slip and the mechanical twinning can be operative

The compression tests were carried out on specimen No.1 which deforms with the basal slip and the mechanical twinning at an early stage of deformation. Fig.11 shows the relations of the shear stress to the shear strain on the basal plane in the static tests ($\dot{\gamma} \approx 10^{-4}/\text{s}$). The shear stress is slightly dependent on the temperature

Table 4 Values of damping constants B_s and B_v

Temperature (°C)	-173	-73	23	123	223
B_s (dyne·s/cm ²)	1.2×10^5	3.8×10^5	6.5×10^5	9.5×10^5	1.3×10^6
B_v (dyne·s/cm ²)	3.3×10^4	5.1×10^4	5.7×10^4	6.0×10^4	6.1×10^4

Fig.10 Damping constant-temperature relation

and the specimens deform up to 50% strain accompanied with only a basal slip. In Fig.12 the relations between the shear stress and the shear strain in the dynamic tests are represented at various temperatures. As shown in the figure, work hardening rate is very low in the small strain range. The amount of the strain where a remarkable work hardening takes place is significantly influenced by the temperature and decreases much more with a rising temperature. The shear stress where the remarkable work hardening commences is estimated at about 1 kg/mm^2 on the basal plane, and is almost independent of the temperature. It is relevant to discuss the behaviour of the stress in some detail.

Bell et al.¹⁸ carried out the static tension tests on zinc single crystals and speculated that mechanical twinning would occur due to a local stress induced by piled-up dislocations in the pyramidal plane. In their experiments, the values of the critical resolved shear stress for the pyramidal slip were obtained as $(1.05 \sim 1.6) \text{ kg/mm}^2$. It is also reported that the work hardening rate is increased due to the operation of the pyramidal slip.

In the present experiment, direct ob-

Fig.11 Shear stress-shear strain relation for basal slip (static)

Fig.12 Shear stress-shear strain relation for simultaneous operation of basal slip and mechanical twinning (dynamic)

Fig.13 Slip traces and mechanical twinning on specimen No.1 (dynamic)

servation under an optical microscope reveals the operation of four twinning systems for both cases of No.1 ($\chi_0=36^\circ$) and No.5 ($\chi_0=55^\circ$) specimens. Homogeneous slip traces in the non-basal planes are observed on No.1 specimen. On No.5 specimen, inhomogeneous slip traces in the non-basal plane are accompanied with mechanical twinning. When No.1 specimen was dynamically deformed up to 30% strain at -45°C , mechanical twinings, basal and non-basal slips are detected as shown in Fig.13. The non-basal slip accompanied with mechanical twinning may be a pyramidal one which is always accompanied with mechanical twinning as the experimental results obtained by Bell et al. demonstrate.

On the assumption that only the basal slip is in operation up to the shear strain where the stress-strain relation with very low rates of work hardening ends, the shear stress on the pyramidal plane is evaluated at the strain and is found to be about $(1.0 \sim 1.2)\text{kg/mm}^2$ as given in Appendix 1. The pyramidal slip may be followed by mechanical twinning. The basal slip will be prevented by the mechanical twinning and the specimens deform with high rates of work hardening. The effect of work hardening due to the pyramidal slip will also contribute to the high rates of work hardening to some extent.

3-3. In the case when both the pyramidal slip and the basal slip can be operative

Fig.14 shows the normal stress vs. the normal strain relations in static tests with crystals where the normal to the basal plane is initially almost perpendicular to the compression axis. Specimen No.7 ($\chi_0=88^\circ, \lambda_0=12.5^\circ$) and specimen No.8 ($\chi_0=89.5^\circ, \lambda_0=6^\circ$) were used for the static tests. In Fig.15 the traces of slips in the basal plane alone are observed on specimen No.7 deformed at -195°C . A number of cracks on the basal plane accompanied with kinks can be observed. The shape of the deformed specimen is illustrated in Fig.16. The cracks in the present case are nucleated by a process schematically represented in Fig.17. Friction will prevent to some extent the specimen from deforming near the contact surfaces to the loading heads. An array of piled-up dislocations on the basal plane may be split in the neighbourhood of the contact surfaces, and consequently a group of cracks will be generated⁹. In the case of specimen No.8, the basal and the pyramidal slips are able to operate as shown in Fig.18. The occurrence of the pyramidal slip may relax the local stress induced by the piled-up dislocations in the vicinity of the contact surfaces. At temperatures above -72°C , both specimens No.7 and No.8 deform with a high rate of work hardening in the small strain range. The work hardening rates decrease markedly with an increasing strain and the stresses decrease remarkably with a rising temperature. The basal and the pyramidal slip traces are detected on both specimens deformed at temperatures above -72°C .

The normal stress vs. normal strain curves at high strain rates are obtained

with specimen No.8. These are shown in Fig.19. At low temperatures the pronounced yielding phenomena are observed at small strain, and the deformation proceeds with high rates of work hardening. Beyond the strain of about $(10 \sim 15)\%$, the specimen deforms with a very low rate of work hardening or with work softening. In the stress-strain relation with the high work hardening rate, the stress usually increases with a rising temperature, and the temperature dependency of the stress is similar to that of the shear stress for the basal slip at high strain rates. The basal and the pyramidal slip traces are observed on the specimen dynamically deformed up to about 20% strain at -45°C as shown in Fig.20. Traces in horizontal direction correspond to the basal slip traces and they run zigzag. Two groups of almost straight traces across the basal slip traces correspond to the pyramidal ones. These observations suggest that the basal slip is the first to come into operation, followed by the pyramidal slip. In a relatively large strain range the pyramidal slip will take the place of the basal one. The Peierls force of dislocations on the basal plane is not strong, while those on non-basal plane may be relatively large. When the deformation is controlled by the thermally activated dislocation motion, the

Fig.14 Stress-strain relation for simultaneous operation of basal slip and pyramidal slip (static)

Fig.15 Cracks and basal slip traces on specimen No.7 (static)

strength decreases with a rising temperature at high strain rates. The mechanical behaviours at the relatively large strains can be understood from this point of view.

To certify the operative slip system during deformation, the rotation of the compression axis relative to the crystal axis is examined. Fig.21 is a stereographic representation of the compression axis which shows how it rotates during deformation. The orientations of specimen No.8 statically and dynamically deformed at room temperature and at -72°C are determined by back reflection x-ray diffraction and are represented in the figure by $P_1 \sim P_6$ and P_d . When the deformation proceeds only by the basal slip, the compression axis initially at P_0 will rotate along the broken line in the figure, and will reach B_1 , B_2 and B_3 on the line corresponding to the normal strains of 5%, 10% and 20%, respectively. The values of the strain given to the specimen are tabulated in Fig.21. The compression axis does not move along the broken line in the experiment, and the operation of non-basal slip will contribute to the rotation of the axis.

Now, it is necessary to know the ro-

Fig.16 Illustration of cracked specimen

Fig.17 Schematic representation of cracks formed in the vicinity of compression surfaces

Fig.18 Slip traces on specimen No.8 (static)

tation of the compression axis on the condition that specimen No.8 is deformed only by the slip in the pyramidal plane. The largest value of the Schmid factor for the pyramidal slip is for the pyramidal plane $\text{Py4}(11\bar{2}2)$, followed by one for the pyramidal plane $\text{Py1}(11\bar{2}2)$. Therefore, the pyramidal slip in the plane Py4 is the first to come into operation, and the compression axis moves from P_0 to Q_1 . When the specimen is deformed up to 1.6% strain, the axis reaches Q_1 which lies on a great circle through the poles of $(11\bar{2}0)$ and $(10\bar{1}0)$. And then, the double slip on the two pyramidal planes Py4 and Py1 will take place and the axis will move towards Q_4 .

Bowen et al.²⁰ have shown that the shear stress τ and the shear strain γ for double slip are related to the normal stress σ_n and the normal strain ϵ_n as follows:

$$\gamma = A \ln[1 + B(\sin\beta_0 / \cos\lambda_0)(\cot\beta - \cot\beta_0)]$$

$$\tau = \sigma_n (1 - \epsilon_n)^2 C \cos\beta [\cos\lambda_0 + B \sin\beta_0 \times (\cot\beta - \cot\beta_0)] \quad (6)$$

$$\sin\beta = (1 - \epsilon_n) \sin\beta_0$$

The values of A , B and C in the above relations are evaluated as shown in Appendix 2, and are given in Table 5. On the condition of the operation of the pyramidal slip alone, the shear stress-strain curves in Fig.22 are obtained by using Eq.(6).

Fig.19 Stress-strain relation for simultaneous operation of basal slip and pyramidal slip (dynamic)

Fig.20 Slip traces on specimen No.8 (dynamic)

The pyramidal slip seems to commence at about $(1\sqrt{2})\text{kg/mm}^2$ independently of the temperature.

4. Conclusions

The temperature and the strain rate effects upon the strength of zinc single crystals were investigated experimentally in the following three cases; (1) when only the basal slip can be operative, (2) when both the basal slip and the mechanical twinning can be operative and (3) when both the basal slip and the pyramidal slip can be operative. Results obtained are summarized as follows:

- (1) The static strength for the basal slip decreases with an increasing temperature and is controlled by a thermally activated process. On the other hand, the dynamic strength increases with a rising temperature and may be controlled by the damping effects of dislocations at high velocity.
- (2) In the dynamic case of the operation of the basal slip followed by the mechanical twinning, the strength also increases with an increase of temperature. The mechanical twinning and the non-basal slip

Fig.21 Stereographic representation of movement of compression axis in the course of deformation

Table 5 Values of constants A, B and C for double slip

A	B	C
4.789	0.237	0.881

Fig.22 Shear stress-shear strain relation for double slip in pyramidal planes

have an important effect on the operation of the basal slip and cause high rates of work hardening.

(3) In the case of the operation of both the basal and the pyramidal slips, the static strength decreases as the temperature is raised. The basal slip plays a predominant role at small strains, while at large strains the pyramidal slip has an important role. The increase of the temperature raises the dynamic strength at small strains and decreases it at large strains.

Appendix 1

When a single-slip system is operative, the formulae expressing the relationships between the shear stress τ and the normal stress σ and between the shear strain γ and the normal strain ϵ_n are given as follows:

$$\begin{aligned}\tau &= \sigma_n \cos \chi_1 \cos \lambda_1 \\ &= \sigma_n (\lambda_1 / l_0)^2 \sqrt{(\lambda_0 / l_1)^2 - \sin^2 \chi_0} \cos \lambda_0 \\ \gamma &= (1 / \cos \lambda_0) \{ \sqrt{(\lambda_0 / l_1)^2 - \sin^2 \chi_0} - \cos \chi_0 \} \quad (7), \\ (\lambda_0 / l_1) &= \sin \chi_0 / \sin \chi_1 = \cos \lambda_0 / \cos \lambda_1\end{aligned}$$

where l_0 and l_1 represent the length of the specimen before and after contraction. In the formulae χ_0 and χ_1 are the angles between the normal to the slip plane and the specimen axis before and after contraction, and λ_0 and λ_1 represent the angles of the specimen axis relative to the slip direction before and after deformation. In Fig.12 the values of the shear stress and the shear strain at which the deformation commences with high rates of work hardening are given in Table 6. Assuming that only the basal slip is operative up to the shear strain given in Table 6, the values of χ_1 and λ_1 in Eq.(7) can be evaluated by using the initial values of χ_0 and λ_0 . Then, the shear stresses for six pyramidal slip systems can be obtained as represented in Table 7. The angles between the compression axis and the normal to the slip plane, and between the axis and the slip direction for the i -th pyramidal slip are denoted as ψ_i and κ_i , respectively. The maximum value of the shear stress on the six slip systems at every temperature is obtained as $(1.0\sim 1.2)\text{kg/mm}^2$ and may be adopted as the critical resolved shear stress for the pyramidal slip.

Appendix 2

Bowen et al.²⁰ gives the expressions of the shear strain γ and the shear stress τ for double slip as:

$$\begin{aligned}\gamma &= (2 / n_1 \cdot s_2) \ln [1 + (n_1 \cdot s_2 / |P|) \\ &\quad \times (\sin \beta_0 / \cos \lambda_0) (\cot \beta - \cot \beta_0)] \\ \tau &= \sigma_n (1 - \epsilon_n)^2 (|P| / 2) \cos \beta \{ \cos \lambda_0 + \\ &\quad (n_1 \cdot s_2 / |P|) \sin \beta_0 (\cot \beta - \cot \beta_0) \}\end{aligned}$$

Table 6 Values of strain and stress used for evaluation of starting stress for pyramidal slip

Temperature (°C)	-195	-45	23	100	200
Shear strain (%)	23	23	15	10	15
Shear stress (kg/mm ²)	1.1	1.1	1.3	1.1	1.3

Table 7 Shear stresses on six pyramidal slip systems

		ψ_1	κ_1	ψ_2	κ_2	ψ_3	κ_3	ψ_4	κ_4	ψ_5	κ_5	ψ_6	κ_6	Max. of τ
100°C	$\chi_1=34.1^\circ$	28°	63°	54.5°	51°	84.5°	27°	88°	7°	77°	34.5°	45.5°	56.5°	
	$\lambda_1=56.3^\circ$	$\tau=0.96(\text{kg/mm}^2)$		0.88		0.21		0.08		0.44		0.93		0.96
23°C	$\chi_1=33.3^\circ$	29°	62°	55°	50°	83.5°	26°	89°	6°	76°	34°	45.5°	55.5°	
200°C	$\lambda_1=57.2^\circ$	1.18		1.06		0.29		0.05		0.58		1.14		1.18
-45°C	$\chi_1=31.9^\circ$	30°	61°	55°	49°	83°	25.5°	90°	5.5°	75.5°	33.5°	46°	54°	
-195°C	$\lambda_1=58.5^\circ$	1.04		0.93		0.28		0		0.52		1.01		1.04

where $\mathbf{P}$ is a unit vector in the direction towards which the compression axis rotates. In Fig.23 the X, Y and Z axes represent the $\langle 11\bar{2}0 \rangle$, $\langle 11\bar{1}00 \rangle$ and $\langle 0001 \rangle$ directions, respectively. The double slip in the pyramidal plane $(11\bar{2}2)$ and its conjugate pyramidal plane $(\bar{1}\bar{1}22)$ is now considered. A unit vector normal to the $(11\bar{2}2)$ plane and a unit vector in the $\langle 11\bar{2}3 \rangle$ direction are denoted as $\mathbf{n}_1$ and $\mathbf{s}_1$, respectively. And the components in X, Y and Z directions of $\mathbf{n}_1$ and $\mathbf{s}_1$ are denoted as α_1 , α_2 and α_3 , respectively. Then, $\mathbf{m}_1$ and $\mathbf{s}_1$ can be written as:

$$\mathbf{m}_1 = \mathbf{m}_1(\alpha_1, \alpha_2, \alpha_3)$$

$$\mathbf{n}_1 = \mathbf{n}_1 \left((c/a)/\sqrt{1+(c/a)^2}, 0, 1/\sqrt{1+(c/a)^2} \right)$$

$$\mathbf{s}_1 = \mathbf{s}_1 \left(1/\sqrt{1+(c/a)^2}, 0, -(c/a)/\sqrt{1+(c/a)^2} \right)$$

where c/a is the axial ratio. Further, the expressions for $\mathbf{n}_2$ and $\mathbf{s}_2$ in Fig.23 are obtained as:

$$\mathbf{n}_2 = \mathbf{n}_2 \left((c/a)/\sqrt{1+(c/a)^2}, 0, -1/\sqrt{1+(c/a)^2} \right)$$

$$\mathbf{s}_2 = \mathbf{s}_2 \left(1/\sqrt{1+(c/a)^2}, 0, (c/a)/\sqrt{1+(c/a)^2} \right)$$

Since, $\mathbf{n}_1 + \mathbf{n}_2 = \mathbf{P}$ and

$$\mathbf{P} = \mathbf{P} \left((c/a)/\sqrt{1+(c/a)^2}, 0, 0 \right)$$

$$|\mathbf{P}| = (c/a)/\sqrt{1+(c/a)^2},$$

$$\mathbf{m}_1 \mathbf{s}_2 = (c/a)/\{1 + (c/a)^2\},$$

Fig.23 Schematic representation of pyramidal slip system

we have

$$|\mathbf{P}| = 1.761, \quad \mathbf{n}_1 \mathbf{s}_2 = 0.148,$$

by using the axial ratio in zinc of $c/a = 1.856$. The values of A , B and C in Eq.(6) are determined as shown in Table 5.

References

- (1) Rosenfield, A.R., et al., Dislocation Dynamics, (1968), McGraw-Hill, p.27.
- (2) Campbell, J.D., Mater. Sci. & Engr., Vol.12. No.1 (1973-7), p.3.
- (3) Gilman, J.J., Trans. A.I.M.E., Vol. 206. (1956-12), p.1326.
- (4) Ferguson, W.G., et al., J. Appl. Phys., Vol.38. No.4. (1967-3), p.1863.
- (5) Yoshida, S. and Nagata, N., J. Jap. Inst. Metal. (in Japanese), Vol.31. No.4. (1968-4), p.444.
- (6) Yoshida, S., et al., J. Jap. Inst. Metal. (in Japanese), Vol.33. No.2. (1970-2), p.271.
- (7) Yoshida, S. and Nagata, N., J. Jap. Inst. Metal. (in Japanese), Vol.31. No.11. (1968-11), p.1237.
- (8) Schmid, E. and Boas, W., Plasticity of Crystals, (1935), p.96, Chapman & Hall.
- (9) Suzuki, H., Introduction to Dislocation Theory, (in Japanese), (1970), p.233, AGNE.
- (10) Gilman, J. J., Trans. A.I.M.E., Vol. 206. (1956-8), p.998.
- (11) Tanaka, K. and Nojima, T., Proc. 1971 Kyoto ICM, (1972), p.176.
- (12) Drouard, R., et al., Trans. A.I.M.E., Vol.197. (1953-9), p.1226.
- (13) Lukac, P., Z. Metallkde., B.57. H.7. (1966-7), S.559.
- (14) Mason, W. P., Appl. Phys. Letters., Vol.6. No.6. (1965-3), p.111.
- (15) Eshelby, J.D., Proc. Roy. Soc. Lond., Ser. A. Vol.197. No.1050. (1949-6), p.396.
- (16) Leibfried, G., Z. Phys., B.127. H.4. (1950-3), S.344.
- (17) Mason, W. P., J. Appl. Phys., Vol.35. No.9. (1964-9), p.2779.
- (18) Bell, R.L. and Cahn, R.W., Proc. Roy. Soc. Lond., Ser. A, Vol.239. No.1219. (1957-4), p.494.
- (19) Stroh, A.N., Phil. Mag., Vol.3. No.30. (1958-6), p.597.
- (20) Bowen, A. N. and Christian, J.W., Phil. Mag., Vol.12. No.116. (1965-8), p.369.