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Analysis of Gain Coefficients for CO₂-N₂-He Nozzle Flows with Boundary Layer*

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Experimental measurements and theoretical calculation of gain coefficients were made for vibrationally nonequilibrium nozzle flows, with relatively low reservoir pressure. A simple and semi-empirical method to estimate the effective cross section of the expansion core was proposed and applied to the calculations. Gain coefficients versus the reservoir pressure, throat height, and the distance along a nozzle were measured. Experimental results agreed well with the theoretical ones.

1. Introduction

High energy lasers have made rapid progress as a means for a large scale metal heating, an isotope separation, and the controlled thermonuclear fusion. CO₂ lasers are drawing attention on account of their output power and efficiency. Particularly, the gasdynamic lasers (GDLs) which attain population inversions by means of aerodynamical cooling of thermally pumped CO₂ mixture, are a subject of much interest to mechanical engineers.

The history of the GDL began with Basov¹ and Hertzberg², who suggested that population inversions could be created by the rapid temperature change of a molecular system which contains various relaxation times. The first oscillation of the GDL was attained by Gerry et al. in 1970. This was a very novel event, because they extracted a very powerful laser beam by purely aerodynamical means. The advantage of the GDL is that it can be scaled up to a large size without any physical complications. In practice, it was reported that a power of 400kw was extracted using a shock tube.³ After that, many methods have been devised for heating, rapid cooling of the laser medium⁴, and mixing of CO₂.

The oscillations of the GDL have been also reported in our country, by using a shock tube^{5,7} or an arc heater.⁸ The GDL has renewed again and again the high power record, but the latest interest of research people is turning to its fundamental processes, and efforts are being made in many aspects⁹, because energy transfer processes which make the population inversion have not been understood clearly.

A special feature of the GDL is that it produces the laser power by a purely thermal, aerodynamical method, therefore, its main part is a supersonic vibrational nonequilibrium flow, which makes the population inversion and refreshes the laser medium.

One method to observe the vibrational nonequilibrium flow is to measure the gain coefficient of a light whose wavelength corresponds to the energy difference between the vibrational energy levels. Small signal gain coefficients, which are affected by the reservoir pressure and temperature, the cross sectional area ratios of the nozzle, and the nozzle size, are the fundamental values for the theoretical analysis of the GDL. The more rapidly the cross sectional area changes the larger the nonequilibrium effect becomes. But boundary layer effects cannot be ignored under these conditions. It is expected that the progress of the boundary layers reduces the effective cross sectional area and promotes the relaxation. On the other hand, when one deals with its effect theoretically^{6,10}, some assumptions are necessary; i.e., about the flow, laminar or turbulent; the wall temperature; the catalysis of the wall, etc. Therefore its effects cannot be clarified simply. Considering these points, we first take a semi-empirical method to estimate the effective cross sectional area of the nozzle expansion.^{11~13}

The infrared gain coefficients were measured with the parameters: the reservoir pressure P_0 , the throat height h_t , and the distance along nozzle axis X . The experimental values showed fairly good agreement with the theoretical estimations.

2. Nomenclature

v_1, v_2, v_3	: Vibrational modes
v_1, v_2, v_3	: Vibrational levels
k	: Rate constant
τ	: Relaxation time
N	: Number density
P	: Pressure
ρ	: Density
R	: Gas constant
T	: Absolute temperature
U	: Flow speed
e	: Specific internal energy
h	: Specific enthalpy
s	: Specific entropy
C_p	: Specific heat at constant pressure
γ	: Ratio of specific heats

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A	: Cross sectional area of a nozzle
A*	: Cross sectional area of a nozzle throat
K	: Boltzmann constant
h	: Planck's constant
c	: Speed of light
π	: Circular constant
ν	: Frequency
λ	: Wavelength
B	: Rotational constant
v	: Vibrational quantum number
J	: Rotational quantum number
g(J), g _v	: Statistical weight
I	: Radiation intensity
σ	: Population inversion
G	: Gain coefficient
α	: Absorption coefficient
X	: Coordinate along flow direction
h	: Nozzle throat height
α	: Constant

Subscripts

*	: Vibrationally excited condition
a	: Active condition
eq	: Equilibrium value
vib	: Vibration
rot	: Rotation
u, l	: Upper and lower levels of the transitions
o	: Reservoir condition, or center of the transition
m	: Measured value
c	: Calculated value

3. Fundamental processes of the GDL

3.1 Principle of the CO₂ GDL

Carbon dioxide is a linear symmetric, triatomic molecule and has three independent vibrational modes; ν_1 , ν_2 , and ν_3 . The vibrational levels of this molecule are characterized by quantum numbers specifying these three modes of vibration (ν_1, ν_2, ν_3), where ν_1 indicates the excitation of the symmetric axial vibration ν_1 ; ν_2 , the excitation of the twofold degenerate mode of vibration ν_2 ; and ν_3 , the excitation of the asymmetric mode of vibration ν_3 , (Fig.1.). Laser oscillations are possible in the transitions from $\nu_3(001)$ to $\nu_1(100)$ and from $\nu_3(001)$ to $\nu_2(020)$. Because of the rotational structure of the molecules, laser oscillations often consist of several vibrational-rotational lines and the former transition is called 10.6 μ band, the latter 9.6 μ band according to their center wavelength. The first energy level of vibrationally-excited

N₂($\nu=1$) molecules is so near to the first energy level of CO₂(ν_3)(001) molecules, that they can exchange vibrational energy resonantly. On the other hand, CO₂(ν_1)(100) modes couples with CO₂(ν_2)(020) mode by Fermi resonance. Therefore, CO₂(ν_3)(001) and N₂($\nu=1$) levels are called the upper laser level, and CO₂(ν_1)(100) and CO₂(ν_2)(020), the lower laser level (Fig.1). A rapid cooling of the thermally excited CO₂-N₂-He mixture gas produces an inversion of number density between the upper and lower levels, namely, the population inversion, as a result of different characteristic time for the vibrational relaxation. Vibrational transitions of the molecules absorb or emit photons corresponding to the energy difference between the energy levels, and this inversion makes the emission much larger than the absorption, which creates the laser condition. Helium is effective to deactivate the lower level and to increase the population inversion. On the other hand, a rapid cooling by the nozzle expansion also enables us to get a high speed flow which continues to send a laser pumped fresh gas actively. Then a high power infrared laser beam can be obtained with an optical resonator.

3.2 Analysis of nonequilibrium nozzle flows

The most important part of the GDL is the supersonic nozzle, which creates the laser active condition of high speed. Therefore, to study the gasdynamic mechanism means to analyze the vibrational nonequilibrium nozzle flows. In the case of a theoretical treatment of the nonequilibrium nozzle flows, one must, at first, pay attention to the applicability of a simple quasi-one-dimensional steady flow model for the expansion of the relaxing gas. This model is available unless the expansion angle of the nozzle is too large. Uncertainties of the relaxation rate constants and boundary layer effects are rather serious for the GDL nozzle flows. Relaxation rate constants are often unreliable and their temperature dependence is sometimes uncertain for the case of rapid cooling processes. In addition, it must be considered that the boundary layer grows up, when the nozzle rapidly changes its cross sectional area.

By the way, the static pressure measurements are admitted to be a sensitive and useful method to observe the nonequilibrium phenomena which contain dissociation. In fact, it is often used as a means to study high enthalpy nonequilibrium flows¹⁴. The dissociation of the CO₂ (or N₂) is little within the temperature range commonly used in the GDL. It is expected that the static pressure change due to the vibrational nonequilibrium is also little¹⁵, because energy stored in the vibrational modes is relatively small compared with its total energy of the mixed gas. From these stands, a semi-empirical method which utilizes the result of the static pressure measurements along the nozzle together with the gas dynamic equations, is proposed to take account of the effective nozzle size without the boundary layer. In this paper, the effective cross-sectional area ratios obtained by this

Fig.1 Energy level diagram for CO₂ and N₂

method were used for calculating the population inversion.

3.3 Relaxation processes and the vibrational model

At the start of a vibrational nonequilibrium flow analysis, we must assume a simplified model of the vibrational energy levels. In general, important collisions of molecules in the CO₂-N₂-He system are

where M means the collision partners (CO₂, N₂, or He). It is assumed that each reaction can be expressed by a simple rate equation containing the rate constants (k_i), which can be measured by experiment and have been collected by Taylor and Bittermann¹⁶. The relation between these (k_i) and so-called relaxation times (τ_i) is expressed by

$$k = \frac{1}{\tau N} = \frac{\alpha k T}{\tau P} \dots\dots (4)$$

where α is a constant whose value is 0.9872 after P(atm) and τ_i (μsec). The values of (k_i) in ref.(16) have been after referred to by many scholars to calculate vibrational energy transfers in the CO₂-N₂-He systems.

However, large scatters exist in much of the data in the ref.(16), while only one set of data are collected for the few rate. In addition, it is not certain whether the values of the vibrational relaxation rate data mainly obtained by rapid heating, are valid also in the rapid cooling phase of the GDL or not. Using the relaxation times estimated by Anderson¹⁷ and also using ref.(16), our numerical results of the population inversion were obtained.

The energy level diagram for CO₂ and N₂ is presented in Fig.1. Energy transfer between CO₂(001) and N₂($\nu=1$) is sufficiently rapid compared with other processes and a local equilibrium is maintained, if there is no strong radiative field. So these two levels can be considered to make one group, which has the same vibrational temperature $T_{\text{vib},1}(\text{Mod.I})$. The lower levels of CO₂(ν_1 and ν_2) are coupled by Fermi resonance, so that these two are also considered to be in one mode (Mod.II). This two mode model and assumption of simple harmonic oscillators lead to the relaxation equation as follows,

$$\frac{d(e_{\text{vib}})_i}{dt} = \frac{1}{\tau_i} [(e_{\text{vib}})_i^{**} - (e_{\text{vib}})_i] \dots\dots (5)$$

where $(e_{\text{vib}})_i$ is the vibrational energy per unit mass and τ_i is the characteristic relaxation time of mode- i ($i=1,2$).

3.4 Fundamental equations

It is assumed that all thermal and flow variables depend only on the distance X along the nozzle center line, and dissipation effects due to viscosity or thermal conductivity are limited to within the boundary layer regions along the nozzle; i.e., we consider that the boundary layer effect is only to reduce the effective cross sectional area of the nozzle wall. Adopting the steady, adiabatic, nonviscous,

quasi-one-dimensional flow assumption for the main flow, three conservation equations will be written as follows,

$$\rho u A = \text{const.} \dots\dots (6)$$

$$dP/dX + \rho u (du/dX) = 0 \dots\dots (7)$$

$$H + u^2/2 = \text{const.} \dots\dots (8)$$

where A is the effective cross-sectional area of the nozzle. As the vibrational energy transfer is only within the internal freedom of the gas, it can be assumed that the main flow is adiabatic. In the analysis, the following two equations are necessary; the equation for the perfect gas,

$$P = \rho R T \dots\dots (9)$$

and the vibrational relaxation equation for the ateady flows,

$$u \frac{d(e_{\text{vib}})_i}{dX} = \frac{1}{\tau_i} [(e_{\text{vib}})_i^{**} - (e_{\text{vib}})_i] \dots\dots (10)$$

where R is a mean gas constant which can be calculated by $1/R = \sum_i \phi_i / R_i$, where ϕ_i is a mole fraction and R_i is a gas constant of the gas species(i).

Nonequilibrium vibrational energy distributions can be calculated by coupling the flow conservation equations with the other two ones. The numerical solutions can be obtained by Runge-Kutta-Gill method¹⁸, under the assumption of an equilibrium flow condition as far as the nozzle throat. In the case of relatively low reservoir pressure conditions ($P_0 < 10\text{atm}$), the freezing of the vibrational energy occurs even at the upstream side of the throat⁹, which means that this assumption cannot be adopted. However this problem is resolved by the Time-Dependent-method²⁰. A calculation by this method starts with an initial distribution of the flow variables and the nonequilibrium quantities which are assumed artificially. When the time derivatives of all quantities obtained from the fundamental unsteady finite-difference equations go to zero after step by step calculations, steady flow solutions can be obtained.

If the flow variables (P, ρ, u, e, T) are expressed for example by $g(X, t)$, $g(X, t + \Delta t)$ is obtained by Taylor's series expansion with time

$$g(X, t + \Delta t) = g(X, t) + \left(\frac{\partial g}{\partial t}\right) \Delta t + \frac{1}{2} \left(\frac{\partial^2 g}{\partial t^2}\right) \Delta t^2 \dots\dots (11)$$

where the time derivatives $\left(\frac{\partial g}{\partial t}\right), \left(\frac{\partial^2 g}{\partial t^2}\right)$ are directly obtained from the non-steady conservation, relaxation equations and the equation of state, in place of eqs.(6)-(10). Steady state solutions are obtained when $\left(\frac{\partial g}{\partial t}\right), \left(\frac{\partial^2 g}{\partial t^2}\right)$ go to zero by iterative calculations. The advantage of this method is as follows. No difficult problems arise at the throat region during the calculation, and a relatively short computer time is enough to get a result.

3.5 Calculation of the population inversion

Denoting the number densities of the upper and lower laser levels with N_u and N_l , the population inversion σ can be written in a normalized form as follows,

$$\sigma = (N_u - N_l) / N_{\text{CO}_2} \dots\dots (12)$$

where N_{CO_2} is the total number density of CO_2 . As N_u and N_l belong to the respective rotational levels, they are written as,

$$N_{u,l} = \frac{N_j g_j \exp \left(\frac{-F_j(J) hc / \kappa T_{rot}}{(P \cdot f)_{rot}} \right)}{(P \cdot f)_{rot}} \dots (13)$$

where N_j is the number density in the vibrational level j , $g(j)$ is the statistical weight of the rotational level J , E_{rot} is the rotational energy of the molecule and $F(J) = E_{rot} / hc$.

On the other hand, N_j is expressed by the equation;

$$N_j = N_{CO_2} \exp \left(\frac{-h\nu_j / \kappa T_{vib,j}}{(p \cdot f)_{vib}} \right) \dots (14)$$

where $(p \cdot f)_{vib}$ and $(p \cdot f)_{rot}$ are the partition functions of the vibration and the rotation, respectively. They are given² for the harmonic oscillator and rigid rotator model) by

$$(p \cdot f)_{vib} = \prod_j \left[1 - \exp \left(\frac{-h\nu_j / \kappa T_{vib,j}}{(p \cdot f)_{vib,j}} \right) \right]^{-1} \dots (15)$$

$$(p \cdot f)_{rot} = \kappa T_{rot} / 2hcB_j \dots (16)$$

where $g_{n,j}$ is the statistical weight of the vibrational level. The vibrational energy $(e_{vib})_i$ of mode- i is given in the next form as a function of the vibrational temperature.

$$(e_{vib})_i = \sum_j N_{CO_2} \frac{h\nu_j}{\left[\exp \left(\frac{h\nu_j / \kappa T_{vib,i}}{(p \cdot f)_{vib,i}} \right) - 1 \right]} \dots (17)$$

where j denotes the vibrational modes contained in the mode- i .

In general, the rotational relaxation of CO_2 is so rapid that T_{rot} is taken to be equal to T , within the temperature and pressure range, considered now.²² After the vibrational temperature is known, the population inversion σ can be calculated from Eq. (17) and Eqs. (13)-(16).

The small signal gain coefficient G_c is estimated by the next equation;²³

$$G_c = \frac{\lambda_0^2}{8\pi\tau_r} g(v_0) N_{CO_2} \sigma \dots (18)$$

where τ_r is the radiative life time of the spontaneous emission, and $g(v)$ is the line shape factor.

4. Experiments

4.1 Apparatus

A pressure driven shock tube used for the experiments is shown in Fig.2. The

Fig.2 Apparatus

Fig.3 Measuring ports and dimension of nozzle section whose width is 120mm.

Port No.	2	3	4	5	6
$A/A^*(ht=3mm)$	1.67	4.45	7.23	10.0	13.0
$A/A^*(ht=1mm)$	3.00	11.44	19.89	28.33	36.78

Table.1. Cross sectional area ratios of the nozzle section.

cross section of the shock tube was $50 \times 120 \text{ mm}^2$ and the length of the high pressure chamber was 1m. The nozzle section was placed 4.085m downstream from the diaphragm. Its size is shown in Fig.3. The expansion angle of the two dimensional wedge nozzle placed within the measuring section of the shock tube was 12° , and its throat height (and cross sectional area ratios) could be changed by replacing the plates between the nozzle blocks and the shock tube walls. There were six measuring ports on the side walls of the nozzle, and the geometrical cross sectional area ratios (A/A^*) at each port are shown in Table 1.

At the beginning of the experiments, the test section was evacuated down to 10^{-2} torr. by two rotary pumps which were placed upstream and downstream of the nozzle. Next, the test gas (10% CO_2 , 40% N_2 , 50% He) was introduced. H_2 gas was used as the driver gas and a mylar diaphragm was ruptured by a needle driven by an electromagnet.

4.2 Pressure measurements

The flow parameters along a nozzle can be solved algebraically by using Eqs. (7), (8), the first and the second laws of thermodynamics, under an isentropic flow condition, if there is no nonequilibrium, or the flow is frozen. Assuming that the flow is isentropic and perfect, the relation between the nozzle effective area ratio (A/A^*) and the static pressure P is written as follows with effective γ value,²⁴

$$\frac{A^*}{A} = \left\{ 1 - \left(\frac{P}{P_0} \right)^{(1-\gamma)/\gamma} \right\}^{1/2} \times \left(\frac{P}{P_0} \right)^{1/\gamma} / \left\{ \left(\frac{1-\gamma}{2} \right)^{1/2} \left(\frac{2}{\gamma+1} \right)^{\gamma+1/(2(1-\gamma))} \right\} \dots (19)$$

Therefore, the effective area of a nozzle can be calculated by this equation when the static pressure is given.

We measured the static pressure to estimate the effective area of the nozzle at the measuring ports in Fig.3. Figure 4 shows the typical signal of a pressure gauge (Kistler Model 201-B).

4.3 Measurements of small signal gain

If the population inversion exists in a gas, the experimental gain coefficient G_m can be obtained by measuring the amplified intensity of the light corresponding to the

Fig.4 Typical signal of pressure measurements (sweep time 1msec/div)

(A) denotes a signal from piezo-electric pressure-pickups. (B) shows a signal from the Kistler pressure gauge at the No.2 port.

Fig. 5 Schematic diagram of gain measurements

energy difference, after the passage of the gas. Denoting the input beam intensity, the amplified beam intensity and the effective optical length respectively by I , ΔI , and L , the observed gain coefficient G_m can be expressed as follows,

$$(I + \Delta I)/I = \exp(GL) \quad (20)$$

where the input beam intensity I must be weak enough not to disturb the population inversion under the influence of its radiative field. We used a CW-CO₂ laser of a glow discharge type to measure G_m . The power level of the probe laser was below 0.1W at the inlet of the optical window (CODAK ITRAN 2). Figure 5 shows a block diagram of the gain measurements. The laser beam was chopped and transmitted through the nozzle and detected by a Hg-Cd-Te photo voltaic detector (Societe Anonyme de Telecommunications). The increment of the laser light ΔI was measured by a synchroscope.

5. Results and discussion

5.1 Results of pressure measurements

Figure 6 shows the results of the pressure measurements at the ports No. 2, 3, 4, 5, and 6. A solid line denotes the theoretical value of P/P_0 obtained by Eq. (19) and the geometrical area ratios (A/A^*). The effective area ratios can be obtained as a dotted line in Fig. 6, in semi-empirical way. Table 2 shows the effective area ratios.

Strictly speaking, the real gas flow containing vibrational nonequilibrium is irreversible, so that its entropy increases.

Fig. 6 Results of pressure measurements with their theoretical values

Port No.	2	3	4	5	6
A/A^* ($\gamma=1.457$)	1.33	3.33	4.67	6.00	7.50
A/A^* ($\gamma=1.370$)	1.65	3.75	5.25	6.80	8.60
A/A^* ($\gamma=1.457$)	2.80	6.70	7.80	8.60	8.40
A/A^* ($\gamma=1.370$)	3.30	7.60	9.00	9.80	9.60

$$(\gamma=1.457; \phi_c=0.1, \phi_{n_2}=0.4, \phi_{he}=0.5, T=400^\circ\text{K})^8$$

$$(\gamma=1.370; \phi_c=0.1, \phi_n=0.4, \phi_{he}=0.5, T=2000^\circ\text{K})$$

Table 2. Effective cross sectional area ratios obtained by pressure measurements.

Assuming that all degrees of freedom except the vibrational modes are in equilibrium, the entropy increase can be considered separately in two parts; the entropy increase by the active modes (translation and rotation) dS_a and that by the vibrational mode dS_{vib} :

$$dS = dS_a + dS_{vib} = c_{pa}(dT/T)$$

$$-R(dP/P) + de_{vib}/T_{vib} \quad (21)$$

If a perfect and adiabatic main flow is assumed (as in our case), it is only the vibrational energy flow that contributes to the entropy increase. Then the energy equation can be written as follows,

$$c_{p,a}T_0 + q = c_{p,a}T + u^2/2 \quad (22)$$

where q is equal to the reduction of the energy stored in the vibrational modes $q = e_{vib}(T_0) - e_{vib}(T_{vib})$. This means that a vibrationally nonequilibrium nozzle flow of real gas can be treated as a perfect gas flow with heat addition q . Two limiting cases are conceivable for this flow; i.e., one is a perfect equilibrium flow of $e_{vib}(T_{vib}) = e_{vib}(T)$ and the other a perfect frozen flow of $e_{vib}(T_{vib}) = e_{vib}(T_0)$ ($q=0$, $\gamma=\text{const}$). Though these flows become isentropic flows, the flow variables in the case of the frozen flow can be calculated easily by using the constant specific heats ratio. As real relaxing gases with finite rate constants behave in a manner between

Fig. 7 Power spectrum of the probe laser

Fig. 8 Typical signal of gain measurements (sweep time 0.5 msec/div)

these two limiting cases, the nonequilibrium flow may be simulated by a frozen flow with an effective γ value. Of course, such a proper value of γ is not known apriori. Therefore, it is necessary to consider the reservoir value of $\gamma(1.370)$ and the downstream equilibrium value of $\gamma(1.457)$, when one attempts to calculate the flow variables in this way.

5.2 Results of the gain measurements

As the gain coefficients have a distribution among the vibrational-rotational transitions, it is necessary to check the oscillation spectrum of the probe laser in advance. Figure 7 shows the output spectrum of the probe laser. In this figure, the abscissa denotes the wavelength and the ordinate the relative intensity of the spectrum. This CRT trace was obtained as follows. The laser beam via the grating monochromator (Japan Spectroscopic CO., LTD. CT-50) was collected on a photo voltaic Hg-Cd-Te infrared detector. From this figure, it is clear that the oscillating line of the probe laser was P(20) only. Then numerical calculations were mainly performed for the P(20) transition.

Next, a typical CRT trace to measure a small signal gain coefficient is shown in Fig. 8. In this figure, (A) is the pressure history at the No.1 port, (B) and (C) are the signals of the infrared detector, where

Fig.9 Results of gain measurements for $h=1\text{mm}$, with their theoretical values.

Fig.10 Vibrational temperature distribution at $P_0=2\text{atm}$.

(C) is enlarged to five times (B), and was used to estimate ΔI .

Figure 9, Figure 11, and Figure 12 show the results of the gain measurements also with their theoretical values. In these figures, solid lines represent the theoretical values obtained by using the geometrical cross-sectional area ratios. Dotted lines ($\gamma=1.457$) and dot-dashed lines ($\gamma=1.370$) are the theoretical values obtained by using the effective area ratios shown in Table 2. Figure 9 shows the result of G_c and G_m for $h_t=1\text{mm}$, at $X=95\text{mm}$. Reservoir pressure P_0 was measured at the No.1 port and reservoir temperature T_0 was held constant (1300°K) by controlling the initial conditions of the shock tube.

The vibrational temperature distribution for the peculiar point ($P_0=2\text{atm}$) in Fig.9 was calculated and shown in Fig.10. Figure 10 reveals that the decrease of the effective area ratio by the boundary layers, increases the translational temperature T and promotes the relaxation of the upper and lower levels, at the same time. Since the effect which promotes the relaxation, promotes the deactivation of the lower level more than the deactivation of the upper one in this condition, the boundary layer effect increases the population inversion compared with the case of no boundary layers.

Figure 11 denotes the result for $h_t=3\text{mm}$, at $X=95\text{mm}$. In this figure, the effect on the gain coefficient is not so clear. But it is said that the increase of P_0 rather decreases the gain coefficient, with an increase of the population density which promotes the relaxation. Figure 12 shows the experimental and

Fig.11 Results of gain measurements for $h=3\text{mm}$, with their theoretical values.

Fig.12 Gain coefficients along the nozzle axis.

theoretical gain coefficients along the nozzle, for $P_0(4\text{atm})$, $T_0(1300^\circ\text{K})$.

On the other hand, Fig.13 shows the result of the similar experiments obtained by change the oscillation wavelength of the probe laser. With different discharge conditions and the flow rate of the laser medium, the oscillation lines of the probe laser could be selected. In this figure, J-dependency of the gain coefficients is not seen.

5.3 Saturation parameter

In general, the gain coefficient in the case of a homogeneous laser medium is expressed as follows,^{2,3}

$$G_m = G_c(1 + I/I_s)^{-1} \quad (23)$$

where G_c is the non-saturating gain coefficient which is equal to the small signal gain coefficient, and G_m is the measured gain coefficient. The symbol I is the radiative energy density in the medium, and I_s is the saturation parameter which is generally larger than $100\text{W}/\text{cm}^2$,^{2,3,25} in the case of high speed flow lasers. The effect of the saturation is less than 3%, when the radius of the probe laser beam is 1mm and its power level is 0.1W (our case).

Fig.13 J-dependence of the gain coefficients.

Fig.14 Example of the gain distributions ($h=3\text{mm}$, $P_0=4.58\text{atm}$, $T_0=1500^\circ\text{K}$), 4 ($X=55\text{mm}$), 5 ($X=75\text{mm}$), 6 ($X=95\text{mm}$) show the port No.

Fig.15 Example of gain distribution along the nozzle axis, ($P_0=4.58\text{atm}$, $T_0=1500^\circ\text{K}$).

5.4 Distribution of gain coefficients

As shown in Eq.(13), CO_2 molecules are distributed among the rotational levels. So the gain coefficients must be calculated with respect to different vibration-rotation transition separately. According to the selection rules of quantum mechanics, only the transition whose rotational quantum number of the lower level is even, is possible, and the difference of the rotational quantum number (ΔJ) concerning the transitions must be ± 1 . It is customary to say that the transitions whose J increase by 1 are the P-branch transitions, and ones whose J decrease by 1, the R-branch transitions. The P-branch transitions can be written as a function of the lower level J as follows,^{2,6}

$$G_{J-1,J} = \frac{h\nu_0}{4\pi} B_{J-1,J} (2J-1) 2D \times [N_J B_{J-1,J} \exp(-F_u(J)D) - N_{J-1} B_{J-1,J} \exp(-F_l(J)D)] \quad (24)$$

where $D=hc/kT$ and $B_{J-1,J}$ is the Einstein B coefficient. Figure 14 and Figure 15 show the gain coefficient calculated by Eq.(24). In these figures, the calculation was performed for the condition of $P_0=4.58\text{atm}$, $T_0=1500^\circ\text{K}$. Figure 14 shows that the gain coefficient under these conditions has a peak at P(20) but the absolute value scarcely varies with J near the peak. Figure 15 shows the gain coefficients along X for the P(10), P(20), P(30) and P(40) transitions.

5.5 Absorption coefficient

One must pay attention to the absorption of the probe laser beam²⁷ by the initial gas in the case of small signal gain measurements. As shown in the next equation, the measured gain coefficients are larger apparently than the proper value due to the absorption by the mixed gas existing in the nozzle section before the nozzle operates.

$$G_m' = G_m + a \quad (25)$$

is the absorption coefficient which can be calculated by an equation similar to Eq. (24), and this value can be obtained simply by assuming thermal equilibrium ($T=T_{\text{rot}}=T_{\text{vib}}$) in the mixture.

Fig.16 Calculated absorption coefficient.

Fig.17 Temperature dependence of the absorption coefficient.

Figure 16 shows the calculated α value corresponding to the P-branch transition, at $T=290^\circ\text{K}$, $P=60\text{torr}$. Figure 17 shows the α value corresponding to the P(20) transition, where the ordinate is the gas temperature T . Absorption coefficients show a strong temperature dependence, but it may be concluded that its effect on the gain coefficient is small enough within the room temperature range ($\alpha/G < 1.6\%$).

6. Conclusions

A small signal gain coefficient is an important fundamental parameter for the discussion of the GDL operations, and it directly corresponds to the population inversion. Therefore, we never fail to estimate its correct value, in the theoretical treatment of the GDLs.

We compared the experimental values of the small signal gain coefficients with the numerical estimations, in the $\text{CO}_2\text{-N}_2\text{-He}$ supersonic flow of vibrational nonequilibrium. A detailed comparison has been done for the relatively low reservoir pressure region to compare with the report in other papers, and our conclusions are as follows.

Gain coefficients were measured for the reservoir condition at $P_0=2\text{-}6\text{atm}$ and $T_0=1300^\circ\text{K}$, with various parameter values of $h\nu$, A , A^* , and X , and comparisons of their values with numerical ones are fairly good.

A semi-empirical method was proposed and used for the analysis to eliminate the displacement effect of the boundary layer in the nozzle.

In the case of the comparison between the gain coefficients, the effects of the rotational level distribution, saturation parameter, and absorption coefficient were discussed in detail.

Ultimately, we got fairly good agreement between the measured values and the theoretical ones after eliminating the boundary layer effect and others. It may be concluded that the vibrational relaxation rate constants presented by Anderson et al., are available in our experimental range.

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