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Analysis of Diametral Growth and Working Forces in Tube Spinning*

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The best method to estimate the accuracy of the diameter of the parts and the working forces in tube spinning process is discussed. At first, the velocity of the metal and the strain rate in the deformation zone are estimated by introducing the parameters ξ and z_0 obtained from the observation of the previous experiments. The optimum values of ξ which has an effect on the accuracy of the diameter of the parts are obtained by minimizing the deformation energy of the tube consumed on the contact area between the roller and the mandrel. Therefore, the proposed method is useful for selecting the working conditions to obtain a good quality of the parts. Furthermore, the working forces in tube spinning can be obtained with good agreement between the experimental and calculated results by this method by knowing the deformation resistance of specimen and the other working conditions.

1. Introduction

The aspects of tube spinning will be clear from the survey of the previous experimental work⁽¹⁾, but the method has to be explored in terms of mechanics in order to receive wide applications in the future. From the observation of the actual operation in the manufacturing of combustion chamber of rocket and centrifugal separator of uranium, it is recognized that the theoretical estimation of the accuracy of the diameter of the parts and the working forces in the tube spinning is important for the selection of the working conditions such as the shape and the feed of the roller as well as for designing of the spinning machine. Sometimes, an optimum working condition is selected after many trials and how to determine the rigidity of machine becomes a problem, since the method being useful to estimate the working forces is insufficient. Although the working forces in the tube spinning were estimated by many researchers^{(2)~(10)}, owing to the three dimensional deformation, they could not solve the problem in detail.

Then, in the following estimation of the velocity of the metal flow and the strain rate in the deformation zone by introducing the parameters obtained from the observation of the previous experiments, the best method is proposed to estimate the accuracy of the diameter of the parts and the working forces in the tube spinning. As we mentioned in the previous report, there are two processes, forward and back-

ward tube spinning. Then, we tried to unify the theoretical expressions about them, considering the characteristic of metal flow in each case. At first, backward tube spinning is discussed to obtain facilities for expressions.

2. Theory of backward tube spinning

2.1 Description of metal flow

(a) The form of the metal flow under the roller

Figure 1(a) shows the aspect of contact between the blank and the roller in

Fig.1 Metal flow in backward tube spinning

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the backward tube spinning. The choice of the co-ordinate axes x, y, z with the origin at 0 on which x axis coincides with the anti-rotational direction of the blank and z axis coincides with the axis of the tube is indicated in the figure. The material coming into the clearance between the roller and the mandrel with the velocity V_0 in the rotational direction of the blank is forced to flow in the direction of the z axis, while the other part of the metal seems to flow in the rotational direction of the blank and its velocity becomes V_z , given by the equation

$$V_z = -(c' + c_1')V_0 / (c' + \xi c_1') \quad (1)$$

where

$$c' = A' + B', \quad c_1' = A_0' + B_0' + B_1'$$

ξ is a parameter to decide the volume of the metal flow in the direction of the z axis. A', B', A_0', B_0' and B_1' are shown in Figure 1(a) and denoted in Appendix in detail.

In the backward tube spinning, the material is thinned and displaced in a direction opposite to the motion of the roller, while some of the material must flow in the same direction as the feed of the roller, in order to maintain the steady deformation keeping the volume A_3' of the wave ahead of the roller. Then, it is reasonable to assume that the material coming under the roller is divided into two parts at an arbitrary distance z_0 from the origin, which is a second parameter to decide the flow of the metal. Therefore, in order to maintain the steady flow of the metal in the direction of the z axis, the following equation must be satisfied.

$$\xi A_0' = A_3' \quad (2)$$

On the other hand, we denoted the size of a wave by

$$\zeta = (t_0' - t_0) / t_0 \quad (3)$$

and pointed out the empirical formulas

$$1/\zeta = a - bR \quad (4)$$

in the previous report⁽¹⁾, so that the height t_0' of the wave which is necessary for the numerical calculation is decided by Eq. (4). In addition, the angle γ of inclination of the wave shown in Figure 1(b) is assumed to be equal to the angle α of the roller, since the calculated results for various values of γ are not affected very much. By the way, R in Eq. (4) denotes the degree of thinning as expressed by

$$R = \ln t_0' / t \quad (5)$$

but in general, it is determined by the dependence

$$R_0 = (t_0 - t) / t_0 \quad (6)$$

(b) Deformation during a half rotation of the blank

When the number of rollers used in the process is denoted by n , the apparent feed of the roller becomes v_n

$$v_n = v / n_0 \quad (7)$$

However, we found experimentally the axial growth of the tube during a half rotation of the blank, so that we took account of the transformation of the form E'F' obtained by the roller before a half rotation of the blank to the form E'F' as shown in Figure 1(b). Since the metal flow in the opposite direction of the feed of the roller is denoted by $\xi B_0'$, we can assume that F'F' becomes $\xi B_0' / t$. Then we obtain

$$\tan \alpha' = \frac{t + (v_n + z_0') \tan \alpha - t_1}{z_0'} = \frac{t_1 - t}{v_n + \xi B_0' / t} \quad (8)$$

We said that the apparent neutral point z_0' of the metal flow in the direction of the z axis during a half rotation of the blank should be between z_0 and $z_0 - v_n$ ⁽¹⁾, but if we assume that $z_0' = z_0$, B_0' is given by Eq. (9) and the neutral point z_0 is obtained from Eqs. (8) and (9).

$$B_0' = (t_1 - t + v_n \tan \alpha) z_0 / 2 \quad (9)$$

$$z_0 = \frac{\sqrt{(\xi c d + 2 t d)^2 - 8 \xi c d \tan \alpha t v_n} - d(\xi c + 2 t)}{2 c \xi \tan \alpha} \quad (10)$$

where

$$c = t_1 - t + v_n \tan \alpha, \quad d = t - t_1 + v_n \tan \alpha$$

Then, the actual feed v_n' of the roller considering the axial growth of the tube becomes

$$v_n' = (t_0' - t) \cot \alpha - z_1 \quad (11)$$

where

$$z_1 = (t_0' - t_1) \cot \alpha' \quad (12)$$

Since these equations contain the parameter ξ , in order to determine the values of z_0 , α' , v_n' , we have to obtain the optimum values of ξ by minimizing the total energy consumed on the deformation zone as developed in the following.

2.2 Velocity of metal flow and strain rate

From the same consideration as Eq. (1), the velocity V_z in the rotational direction of the blank at an arbitrary distance x from the origin in Figure 2(b) is expressed by Eq. (13).

$$V_z = -(c + c_1)V_0 / (c + \xi c_1) \quad (13)$$

In the following, symbols without dash such as c and c_1 denote the values at a distance x from the origin. Then, the strain rate in the direction of the x axis is obtained from $\partial v_x / \partial x$.

$$\dot{\epsilon}_x = \frac{(1 - \xi)(c_1 c_x - c c_{1x})}{(c + \xi c_1)^2} \frac{dt_y}{dx} V_0 \quad (14)$$

When $\xi = 1$, the metal compressed under the roller flows completely in the direction of the z axis and therefore, $V_z = -V_0$, $\dot{\epsilon}_x = 0$. Suffix x of c_x and c_{1x} in Eq. (14) means

the differentiation with respect to x . On the other hand, the velocity V_x in the direction of the x axis at an arbitrary distance z from the origin is obtained from the constancy of the volume of metal flow. Namely, when $z > 0$

$$V_x = \frac{\xi(z-z_0)}{t_{zf}} \frac{c+c_1}{c+\xi c_1} \frac{dt_y}{dx} V_0 \dots\dots\dots(15)$$

and when $z < 0$

$$V_x = \frac{-\xi(z_0+|z|)}{t_{zb}} \frac{c+c_1}{c+\xi c_1} \frac{dt_y}{dx} V_0 \dots\dots\dots(16)$$

Then, the strain rate in the direction of the z axis becomes as follows. When $z > 0$

$$\dot{\epsilon}_z = \frac{\xi(t_y+z_0 \tan \alpha)}{t_{zf}^2} \frac{c+c_1}{c+\xi c_1} \frac{dt_y}{dx} V_0 \dots\dots\dots(17)$$

When $z < 0$

$$\dot{\epsilon}_z = \frac{\xi(t_y-z_0 \tan \beta)}{t_{zb}^2} \frac{c+c_1}{c+\xi c_1} \frac{dt_y}{dx} V_0 \dots\dots\dots(18)$$

Furthermore, the strain rate in the direction of the y axis is given by

$$\dot{\epsilon}_y = -(\dot{\epsilon}_z + \dot{\epsilon}_x) \dots\dots\dots(19)$$

and then, from the boundary condition that $V_y = 0$ at $y = 0$, we obtain

$$V_y = \dot{\epsilon}_y y \dots\dots\dots(20)$$

The symbols used in these equations such as c , c_1 , c_2 , c_{12} , t_{zf} and t_{zb} are described in Appendix in detail. In addition, suffixes f and b mean the values in the ranges of $z > 0$ and $z < 0$ respectively.

When we denote the maximum diameter of the roller by D_R , t_y and dt_y/dx are given by the following equations.

$$t_y = D_R/2 + t - \sqrt{D_R^2/4 - x^2} \dots\dots\dots(21)$$

Fig.2 Mechanism of deformation of tube spinning

$$dt_y/dx = 2x/\sqrt{D_R^2 - 4x^2} \dots\dots\dots(22)$$

2.3 Estimation of the total energy consumed on the deformation zone and its minimum value

The most remarkable feature of tube spinning is that the length of the tube is increased as a result of the local deformation by the indentation of the roller which is influenced by the surroundings. Then, we have to consider the energy constrained by the surroundings in addition to the deformation energy consumed under the roller. Each energy indicated on the xz plane as shown in Figure 2(b) is expressed as follows. The new symbols, the advanced expressions of each equation and the strength of the tube are described in Appendix.

(a) Plastic deformation energy U_i under the roller

The equivalent plastic strain is obtained as

$$\dot{\epsilon} = \sqrt{2[(\dot{\epsilon}_x - \dot{\epsilon}_y)^2 + (\dot{\epsilon}_y - \dot{\epsilon}_z)^2 + (\dot{\epsilon}_z - \dot{\epsilon}_x)^2]} / 3 \dots\dots\dots(23)$$

so that, considering the mean deformation resistance σ_m which is given in Eq.(54), the plastic deformation energy U_i is given by when $z > 0$

$$U_{if} = \sigma_m \int_0^{z_0} \int_0^{t_y} \dot{\epsilon} t_{zf} dz dx \dots\dots\dots(24)$$

when $z < 0$

$$U_{ib} = \sigma_m \int_0^{z_0} \int_0^{t_y} \dot{\epsilon} t_{zb} dz dx \dots\dots\dots(25)$$

(b) Energy due to the velocity discontinuities U_a along the entrance HE'

$$U_a = \frac{\sigma_m}{\sqrt{3}} \int_0^{z_0} \int_0^{t_y} \sqrt{V_{yz}^2 + V_{za}^2} dy dz \dots\dots\dots(26)$$

V_{yz} , V_{za} and t_y' mean the values of V_y , V_z and t_y at $x = x_a$.

(c) Energy due to the velocity discontinuities U_g and energy due to the indentation U_{inf} on the E'E plane

$$U_g = \frac{\sigma_Y}{\sqrt{3}} \int_0^{z_0} \int_0^{t_y} \sqrt{(V_{zg} - V_0)^2 + V_{yg}^2} dy dz \dots\dots\dots(27)$$

$$U_{inf} = \sigma_Y \int_0^{z_0} V_{zg} t_0' dx \dots\dots\dots(28)$$

Suffix g means the values at $z = z_g$. On this plane, material is not work hardened, so that the yield stress is used as the deformation resistance.

(d) Energy due to the velocity discontinuities U_n and energy due to the indentation U_{inb} on the HL plane.

$$U_n = \frac{\sigma_b}{\sqrt{3}} \int_0^{z_b} \int_0^{t_n} \sqrt{(V_{zn} - V_0)^2 + V_{yn}^2} dy dx \quad (29)$$

$$U_{inb} = \sigma_b \int_0^{z_b} |V_{zn}| t_n dx \quad (30)$$

Suffix n means the values at $z = z_n$. On this plane, the material is work hardened, so that σ_b given by Eq.(53) is used as the deformation resistance.

- (e) Frictional energy consumed on the contact surface between the roller and the blank, and on the contact surface between the mandrel and the blank.

$$U_{ff} = \int_0^{z_0} \int_0^{t_0} \frac{\sigma_m}{\sqrt{3}} (m' + m \sec^2 \alpha) |V_z| dz dx \quad (31)$$

$$U_{fb} = \int_0^{z_b} \int_0^{t_n} \frac{\sigma_m}{\sqrt{3}} (m' + m \sec^2 \beta) |V_z| dz dx \quad (32)$$

m and m' mean the coefficients of friction on the surface between the roller and the blank and on the surface between the mandrel and the blank respectively. In the numerical calculation, we assume that $m = m' = 1.0$. The frictional energy in the direction of the x axis is neglected because of the rolling contact between the roller and the blank.

- (f) Constrained energy along the exit EL

The metal flows out the roller at $x = 0$ with the velocity V_{x0} in the direction of the x axis, while the material in the range of $z > 0$ in the backward tube spinning is constrained by the mandrel collar. Then, we consider the constrained energy U_{rf} in the backward tube spinning and conversely, in the forward tube spinning, we consider U_{rb} in the range of $z < 0$.

$$U_{rf} = \sigma_m (V_{x0} - V_0) (t_0'^2 - t^2) \cot \alpha / 2 \quad (33)$$

$$U_{rb} = \sigma_m (V_{x0} - V_0) (t^2 - t_0'^2) \cot \beta / 2 \quad (34)$$

- (g) Total energy U_t consumed on the deformation zone

U_f and U_b mean the energy consumed in the ranges of $z > 0$ and $z < 0$ respectively, so that the total energy U_t consumed on the total deformation zone will be

$$U_t = U_f + U_b \\ U_t = (U_{if} + U_a + U_g + U_{inf} + U_{ff} + U_{rf}) + (U_{ib} + U_n + U_{inb} + U_{fb}) \quad (35)$$

If we assume the value of ξ , the total energy U_t can be calculated only by knowing the working conditions and the dimensions of the tools. Then, it is easy to obtain the optimum value of ξ which gives the minimum value of U_t .

2.4 Estimation of working forces

The equations to estimate the three components of the working forces - radial force P_r in the direction of the y axis, feed force P_f in the direction of the z axis and tangential force P_θ in the direction of the x axis - are given as under. When we integrate the velocity of the displacement in the direction of the y axis over the contact area, we have in the range of $z > 0$

$$Y_f = \int_0^{z_0} \int_0^{t_0} V_y dz dx \quad (36)$$

and in the range of $z < 0$

$$Y_b = \int_0^{z_b} \int_0^{t_n} V_y dz dx \quad (37)$$

Furthermore, the contact area projected on to the xz plane is obtained by the equations described in Appendix. Then, the radial forces P_{rf} in the range of $z > 0$ and P_{rb} in the range of $z < 0$ can be written as follows.

$$P_{rf} = (U_f / Y_f) S_f \quad (38)$$

$$P_{rb} = (U_b / Y_b) S_b \quad (39)$$

Therefore, the total radial force component P_r over the total contact area is given by

$$P_r = P_{rf} + P_{rb} \quad (40)$$

On the other hand, we determine the feed force and the tangential force by considering the radial force P_{rf} in the range $z > 0$ only. Especially, we regard the feed force component acting on the metal piled up on the outer surface of the tube as zero. Then, we can propose the following equations.

$$P_f = P_{rf} \tan \alpha (t_0 - t) / (t_0' - t) \quad (41)$$

$$P_\theta = P_{rf} \sqrt{v_n'} \tan \alpha / D_R \quad (42)$$

Fig.3 Metal flow in forward tube spinning

3. Theory of forward tube spinning

In the forward tube spinning shown in Figure 3, it seems that some volume of material flows in the direction opposite to the feed of the roller, but we assume that the metal coming under the roller is divided at $z_0 = 0$ and the neutral point z_0' during a half rotation of the blank becomes $-v_n$. These assumptions are made by considering the fact that when $z_0 = z_0'$, a minimum value of the total energy is obtained after the calculation of the total energy for various values of z_0 . In these cases, it is necessary that the height of the wave be constant to keep the steady deformation during the process. Assuming a free axial growth of the tube end, the following equation is given as the conditions of the metal flow in the forward tube spinning

$$\xi A_1' = A_2' \quad (43)$$

From Figure 3(b) expressing the assumption on the deformation during a half rotation of the blank,

$$\tan \alpha' = (t_1 - t) / v_n \quad (44)$$

This equation can also be obtained by substituting $B_0' = 0$ into Eq.(8). Furthermore, we can assume Eq. (45) as the other condition of the metal flow

$$\xi A_1' = t_0'(v_n - v_n') \quad (45)$$

Then, the values of γ can be found from Eqs. (43), (44) and (45)

$$\tan \gamma = \frac{t_0' - t - z_1 \tan \alpha}{((t_0' \cot \alpha)^2 - 2t_0' \cot \alpha (v_n - v_n'))^{1/2} - t \cot \alpha - z_1 - \tan \alpha} \quad (46)$$

Of course, the actual feed of the roller is given by the same equation as Eq.(11).

Therefore, we can express the characteristic of the metal flow in the forward tube spinning in a different form from the backward tube spinning and recognize that the expressions about the factors of mechanics in the forward tube spinning are obtained by substituting $z_0 = 0$ into the equations given in chapters 2.2, 2.3 and 2.4. As we said before, we took account of the energy U_{rb} in the range of $z < 0$ in the equation which expresses the total energy, because the deformed part of the tube is

Table 1 Working conditions

Diameter of mandrel	d mm	36
Diameter of roller	D_R mm	74
Angle of roller	$\alpha = \beta$ deg.	15, 20, 30
Number of roller	n_0	2
Number of rotation of mandrel	N rpm	210
Feed of roller	v mm/rev.	0.124, 0.249, 0.378
Thickness of tube wall	t_0 mm	2
Yield point of tube	σ_Y kg/mm	23

Stress and strain curve of tube $\sigma = 82 \times 10^{-3} \text{ kg/mm}^2$

constrained by the tailstock. Then, the total energy becomes as follows.

$$U_t = (U_{if} + U_a + U_g + U_{in} + U_{ff}) + (U_{ib} + U_n + U_{inb} + U_{fb} + U_{rb}) \quad (47)$$

Thus, we can unify the theoretical expressions, considering the characteristic of metal flow in the backward and forward tube spinning.

4. Comparison of calculated and experimental results

4.1 Discussion about the calculated results of ξ and z_0

In order to compare with the calculated results, the experimental results described in the previous report are used. The experimental conditions and the strength of the steel tube are given in Table 1.

At first, if we decide the value of t_1 such as to satisfy the conditions of metal flow of Eqs. (2) or (43), the form of the metal flow in the forward and backward tube spinning is obtained easily. Then, an optimum value of ξ is determined by minimizing the total energy calculated from the equations described in chapter 2.3 and Appendix.

The optimum values of ξ for the various working conditions are shown in Figure 4 and Figure 5 about the degree of thinning R_0 , the feed of the roller v and the angle of the roller α . The decreasing of ξ is associated with the increasing of R_0 as well as with the decreasing of v and α . When the value of ξ is small, the metal flow in the direction of the x axis increases and then, the fit between the finished part and the mandrel will be loose.

Now, we denote the number of turnings of the blank until a part of the tube passes through the roller, by N_0 .

$$N_0 = (t_0' - t) \cot \alpha / v_n' \quad (48)$$

Fig. 4 Relationship between calculated coefficient ξ and R_0 (forward)

If we calculate the mean value ε_{xm} of the strain rate in the direction of the x axis over the contact length between the roller and the blank at the same time as calculation of ξ , we can compare the diametral growth Δd of the part obtained in the previous experiments with $\varepsilon_{xm}N_0$ as shown in Figure 6. The calculated results of $\varepsilon_{xm}N_0$

seem to be closely related to the diametral growth Δd , but we can not explain the existence of the minimum values of Δd observed in the experiments.

Fig. 5 Relationship between ξ and α ($R_0 = 0.2$)

Fig. 6 Comparison of diametral growth Δd and $\varepsilon_{xm}N_0$

Fig. 7 Practical expression between working conditions and $\varepsilon_{xm}N_0$ ($\alpha = 20^\circ$, forward)

Fig. 8 Nondimensional values of neutral point z_0 ($\alpha = 20^\circ$, backward)

Fig. 9 Comparison of calculated values of τ and shape of wave (forward)

Fig. 10 Relationship between v_n'/v_n and R_0 (forward)

In order to realize the above results more practically, we denote the form of contact between the roller and the blank by z_m/x_m . x_m denotes the contact length in the direction of the x axis and z_m the contact length in the direction of the z axis. They are given by the following equations.

$$x_m = \sqrt{D R v_n' \tan \alpha} \quad (49)$$

$$z_m = (t_0' - t) \cot \alpha - v_n' / 2 \quad (50)$$

As shown in Figure 7, we can find that the values of $\varepsilon_{sm} N_0$ decrease with a decreasing of z_m/x_m . Then, it is possible to obtain a tight fit between the part and the mandrel by using small values of z_m/x_m .

The neutral point z_0 obtained from Eq. (10) in the backward tube spinning is shown in Figure 8 as a nondimensional value divided by z_1 . It is determined by the equilibrium of the metal flow in the direction of the x axis and the z axis. If we consider that the larger the values of z_0/z_1 are, the more effectively the parts

are made, the feed of the roller should not be selected as small, for example, as $v = 0.249$ mm/rev.

In the forward tube spinning, the angle γ of the wave can be obtained from Eq. (46), so that we compared the calculated value of γ with the shape of the wave as shown in Figure 9. The calculated values of γ increase with an increasing angle α of the roller, agreeing with the tendency of the experiments.

Figure 10 shows an example of the actual feed of the roller v_n' calculated from Eq. (11). We can find that v_n' depends not only on the degree of thinning, but also on the angle of the roller and its feed. In the forward tube spinning, it has been suggested, v_n' is given by Eq. (51):

$$v_n' = v_n t / t_0 \quad (51)$$

However, it is interesting that the calculated values of v_n' shown in Figure 10 are different from the results of Eq. (51).

Fig. 11 Relationship between P_r , P_z and R_0 ($\alpha = 20^\circ$, forward)

Fig. 12 Relationship between P_r , P_z and R_0 ($\alpha = 20^\circ$, backward)

Fig. 13 Relationship between P_θ and R_0 ($\alpha = 20^\circ$)

Fig. 14 Relationship between P_r , P_z , and α ($R_0 = 0.2$, forward)

4.2 Discussion of the working forces

Figure 11 and Figure 12 are the comparison of the calculated and experimental results about P_r and P_s in the forward and backward tube spinning respectively. The agreement between them is fairly good. Furthermore, the calculated results of P_θ also agree very well with the experimental results as shown in Figure 13. Then, the new suggestion of Eqs. (41) and (42) for the calculation of P_r and P_θ seems to be reasonable. From the agreement of the above figures and the comparison for $\alpha = 15^\circ$ shown in the previous report, it is concluded that the theory of the tube spinning developed in this paper is useful and reliable.

In order to investigate the effect of the angle α of the roller on the working forces, the variation of the radial forces and the feed forces with the angle α of the roller is shown in Figure 14. We can recognize the minimum values of P_r in the figure and prove that an angle about 20° or 25° is the optimum angle of the roller from the experimental results described in the previous report.

5. Conclusions

(1) We developed a unified theory for the forward and backward tube spinning, considering the characteristic of metal flow in both processes and then, suggested a best method to estimate the accuracy of the diameter of the parts and the working forces.

(2) The agreement between the calculated and experimental results is very good over a wide range of working conditions such as the angle of the roller, the feed of the roller and the degree of thinning. The experimental facts described in the previous report are proved by the theoretical procedure.

(3) It is confirmed that the accuracy of the diameter of the parts is assured by the optimum value of ξ and is determined by the contact form between the roller and the blank z_m/x_m . Thus, the proposed method is found useful for selecting the working conditions to obtain the good quality of the parts.

(4) The forward tube spinning with the working conditions of $\alpha = 20^\circ \sim 25^\circ$, $v = 0.249$ mm/rev. is recommended for use in practice on the basis of the theoretical and experimental results.

The method proposed here will be useful for predicting the working forces and for selecting the working conditions in the industry.

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Appendix

(1) Equations on coordinates

$$\begin{aligned}
 z_1 &= (t_0' - t_1) \cot \alpha' \\
 |z_2| &= (t_1 - t_y) / (\tan \alpha' + \tan \beta) \\
 \Delta z &= (t_0' - t_y - z_1 \tan \alpha) / (\tan \alpha + \tan \gamma) \\
 \Delta z_0 &= (t_0' - t - z_1 \tan \alpha) / (\tan \alpha + \tan \gamma) \\
 z_y &= (t_0' - t_y) \cot \alpha \\
 |z_2'| &= (t_1 - t) / (\tan \alpha' + \tan \beta) \\
 |z_n| &= (\sqrt{t_y^2 + 2\{\xi(B_0 + B_1) + B\} \tan \beta} - t_y) \cot \beta \\
 t_{zf} &= t_y + z \tan \alpha, \quad t_{ts} = t_y + |z| \tan \beta \\
 t_z' &= t_1 + z \tan \alpha', \quad t_1' = t + v_n \tan \alpha \\
 t_n &= t_y + |z_n| \tan \beta \\
 t_i &= t + (t_1 - t) \tan \beta / (\tan \alpha' + \tan \beta) \\
 x_a &= \sqrt{(D_R - 2z_1 \tan \alpha)(t_0' - t - z_1 \tan \alpha)} \\
 x_s &= \sqrt{D_R(t_1 - t)}
 \end{aligned}$$

(2) Equations on metal flow

$$\begin{aligned}
 C &= A + B, \quad C_1 = A_1 + B_1, \quad C' = A' + B' \\
 C_1' &= A_0' + B_0' + B_1', \quad C_z = A_z + B_z \\
 C_{1z} &= A_{1z} + B_{1z} \\
 A &= \{2t_y + (z_1 + \Delta z) \tan \alpha\} (z_1 + \Delta z) / 2 \\
 B &= (|z_2| \tan \beta + 2t_y) |z_2| / 2 \\
 A_1 &= \{\Delta z^2 (\tan \alpha + \tan \gamma) \\
 &\quad + (t_0' + t_1 - 2t_y - z_1 \tan \alpha) z_1\} / 2 \\
 B_1 &= (t_1 - t_y) |z_2| / 2, \quad B_1' = (t_1 - t) |z_2| / 2 \\
 A' &= \{2t + (z_1 + \Delta z_0) \tan \alpha\} (z_1 + \Delta z_0) / 2 \\
 B' &= (|z_2'| \tan \beta + 2t) |z_2'| / 2 \\
 A_1' &= \{\Delta z_0^2 (\tan \alpha + \tan \gamma) \\
 &\quad + (t_0' + t_1 - 2t - z_1 \tan \alpha) z_1\} / 2 \\
 A_2' &= (t_0'^2 - t^2) \cot \alpha / 2 - A' \\
 A_3' &= (t_0' - t_0) v_n' - \Delta z_0 v_n' \tan \gamma / 2 \\
 A_0' &= A_1' - B_0' \\
 B_0' &= \{2t_1 - 2t + z_0 (\tan \alpha' - \tan \alpha)\} z_0 / 2 \\
 A_z &= \{(z_1 + \Delta z) \tan \gamma - t_y\} / (\tan \alpha + \tan \gamma) \\
 B_z &= (|z_2| \tan \alpha' - t_y) / (\tan \alpha' + \tan \beta) \\
 A_{1z} &= -(z_1 + \Delta z), \quad B_{1z} = -|z_2|
 \end{aligned}$$

- (3) Equations on deformation resistance
When the wall is thinned from t_0 to t ,
the total strain is denoted by ϵ_0

$$\epsilon_0 = \ln \frac{t_0(t_0 + d)}{t(t + d)} \quad (52)$$

d is the inside diameter of the blank.
Yield stress of the tube is σ_Y kg/mm² and
the deformation resistance σ_b at $\epsilon = \epsilon_0$ is
given by

$$\sigma_b = F\epsilon_0^n \quad (53)$$

Mean deformation resistance σ_m will be

$$\sigma_m = F\epsilon_0^n / (1 + n) \quad (54)$$

- (4) Equations on deformation energy

$$K = \frac{C + C_1}{C + \xi C_1} \frac{dt_y}{dx} V_0$$

$$L = \frac{(1 - \xi)(C_1 C_2 - C C_{12})}{(C + \xi C_1)^2} \frac{dt_y}{dx} V_0$$

$$(b) \quad U_a = \frac{\sigma_m}{\sqrt{3}} \int_0^{x_1} (e/2) [t_y' \sqrt{t_y'^2 + (f/e)^2} + (f/e)^2 \ln \{ |t_y' + \sqrt{t_y'^2 + (f/e)^2} | / |f/e| \}] dz$$

where

$$e = K_a \xi (t_1' + z_0 \tan \alpha) / (t_1' + z \tan \alpha)^2 + L_a$$

$$f = K_a \xi |z - z_0| / (t_1' + z \tan \alpha)$$

Suffix a of K and L means the value at $x = x_a$

$$(c) \quad U_g = \frac{\sigma_Y}{\sqrt{3}} \int_0^{x_a} (g/2) [t_0' \sqrt{t_0'^2 + (h/g)^2} + (h/g)^2 \ln \{ |t_0' + \sqrt{t_0'^2 + (h/g)^2} | / |h/g| \}] dx$$

where

$$g = K \xi (t_y + z_0 \tan \alpha) / t_0'^2 + L$$

$$h = \{ (C + C_1) / (C + \xi C_1) - 1 \} V_0$$

$$U_{inJ} = \sigma_Y \int_0^{x_a} \xi K (z_y - z_0) dx$$

$$(d) \quad U_n = \frac{\sigma_b}{\sqrt{3}} \int_0^{x_b} (i/2) [t_n \sqrt{t_n^2 + (h/i)^2} + (h/i)^2 \ln \{ |t_n + \sqrt{t_n^2 + (h/i)^2} | / |h/i| \}] dx$$

where

$$i = K \xi (t_y - z_0 \tan \beta) / t_n^2 + L$$

$$U_{inb} = \sigma_b \int_0^{x_b} \xi K (z_0 + |z_n|) dx$$

$$(e) \quad U_{fJ} = \frac{\sigma_m}{\sqrt{3}} (m' + m \sec^2 \alpha) \int_0^{x_a} K \xi \cot^2 \alpha \times [(z_0 - 2z_0) \tan \alpha - (t_y + z_0 \tan \alpha) \times \ln \{ t_y t_0' / (t_y + z_0 \tan \alpha)^2 \}] dx$$

$$U_{fb} = \frac{\sigma_m}{\sqrt{3}} (m' + m \sec^2 \beta) \int_0^{x_b} K \xi \cot^2 \beta \times \{ |z_n| \tan \beta - (t_y - z_0 \tan \beta) \ln (t_n / t_y) \} dx$$

- (5) Velocity of deformation in the direction of the y axis and contact area projected on the xz plane

$$Y_f = \int_0^{x_a} \{ j \cot \alpha \ln (t_0' / t_y) + L z_y (t_y + z_y \tan \alpha / 2) \} dx$$

$$Y_b = \int_0^{x_b} \{ l \cot \beta \ln (t_n / t_y) + L z_n (t_y + |z_n| \tan \beta / 2) \} dx$$

where

$$j = \xi (t_y + z_0 \tan \alpha) K$$

$$l = \xi (t_y - z_0 \tan \beta) K$$

$$S_f = \int_0^{x_a} z_f dx, \quad S_b = \int_0^{x_b} z_n dx$$