Dynamic Behavior of Planetary Gear

(5th report, Dynamic Increment of Torque)*

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The authors carried out an experimental investigation about the torque variation in a single-stage Stoecklcht planetary gear (Type 2K-H) constructed with spur gears with the following results:

(i) The torque variation in the planetary gear is caused by the errors per mesh and the run-out errors of each gear.
(ii) When the number of error cycles per mesh coincides with all kinds of natural frequencies of the planetary gear, the torque variation of higher-speed shaft increases.
(iii) A resonance is caused in the gear testing machine with a planetary gear by the run-out error of the sun gear. Consequently, the torque variation of lower-speed shaft increases.

1. Introduction

It has been known from the previous study that the torque in the planetary gear varies in driving, and its variation rate is comparatively large and changes with a changing of the gear speed. Furthermore, the authors have already reported that even if the load distribution of the planetary gear is equal, it happens that the dynamic tooth load varies largely due to the torque variation. We can generally consider the exciting sources in the planetary gear based on the random errors of manufacture and assembly in addition to the torque variation of the driving motor and the variation of loading. Therefore, there have been many studies on the influence of the pitch error, the tooth profile error and the run-out error of the gear on the vibration. These studies are done under the static condition and have a chief aim to reduce the influence of the random errors of manufacture on the vibration by different meshing-Phases among the planet gears and by such assembly methods that the influence of the run-out errors of each planet gear can be counteracted. It is difficult to estimate the torque variation in driving from the random errors of manufacture of the planetary gear because of its complicated construction. However, when the planetary gear is designed, the vibration characteristic of the planetary gear has already had to be made clear before this. Therefore, in the present study, the authors have carried out experimental investigations to make clear the vibration characteristic of the planetary gear. In this experiment a Stoecklcht type single-stage planetary gear (Type 2K-H) was used, and it was the same as the planetary gear shown in the previous paper.

The measured data are those on the torque variations of the higher-speed shaft and the lower-speed shaft, the dynamic displacement of the sun and the ring gear. These data were collected at different gear speeds under a constant load. The frequency characteristics of the torque variation and the dynamic displacement were obtained by a frequency analysis of them. These obtained frequency characteristics were compared with the vibration characteristic of the torsional system of the power circulating type gear testing machine including the planetary gear, and the relation between these two characteristics was investigated.


The experiment on the torque variation in the planetary gear was carried out using a power circulating type gear testing machine shown in the 1st report. The dimensions and the accuracies of the planetary gear used is already shown in the 2nd report.

The torque variations of the higher-speed shaft (sun gear shaft) and the lower-speed shaft were measured at a static tooth load 130 kgf (a lower-speed shaft torque 46.3 kgf.m) and in the speed range of the higher-speed shaft from 1720 rpm to 8230 rpm (the mesh frequency range from 334 Hz to 1600 Hz) for the speed reduction drive.

Fig. 1 shows an electric circuit diagram for torque measurement. The torque on the sun gear shaft and the sun gear and that on the lower-speed shaft, namely, output shaft connected with the planet carrier were estimated from the strains in their shafts. The strains were...
detected by means of the cross type resistance-wire strain gages which were stuck on the two shafts. As shown in Fig. 1, an electric signal from the strain gages is transmitted through the slip-ring, the bridge circuit and the DC amplifier. The output signal from the DC amplifier is once recorded on the magnetic tape of the data-recorder. During the recording the signal is observed by means of an oscilloscope. The strain data in the data-recorder are recorded again on the pen type oscillograph, after the time scale of the strain has been extended to 400 times by using two data-recorders. In order to know the meshing position of the planetary gear, the pitch signal and the synchronizing signal also are recorded at the same time as the recording of the strain. A frequency analysis of the strain is also carried out at the band width of 2.5 Hz by means of the Akashi automatic vibration analyzer (AVA-1). From the result of the frequency analysis, a circular analysis diagram of the torque variation is drawn up, and the frequency characteristic of the torque variation is investigated from this diagram. Furthermore, the torque variation of the higher-speed shaft lying between the gear coupling annexed to the sun gear shaft and the speed change gear (see Fig. 2) is also measured. When this torque variation is compared with that of the sun gear shaft, it is found that the torque variation of the sun gear shaft is influenced by the meshing of the speed change gear smaller than that of the higher-speed shaft. Therefore, in this paper, the torque variation of the sun gear shaft will be adopted, and then it will be named the torque variation of the higher-speed shaft. The experimental procedure for the dynamic displacements of the sun gear and the ring gear was the same as the 2nd report, and the frequency analysis of their dynamic displacements also was carried out by the same procedure as the torque variation.

3. Dynamic characteristic of gear testing machine

Since a power circulating type gear testing machine was used in the present experiment, the dynamic characteristic of this gear testing machine was investigated before investigating the torque variation in the planetary gear. As shown in Fig. 2 (a), the power circulating type gear testing machine can be replaced with the lumped masses and the torsional springs, if it is considered as the torsional vibration system. In Fig. 2(a), the gears and the couplings are represented by the mass polar moment of inertia, and the shafts are represented by the torsional stiffness. Fig. 2(b) shows an equivalent vibration model for Fig. 2(a). This model was obtained by assuming a portion enclosed with the broken line to be one moment of inertia, and this assumption may be proper because the tooth stiffness is larger than the torsional stiffness of shaft. Furthermore, the moment of inertia of the driving motor was omitted from the model because of the small stiffness of the rubber coupling connecting the motor. The equivalent

![Fig. 1 Diagram of electric circuit for measurement of torque variation](image1)

![Fig. 2 Equivalent vibration model of power circulating type gear testing machine](image2)
moment of inertia and the equivalent torsional stiffness of the model were converted into the values on the lower-speed shaft. The natural frequency and the vibration mode of the equivalent vibration model in Fig.2(b) were calculated by Holzer's method. Fig.3 shows the vibration mode which is obtained by putting the angular displacement of $\psi_1$ as unity.

4. Experimental results

4.1 Example of measured data

Fig. 4 shows an example of the measured torque variation. It is seen from this figure that the torque in the planetary gear varies in driving even if the static torque is constant. Also, the reproducibility of the torque variation is confirmed for each synchronous interval at which the meshing conditions are completely the same, and which corresponds to twice rotations of the lower-speed shaft. The torque variation of the lower-speed shaft has a component of the lower-frequency mainly, and the ratio of the high-frequency component is very small. On the other hand, the torque variation of the higher-speed shaft has both a high-frequency component and a low-frequency component of which the period is the same as the torque variation of the lower-speed shaft. Figs. 5 and 6 show the examples of the frequency analysis of the torque variation of the higher-speed shaft and the lower-speed shaft. The ordinate of these figures stands for the half amplitude of each component in the torque variation. It is found from these figures that the low-frequency components have the frequencies related to the revolution per second of the planet carrier, the planet gear and the sun gear, and the high-frequency components have the frequencies related to the mesh frequency of each gear set. Therefore, the torque variation will be investigated to distinguish between the low-frequency component and the high-frequency component, and the relation between the torque variation and the dynamic tooth load, the dynamic characteristic of the gear testing.

Fig. 3 Natural frequency and vibration mode of equivalent model

| Natural frequency | Angular displacement | 1st | 68.0 Hz | 4th | 202 Hz | 2nd | 81.3 Hz | 5th | 276 Hz | 3rd | 138 Hz |

Fig. 4 Example of torque variation at mesh frequency 546 Hz

Fig. 5 Example of frequency analysis of higher-speed shaft torque

Rotation speed of lower-speed shaft \( \text{rpm} \)

Mesh frequency of planetary gear Hz \( f_1 \)

Mesh frequency of power return gear Hz \( f_3 \)

Mesh frequency of speed change gear Hz \( f_5 \)

Fig. 6 Example of frequency analysis of lower-speed shaft torque
machine will be explained.

4.2 Dynamic tooth load and torque variation

Fig. 7 shows the torque variation rates at different gear speeds, which are defined as the ratio of the maximum half amplitude of the dynamic torque to the static torque. It is seen from the figure that the torque variation rate of the lower-speed shaft is smaller that of the higher-speed shaft. This tendency is the same as the result of the experiment carried out by Endo for the IMS type planetary gear. The low-frequency component in the torque variation of the higher-speed shaft was measured by means of a low-pass-filter. This torque variation rate is also shown in Fig. 7. When Fig. 7 is compared with Fig. 16 in the 2nd report, it may be found that the torque variation of the higher-speed shaft has the same tendency as the coefficient of variation of the dynamic load factor of the sun gear, though these differ from each other in the value. The larger variation of the torque on the higher-speed shaft in the speed range above 1000 Hz is due to the larger high-frequency component in the torque variation. On the other hand, the torque variation rate of low-frequency component of the higher-speed shaft and the torque variation rate of the lower-speed shaft do not change so largely with an increasing gear speed. These, however, have the maximum values at the mesh frequencies 550 Hz, 1100 Hz and 1600 Hz. Fig. 8 shows the torque waves and the dynamic load factors of the sun gear vs. the rotation angle of the lower-speed shaft at 400 Hz, 550 Hz and 1600 Hz respectively. It is seen from this figure that the wave form and the amplitude change complexly with an increasing gear speed, and the high-frequency torque variation of the lower-speed shaft is very small. The reason can be explained from the fact that the high-frequency torque is absorbed by the planet carrier, because its mass is larger than the sun gear mass.

4.3 Relation between torque variation and dynamic characteristic of gear testing machine

4.3.1 Measuring result of torque variation Figs. 9-12 shows the circular analysis diagrams of the torque variation of the higher-speed shaft and the lower-speed shaft, which are drawn up from the frequency analysis diagrams of Figs. 5 and 6 etc. The low-frequency components in the torque are shown in Figs. 9 and 10, and the high-frequency components are also shown in Figs. 11 and 12. The circular analysis diagram consists of the abscissa representing the mesh frequency of the planetary gear, the ordinate representing the frequency of the torque variation and the circle of which the diameter represents the half amplitude of the torque variation. The symbol σ in these figures indicates the rotation per second of the lower-speed shaft, therefore, 6σ is that of the higher-speed shaft because the speed ratio of the planetary gear is 6. Also, the symbols $f_1$ and $f_2$ ($=2f_1$) indicate the mesh frequencies of the planetary gear, the power return gear, and the speed change gear respectively. It is found from Figs. 9-12 that each frequency component in the varying torque stands in the straight lines drawn through the origin of coordinate system at different gear speeds. Therefore, it can be explained that the torque variation in the planetary gear has the frequen-

![Fig. 7 Torque variation rates of higher-speed and lower-speed shafts](image)

![Fig. 8 Dynamic load factor and torque variation vs. rotation angle of lower-speed shaft](image)
cies proportional to the speed. From the fact, it is considered that this vibration is a forced vibration.

4.3.2 Low-frequency component in torque variation The torque variation of the lower-speed shaft shown in Fig. 9 has the principal components of $C$, $2C$, $(11/3)C$, $(11/2)C$, $5C$, $6C$, and $11C$. Also, as shown in Fig. 10, the torque variation of the higher-speed shaft has many components, such as $(5/2)C$, $(11/3)C$, $5C$, $(11/2)C$, $6C$, $(15/2)C$, $10C$, $11C$, $(25/2)C$ and $13C$. When the frequencies of the components $11C$, $6C$, $(11/2)C$ and $(11/3)C$ coincide with the frequencies 86 Hz, 177 Hz and 265 Hz respectively, the amplitudes of these components have a tendency to increase rapidly. This tendency appears remarkably especially in the components of $11C$, $(11/2)C$ and $(11/3)C$. The frequencies 86 Hz, 177 Hz and 265 Hz coin-

Fig. 9 Circular analysis diagram of torque variation of lower-speed shaft (low-frequency component)

Fig. 10 Circular analysis diagram of torque variation of higher-speed shaft (low-frequency component)

Fig. 11 Circular analysis diagram of torque variation of lower-speed shaft (high-frequency component)

Fig. 12 Circular analysis diagram of torque variation of higher-speed shaft (high-frequency component)
cide nearly with the 2nd, 3rd and 5th torsional natural frequencies of the power circulating type gear testing machine, respectively. It is found from the vibration mode in Fig.3 that the power return gears (J2) lying on the side of the lower-speed shaft and the planetary gear (J3) vibrate with the reverse phase to each other at the 2nd, 3rd and 5th natural frequencies. At the 1st and 4th natural frequencies where \( J_2 \) and \( J_3 \) vibrate with the same phase, the torque variation does not grow large as shown in Figs.9 and 10. The natural frequencies of a torsional system, which is composed only of a planetary gear and two annexed rotating elements, are calculated. Namely, this torsional system is composed of \( J_3, K_2, K_3 \) and \( J_1, J_2 \), which are handled as one moment of inertia in the calculation, as shown in Fig.2. The calculated natural frequencies are 86.8 Hz and 293 Hz. Therefore, the 2nd natural frequency of the power circulating type gear testing machine is equivalent to the 1st natural frequency of the torsional system composed only of a planetary gear and two annexed rotating elements. The torque variation grows particularly large by the coincidence of the frequency of the torque variation with the frequency of 86.8 Hz. Consequently, when the low-frequency torque variation at the resonance is discussed, in practical use the 1st natural frequency of the torsional system composed only of the planetary gear and the two annexed rotating elements should be discussed. Though the load distribution was equal at the mesh frequency of about 560 Hz as reported in the 1st report, the torque variation increased due to the above-mentioned reason.

4.3.3 High-frequency component in torque variation

It is clear from Fig.11 that the high-frequency component in the torque variation of the lower-speed shaft is very small as compared with the static torque, and the torque variation has the components of the mesh frequencies of a planetary gear and a power return gear and their higher harmonic components. The increment of the torque amplitude at the mesh frequency 500 Hz results from the resonance of the ring gear, which occurs by the coincidence of the mesh frequency with a natural frequency 500 Hz of the ring gear. The circular analysis diagram of the torque variation of the higher-speed shaft is shown in the two figures of Fig.12 for the torque variations below 4000 Hz and above 4000 Hz respectively. It is clear from Fig.12 that the high-frequency torque variation of the higher-speed shaft has the components of various mesh frequencies and their higher harmonic components. Among these mesh frequencies, there are not only the mesh frequency of the planetary gear but also the mesh frequencies of the power return gear and the speed change gear. It is seen from Figs.11 and 12 that the torque variation increases when its frequency coincides with 1640 Hz. This increment of the torque variation is caused by the resonance of a partial torsional vibration system composed of the planetary gear \( J_3 \), the gear coupling annexed to the sun gear shaft \( J_3 \), and the sun gear shaft \( K_3 \), where \( J_3 \), \( K_3 \), and \( K_3 \) indicate the moment of inertia and the torsional stiffness respectively. This consideration is based on the fact that the calculated natural frequency of this torsional vibration system is in good agreement with 1640 Hz. Fig.13 shows an equivalent vibration model of this system. The equivalent moment of inertia and the equivalent torsional stiffness shown in Fig.13 were converted into the values on the lower-speed shaft as in Fig.2.

It is seen from Fig.12 that the torque variation of the higher-speed shaft increases again when its frequency coincides with 8800 Hz. This increment of the torque variation appears remarkably at the mesh frequency 1470 Hz. This phenomenon can be explained from a rapid increase of the dynamic tooth load at this time, which occurs due to the coincidence of the 6th frequency of the mesh frequency with the 3rd natural frequency (8800 Hz) of the planetary gear.

5. Gear errors and principal vibration components in the torque variation

As above-mentioned, the high-frequency components in the torque variation include the mesh frequency component and its higher harmonic components. The mesh frequency component is produced from a change of tooth stiffness with mesh progress, the gear errors and the interfering contact at the tooth tip edge caused by the approach of the neighboring tooth based on the tooth deformation. There is no great difference between the mesh frequency component for the ordinary parallel-shaft gear set and the one for the planetary gear. However, in the planetary gear many resonance phenomena appear as compared with the parallel-shaft gear set, because it has many degrees of freedom.

The low-frequency torque variation has many components as shown in section 4.3.1. These components are produced mainly from the run-out errors of the sun gear, the planet gear and the ring gear. The equivalent tooth surface errors for the run-out errors of these gears under the static condition were derived by Hayashi, et al. (18).

\[
\begin{array}{cccc}
(K_3) & J_3 & 12.1 \\
(K_3) & J_9 & 1.58 \\
K_3 & 153 \times 10^6 \\
K_3 & 28.3 \times 10^6 \\
K_3 & 8.27 \times 10^6 \\
\end{array}
\]

J: Equivalent moment of inertia (Kgf mm m²)
K: Equivalent torsional stiffness (Kgf mm/rad)

Fig.13 Equivalent vibration model
For the planetary gear, since the planet gear revolves at the same rotating speed as the lower-speed shaft, two lines of action where the planet gear engages with the sun gear or the ring gear revolve at the same rotating speed and in the same direction as the lower-speed shaft. Therefore, the meshing errors based on the run-out errors of each gear appear in meshing at periods coincident with the relative rotating speeds of each gear with respect to the lower-speed shaft. In the case of the present paper, when the rotating speed of the lower-speed shaft is \( C \), then the relative rotating speeds of the sun gear, the planet gear and the ring gear become \( 5C, (5/2)C \) and \( C \) respectively from the relation between their numbers of teeth. Now, let us assume that an imaginative band plate, which is equivalent to the run-out error of gear, is inserted at the meshing point at right angles to the line of action and in the same direction as the gear rotation. Also, let the two meshing points, where the planet gear engages with the sun gear or the ring gear, be named the meshing points \( M_{sp} \) and \( M_{rp} \) respectively. The insertion directions of the two plates, namely, the run-out errors of the sun gear and the ring gear at the meshing points \( M_{sp} \) and \( M_{rp} \) are opposite to each other, because the relative rotating directions of the sun gear and the ring gear with respect to the planet carrier are opposite to each other. In the same manner, the insertion directions of the run-out error of the planet gear at the meshing points \( M_{sp} \) and \( M_{rp} \) are opposite to each other. Therefore, below, the frequencies of the run-out errors of gears will be expressed, including the signs of plus and minus. It can be explained from the above that when the rotating speed of the lower-speed shaft is \( C \), the run-out errors of the sun gear, the planet gear and the ring gear appear in meshing with the frequencies of \( 5C, (5/2)C \) and \( C \) respectively. If these run-out errors are observed from the stationary coordinate system, the frequencies of them apparently become \( G(=5C+C), (7/2)C(=5C+C) \) or \( -(3/2)C(=5C+C) \) and \( 0C(=-C+C) \) respectively, because then the rotating speeds of the sun gear, the planet gear and the ring gear become larger than the relative rotating speeds of them at the rate of one rotation per one rotation of the lower-speed shaft. The influence of the eccentric error of planet carrier also appears in meshing, since the planet gear is annexed to the planet carrier. The frequency of the eccentric error at the meshing point becomes \( C(=0+C) \) in the case of observation from the stationary coordinate system. The Stoecklitch type single-stage planetary gear has the floating sun and ring gears for the purpose of equalizing the loads, and it is already clear that the rigid-body displacement of the ring gear is very small, and that the load distribution is about equal in the mesh frequency range below 1200 Hz. Therefore, it can be supposed that the influence of the run-out error of each gear is counteracted by only the displacement of the sun gear, which can be beshown with a reproducible locus on a plane.

Figs. 14 and 15 show the circular analysis diagrams for the low-frequency components in the dynamic displacements of sun gear and ring gear. These diagrams were obtained by the same means as the torque variation, and these displacements have already been explained in detail in the 2nd report. It is seen from these figures that the displacement amplitude of each component has a nearly constant value regardless of the gear speed. The displacement of ring gear in Fig. 15 has
the component $3C$ based on the elastic deformation of ring gear mainly. The influence of this component $3C$ appears in the displacement of sun gear on Fig.1.a, but it scarcely appears in the torque variation shown in Figs.9 and 10. Except the component $3C$, the principal components in the sun gear displacement are $C$, $(-3/2)C$, $(7/2)C$ and $6C$. The frequencies of these components respectively coincide with those of the run-out errors of each gear at each meshing point for the case of observation from the stationary coordinate system. The displacement amplitudes of the components $C$ and $6C$ are supposed to be four times as much as the eccentric amount of planet carrier and two times as much as the run-out error amount of sun gear, respectively. The components $(-3/2)C$ and $(7/2)C$ exist in the sun gear displacement with their amplitudes which are the total amounts of the run-out errors of three planet gears respectively. The loci of each component in the sun gear displacement were drawn by means of an x-y recorder. The minus sign in $(-3/2)C$ was marked down due to the fact that the progressive direction of its locus was reverse to that of the other components.

For the ordinary parallel-shaft gear set, we can consider that the run-out error of gear results only in a variation of center distance. Therefore, the influence of the run-out error of gear on the dynamic tooth load is small. This may be attributed to the geometric property of involute gear. On the other hand, in the case of the planetary gear, since the sun gear engages with some planet gear at the same time, the meshing condition at one meshing point influences directly that at other meshing points. Therefore, it can be considered that the influence of the run-out error of the dynamic tooth load for the planetary gear is larger than that for the parallel-shaft gear set. This influence of the run-out error appears also in the torque variation.

Since the run-out error of each gear induces the meshing error which has the frequency coincident with the relative rotating speed of each gear, in the case of this paper it is clear that the torque variation having the frequencies of $C$, $(5/2)C$ and $5C$ occurs due to the components of $C$, $(-3/2)C$ or $(7/2)C$ and $6C$ in the sun gear displacement. It has already been explained that the sun gear displaces for the compensation of the non-equality load distribution caused by the run-out error of each gear. However, this compensation becomes sometimes too little or too much due to the frictional and inertia resistances. Since the sun gear has the compensating movement as above-mentioned, the planetary gear may be handled by the same consideration as the universal joint. Thus, it can be explained that a stemming moment, of which the frequency is two times as large as the relative rotating speed of each gear, appears at the meshing point of the sun gear. Further, there is a probability that a torque variation, of which the frequency is three times as large as the relative rotating speed of each gear, occurs. Thus, in the case of this paper the torque variations having the frequencies of $(15/2)C$ and $15C$ which are three times as large as $(5/2)C$ and $5C$, may occur. In fact this can be proved from the fact that the components $(15/2)C$ and $15C$ in Fig.10 have the larger torque amplitude. In addition to the above components the torque variation has many components, such as $11C$, $(11/2)C$ and $(11/3)C$ and so on, which can not be explained as the higher harmonic component of $C$, $(5/2)C$ and $5C$.

Now, for explanation the following notations are used: $T$: torque acting on sun gear. $I_s$: moment of inertia of sun gear. $\omega_y$: angular displacement of sun gear. $x_{RP}$: run-out error of sun gear. $r_{PB}$: radius of base circle of sun gear. $k_{PB}$, $k_{PBx}$, $k_{PBz}$: composite tooth stiffness of sun gear and each planet gear. $x_{PB}$, $x_{PBx}$, $x_{PBz}$: relative displacements between sun gear and each planet gear on the line of action. $C_j$: damping factor. Thus, the equation of torsional motion for the sun gear can be given as follows:

$$I_s \frac{d^2 \theta}{dt^2} + C \frac{d \theta}{dt} + \frac{3}{4} k_{PB} \theta = T$$

As seen from the above equation, the torsional vibration of the sun gear can be explained by the equation with a term which shows the forced vibration proportional to $\sin \omega_t$. The angular velocity of the sun gear shaft has the varying angular velocity components of $\omega_t \omega_s$, $(5/2)\omega_s$, $5\omega_s$ in addition to the mean angular velocity $\omega_0$ of the sun gear. Therefore, putting the amplitude of the varying angular velocity $\omega_t$ as $\omega_t'$, $\omega_t''$ is written as follows:

$$\omega_t = 6 \omega s t + \frac{1}{4} \omega_t' \sin \omega_t' t$$

where $t$ is time, and if $\omega_t'$ is very small, the following relations are obtained:

$$\cos \left( \frac{1}{4} \omega_t' \sin \omega_t' \right) \approx 1$$

$$\sin \left( \frac{1}{4} \omega_t' \sin \omega_t' \right) \approx \frac{1}{4} \omega_t' \sin \omega_t'$$

Taking account of these relations, $\sin \theta$ can be written as follows:

$$\sin \theta = \sin \omega_0 t + \frac{1}{2} \omega_t' \left[ \sin \left( \omega_0 + \omega_t' \right) t - \sin \left( \omega_0 - \omega_t' \right) t \right]$$

Therefore, it can be explained from Eq.(4) that the forced vibrations having the frequencies of $(6\omega + \omega_t')$, $(6(5/2)\omega + \omega_t')$, $(65\omega + \omega_t')$ and so on occur in the sun gear, also in the higher-speed shaft, where $\omega_0$ in Eq.(4) is replaced by $C$. In the same manner, the forced vibrations of $(1\omega + \omega_t')$, $(1(5/2)\omega + \omega_t')$, $(15\omega + \omega_t')$ and so on occur in the lower-speed shaft. Considering the run-out errors of gears and the forced vibration expressed by Eq.(1), the principal forced vibrations of low-frequency components in the torque variation can be explained qualitatively. Further, in the case of this paper, it is expected that the component of $11C$ is larger.
chichih type single-stage planetary gear, and the measured results were considered. The conclusions from this consideration are summarized as follows:

(1) The torque variation in the planetary gear has the high-frequency and low-frequency components. The high-frequency torque variation appears mainly in the higher-speed shaft. The torque variation rate of the high-frequency component increases rapidly with an increasing gear speed. The torque variation rate of the low-frequency component does not show this tendency, and it is smaller than that of the high-frequency component.

(2) The high-frequency torque variation is a forced vibration caused by the meshing errors of which the periods coincide with those of the meshings of teeth. And when the frequencies of these meshing errors coincide with all kinds of the natural frequencies of the planetary gear, the torque variation rate of the high-frequency components increases suddenly.

(3) The low-frequency torque variation occurs with frequencies which are made by the rotating speed of each gear added to the frequency of its run-out error or by the former subtracted from the latter. When the low-frequency torque variation at the resonance is discussed, in practical use the last natural frequency of a torsional system composed only of a planetary gear and two annexed rotating elements should be discussed.

(4) There is a necessity for the planetary gear especially to be designed such as to have a smaller run-out error of each gear as compared with the parallel-shaft gear set.

References