621.852.052.44

Paper No. 169-11

A Study on Strength of Toothed Belt*

(2nd report, Influence of Pitch Difference on Load Distribution)

By Tomio KOYAMA**, Masanori KAGOTANI***,
Toshinobu SHIBATA† and Takeshi HOSHIRO††

In this paper, the load distribution on toothed belt whose pitch is different from that of pulley is discussed theoretically and experimentally.

From this study the following conclusions have been obtained:

- 1) The pitch difference exerts an important influence on the load distribution, so the most suitable pitch difference should be chosen in consideration of the transmissible load, the modulus of elasticity of the belt, the compliance of the belt tooth, the coefficient of friction between belt and pulley, etc.
- 2) The effectiveness of the theoretical prediction on load distribution is proved by the experimental results.

1. Introduction

In order to transmit power smoothly by toothed belt drives, the belt pitch needs to be equal to the pulley pitch when in meshing. In general, belt pitch is manufactured in such a way that it is a little less than the pulley pitch. Because the meshing condition of them exerts a great influence on belt life, it is very important to know the load distribution on the belt teeth of toothed belt drives with different pitches. In the first report⁽¹⁾, we discussed on the load distribution of toothed belt drives with the same pitch. In this paper, the load distribution in meshing of belt and pulley which have different pitches is analyzed, and the influences of the pitch difference, the tension, the compliance of the belt tooth, the modulus of elasticity of the belt, the coefficient of friction between belt and pulley, etc. on load distribution are discussed. And then the theoretical results of the load distribution are compared with the experimental ones.

Fig. 1. Meshing toothed belt with pulley

2. Theoretical Analysis

Fig. 1 shows the meshing of belt and pulley. In the figure, T_0 is the tight side tension and $T(N+1)$ is the slack side tension. The belt teeth are numbered 1, 2, ..., N , $(N+1)$, and the belt tension between each tooth is expressed by $T(1)$, $T(2)$, ..., $T(N)$. T_0 and $T(N+1)$ are arbitrarily defined and the angles as created by their directions are respectively represented by ϱ and ψ as shown in the figure. The load on each tooth of the pulley is expressed by $Q(1)$, $Q(2)$, ..., $Q(N)$, $Q(N+1)$ and their directions are equal to the pressure angle α . The other angles β , γ , and ϕ are represented in the figure. The pitch difference Δt is presented as:

$$\Delta t = t_B - t_P \dots \dots \dots (1)$$

where t_B is the belt pitch and t_P is the pulley pitch (see Fig. 2). Δt has two kinds of signs, plus and minus; i.e., if the belt pitch is greater than that of pulley, the pitch difference Δt is defined as plus and in the opposite case Δt becomes minus.

Fig. 2 shows the standard meshing condition when the belt is lightly wound on the pulley; it is assumed that the belt teeth mesh rectilinearly with the pulley. Fig. 2(a)

Fig. 2. Standard position of meshing due to pitch difference

* Received 7th November, 1977.

** Lecturer, Osaka Institute of Technology, Osaka.

*** Assistant, Osaka Industrial University.

† Associate Professor, Kyoto University.

†† Chief Engineer, Mitsubishi Belt Co., Ltd.

shows a plus pitch difference and (b) shows a minus one. In both figures, the dotted line represents the condition in which the belt tooth is already meshed with the pulley tooth. In the longitudinal direction of the belt, the clearance C between the tight side tooth flank of the belt and the pulley tooth flank is given a "plus" value if they are not yet in contact with each other as shown by the $(m+1)$ th belt tooth in Fig. 2(a), and in the case of the $(m-1)$ th belt tooth which is already in interference with the pulley tooth, the clearance C is given a "minus" value. The method of defining the sign for clearance B at the slack side tooth flank of the belt tooth is the same as was with clearance C . As shown in Fig. 2, when the m th tooth has no clearance (i.e., $C=0$), the number of belt tooth where the clearance B begins to become equal to 0 or minus is the $(m+BL/\Delta t)$ th tooth, where BL is the backlash between the belt tooth and pulley tooth. When $C(m)=0$, both clearances $C(K)$ and $B(K)$ of the K th belt tooth are expressed by the following equations:

$$\begin{cases} C(K) = (K-m)\Delta t \\ B(K) = BL - C(K) \end{cases} \quad (2)$$

where $K=1, 2, \dots, N, (N+1)$, and $m=1, 2, \dots, N, (N+1)$.

Now, the relation between the deflection of belt tooth and the deformation of belt tooth and groove due to belt tension will be explained. When the belt is lightly wound on the pulley, there are three kinds of belt tooth positions determined by pitch difference. One of the typical examples is shown in Fig. 3. Now, consider that the belt deflects due to tension T_0 acting on the belt from the tight side as the dotted lines shown in Fig. 3. At that point, the thickness of the K th groove S_B becomes $\{S_B + S(K)\}$ and the tooth length l_B becomes $\{l_B + l(K)\}$ by elongation and the K th and $(K+1)$ th belt teeth deflect $\delta(K)$ and $\delta(K+1)$ respectively. As pulleys are generally made of steel, we can assume that deformation of pulley teeth is negligible. Also for simplification of the analysis, we assume that load $Q(K)$ acting on a belt tooth is a concentrated load, the direction of which coincides to the pressure angle, and belt teeth deflect in parallel to it except near the bottom of the tooth. If the tooth deflects to the tight side, $\delta(K)$ is given a plus value. If the tooth deflects to the slack side, it is minus. Fig. 3 shows the case of $\delta(K) > 0$. The $(K+1)$ th tooth deflection $\delta(K+1)$ is

Fig. 3. Deformation of belt

Fig. 4. Deflection of belt tooth from standard position

expressed in Eq. (3).

$$\delta(K+1) = \delta(K) - \{S(K) + l(K)\} \quad (3)$$

where $K=1, 2, \dots, N$.

Namely, during one belt pitch in Fig. 3, the deflection of the $(K+1)$ th tooth is equal to the difference between the deflection of the K th belt tooth and the sum of elongation of the K th belt tooth and groove.

The belt tooth deflection after contacting the pulley tooth is considered from the above mentioned standard meshing position. There are three kinds of deflections when the belt is lightly wound on the pulley. They are shown in Fig. 4(a)-(I) and (b)-(IV), (a)-(II) and (b)-(V), and (a)-(III) and (b)-(VI), respectively. In the case of (a)-(I) and (b)-(IV), both of the K th tooth flanks are not in contact with pulley teeth, i.e., $C(K) > 0$ and $B(K) > 0$. In the case of (a)-(II) and (b)-(V), $C(K)$ is longer than backlash, i.e., $C(K) > 0$ and $B(K) < 0$. In the case of (a)-(III) and (b)-(VI), the tight side tooth flank of the K th tooth is already in contact with the pulley tooth, i.e., $C(K) < 0$, $B(K) > 0$. The deflection $\lambda(K)$ of the K th tooth by belt tension, as shown by the dot-dash-line in Fig. 4, is defined as "plus" when the tooth deflects to the tight side (see Fig. 4(a)), and in the case of the opposite deflection it is defined as "minus". Also, the belt tooth is assumed to deflect symmetrically, and $\lambda(K)$ is expressed by the following equation Eq. (4).

- [I] In the case of $\delta(K) > 0$:
- (1) For $C(K) > 0$, $B(K) \geq 0$:
[Fig. 4(a)-(I)]
 $\delta(K) > C(K)$: $\lambda(K) = \delta(K) - C(K)$
 $\delta(K) \leq C(K)$: $\lambda(K) = 0$
 - (2) For $C(K) > 0$, $B(K) < 0$:
[Fig. 4(a)-(II)]
 $\delta(K) > |B(K)|$, $\delta(K) > C(K)$:
 $\lambda(K) = \delta(K) - C(K)$
 $\delta(K) \geq |B(K)|$, $\delta(K) \leq C(K)$:
 $\lambda(K) = 0$
 $\delta(K) < |B(K)|$: $\lambda(K) = \delta(K) + B(K)$
 - (3) For $C(K) < 0$, $B(K) > 0$:
[Fig. 4(a)-(III)], $\lambda(K) = \delta(K) - C(K)$
- [II] In the case of $\delta(K) < 0$:
- (1) For $C(K) > 0$, $B(K) \geq 0$:
[Fig. 4(b)-(IV)]
 $|\delta(K)| > B(K)$: $\lambda(K) = \delta(K) + B(K)$
 $|\delta(K)| \leq B(K)$: $\lambda(K) = 0$
 - (2) For $C(K) > 0$, $B(K) < 0$:
[Fig. 4(b)-(V)], $\lambda(K) = \delta(K) + B(K)$
 - (3) For $C(K) < 0$, $B(K) > 0$:
[Fig. 4(b)-(VI)]
 $\delta(K) < C(K)$, $|\delta(K)| > B(K)$:
 $\lambda(K) = \delta(K) + B(K)$
 $\delta(K) \leq C(K)$, $|\delta(K)| \leq B(K)$:
 $\lambda(K) = 0$
 $\delta(K) > C(K)$: $\lambda(K) = \delta(K) - C(K)$
- where $K=1, 2, \dots, N, (N+1)$.

Then, let us consider the elongation of belt tooth and groove. For simplification of the analysis, we assume the lengths of belt groove S_B and tooth l_B are equal to the length of the top land of the pulley tooth and the length of the pulley groove, respectively. First of all, let us consider the elongation $S(K)$ of the K th belt groove. As shown in Fig. 5, the groove between the K th and the $(K+1)$ th belt teeth is wound on the top land of the pulley tooth with a wound angle 2β . When belt tension $T(K)$ acts at A, the tension at B changes $T(K)'$ by the frictional force between the bottom land of the belt groove and the top land of the pulley tooth, where, in the case of $T(K) > T(K)'$, $\delta(K)$ becomes plus and

in the case of $T(K) < T(K)'$, $\delta(K)$ becomes minus. The tension $T(K)'$ at B is expressed by Eq. (5):

$$\left. \begin{aligned} \delta(K) > 0: T(K)' &= e^{-2\mu\beta} T(K) \\ \delta(K) < 0: T(K)' &= e^{2\mu\beta} T(K) \end{aligned} \right\} \dots\dots\dots (5)$$

where μ is the coefficient of friction. Therefore the elongation $S(K)$ is expressed by the following equation regardless of the sign of $\delta(K)$.

$$S(K) = \frac{r}{AE\mu} (1 - e^{-2\mu\beta}) T(K) \dots\dots\dots (6)$$

where E : modulus of elasticity of belt wound on pulley
 A : cross sectional area of load-carrying cords
 r : radius of outside diameter of pulley
 Z : number of pulley teeth

$$\beta = \frac{\pi S_p}{Z l_p}$$

$$K = 1, 2, \dots, N.$$

Now let us consider elongation $l(K)$ of the belt tooth. As shown in Fig. 6, when the load $Q(K)$ acts on the tight side tooth flank, it is defined as plus, and in the opposite case, it is defined minus. Also the tension $T(K)$ at A is equal to the force $\{T(K-1)' - Q(K) \cos \alpha\}$ at C. The tooth elongation $l(K)$ of the belt tooth between A and C is given by Eq. (7):

$$\left. \begin{aligned} \delta(K) > 0: \\ l(K) &= \frac{l_p}{AE} \{e^{-2\mu\beta} T(K-1) - Q(K) \cos \alpha\} \\ \delta(K) < 0: \\ l(K) &= \frac{l_p}{AE} \{e^{2\mu\beta} T(K) + Q(K) \cos \alpha\} \end{aligned} \right\} \dots\dots\dots (7)$$

where $K = 2, 3, \dots, N$, and the elongation $l(1)$ of the first tooth is expressed by the following equation:

$$l(1) = \frac{l_p}{AE} \{T_0 - Q(1) \cos \alpha\} \dots\dots\dots (7')$$

The load $Q(K)$ acting on the K th tooth is expressed by Eq. (8):

$$Q(K) = \frac{\lambda(K)}{f(K) \cos \alpha} \dots\dots\dots (8)$$

where $K = 1, 2, \dots, N, (N+1)$, and $f(K)$ is a proportional constant concerning the deflection of belt tooth. $f(K)$ is called "compliance", and it can be obtained by use of the experimental apparatus which is described in the 1st report. The experimental equation of compliance $f(K)$ is expressed by Eq. (9):

$$f(K) = a \{\lambda(K)\}^3 + b \{\lambda(K)\}^2 + c \lambda(K) + d \dots\dots\dots (9)$$

where $K = 1, 2, \dots, N, (N+1)$, and a, b, c and d are the experimental constants.

In order to obtain the belt tension on each tooth, let us consider the force balance of belt and pulley in tangential direction. The tight side tension T_0 , which is arbitrarily determined, acts in $\mathcal{Q}$ direction (see Fig. 1). In the case of $(0 \leq \mathcal{Q} < \phi)$, as the frictional force does not act on the top land of the 1st pulley tooth, belt tension $T(1)$ at the 1st tooth is expressed by Eq. (10):

Fig. 5. Elongation of belt groove

Fig. 6. Elongation of belt tooth

$$T(1) = \{T_0 \cos \mathcal{Q} - Q(1) \cos \alpha\} / \cos \gamma \dots\dots\dots (10)$$

Tension $T(K+1)$ at the $(K+1)$ th tooth is expressed by Eq. (11):

$$\left. \begin{aligned} \text{In the case of } \delta(K+1) > 0: \\ T(K+1) &= \{e^{-2\mu\beta} T(K) \cos \phi \\ &\quad - Q(K+1) \cos \alpha\} / \cos \gamma \\ \text{In the case of } \delta(K+1) < 0: \\ T(K+1) &= \{e^{2\mu\beta} T(K) \cos \phi \\ &\quad - Q(K+1) \cos \alpha\} / \cos \gamma \end{aligned} \right\} \dots\dots\dots (11)$$

where $K = 1, 2, \dots, (N-1)$. When the slack side tension $T(N+1)$ acts in Ψ direction, $(0 \leq \Psi < \gamma)$, $T(N+1)$ can be expressed by Eq. (12):

$$\left. \begin{aligned} \text{In the case of } \delta(N+1) > 0: \\ T(N+1) &= \{e^{-2\mu\beta} T(N) \cos \phi \\ &\quad - Q(N+1) \cos \alpha\} / \cos \Psi \\ \text{In the case of } \delta(N+1) < 0: \\ T(N+1) &= \{e^{2\mu\beta} T(N) \cos \phi \\ &\quad - Q(N+1) \cos \alpha\} / \cos \Psi \end{aligned} \right\} \dots\dots\dots (12)$$

$$\text{where } \phi = \frac{\pi}{Z} \frac{l_p - BL}{t_p}, \gamma = \frac{\pi}{Z} \frac{l_p + BL}{t_p}$$

$(N+1)$ is the number of meshing belt teeth on the pulley and $T(N+1) \geq 0$. The direction of frictional force between the $(K+1)$ th belt groove and the top land of the pulley tooth is determined by the sign of $\delta(K+1)$. Frictional force $R(K+1)$ can be expressed by Eq. (13):

$$\left. \begin{aligned} \text{In the case of } \delta(K+1) > 0: \\ R(K+1) &= (1 - e^{-2\mu\beta}) T(K) \\ \text{In the case of } \delta(K+1) < 0: \\ R(K+1) &= (1 - e^{2\mu\beta}) T(K) \end{aligned} \right\} \dots\dots\dots (13)$$

where $K = 1, 2, \dots, N$.

Load $W(K)$ acting on the K th pulley tooth, is the sum of the load $Q(K)$ acting on the belt tooth flank and the frictional force $R(K)$ acting on the top land of the pulley tooth, and it is expressed by Eq. (14):

$$W(K) = Q(K) + R(K) \dots\dots\dots (14)$$

where $K = 1, 2, \dots, N, (N+1)$, and $R(1) = 0$.

The transmitting load P_w can be expressed by Eq. (15) using Eq. (14) as follows:

$$P_w = T_0 - T(N+1) = \sum_{K=1}^{N+1} W(K) \dots\dots\dots (15)$$

Thus the load distribution of toothed belt can be obtained by substituting Eq. (1)-(7) into Eq. (8)-(15); i.e., given the numerical values of $T_0, t_B, t_p, m, S_p, l_p, A, E, \mu, a, b, c, d, \mathcal{Q}, \Psi, BL, \alpha, r, Z, (N+1)$ and $T(N+1)$ as the data, $Q(K)$ and $T(K)$ can be calculated. In numerical calculation, the computer IBM System/370 Model 125 in Osaka Institute of Technology was used.

Also, the load distribution can be continuously calculated under consideration of frictional force conditions at the beginning and end of meshing.

3. Results of Numerical Calculation

In this section, the influences of pitch difference, tight side tension, slack side tension, modulus of elasticity of the belt, compliance of belt tooth, coefficient of friction, etc., on load distribution of toothed belts are discussed by using the theoretical analysis obtained in the preceding section.

The following numerical data are used in the calculation: belt width $b = 19$ mm, number of pulley teeth $Z = 36$, number of belt teeth meshing with the pulley $(N+1) = 19$. Other numerical data are shown in the figure. The compliance $f(K)$ is assumed to be constant in order to simplify the comparison with the results of numerical calculation.

Fig. 7 shows the influence of pitch difference on the load distribution of toothed belts. When the pitch difference Δt is 0 or plus, the load $Q(K)$ acting on the belt tooth decreases with an increasing tooth number. Namely, on the drive pulley, $Q(K)$ reaches a maximum level on the 1st meshing tooth and a minimum level on the last meshing tooth. In the case of minus pitch differences, the maximum level of

$Q(K)$ occurs at the 1st or the last meshing tooth and the location of minimum $Q(K)$ shifts toward the 1st tooth with a decreasing Δt . From the view point of wear on belt teeth at the beginning of meshing with a pulley, the lower $Q(1)$ or $Q(19)$ are, the less wear on belt teeth due to the interference may be. So it may be advantageous to set $\Delta t < 0$ at the drive pulley and $\Delta t > 0$ at the driven pulley. For the shearing strength of the belt, it may be better for the load distribution to be uniform and the maximum load to be low.

Even if the materials and the dimensions of the belt are identical, the load distribution is different depending on tight and slack side tension or the number of meshing teeth. Therefore it is important to select the most suitable minus pitch difference which is appropriate to the service conditions for longer belt life.

Fig. 8 shows the influence of tight side tension T_0 on load distribution. In the case of $T_0 = 30$ kg, the loads acting on the 1st to 12th teeth are nought. This means that these teeth flanks are not in contact with the pulley teeth flanks, and the transmissible loads on the pulley teeth in these regions are only the frictional forces between the top of the pulley teeth and the bottom land of the belt grooves.

The load acting on the 1st tooth increases with an increasing tight side tension, but the rate of increase of $Q(19)$ is less than $Q(1)$. It is known that under conditions shown in Fig. 8, the changes of the tight side tension play hardly any role in the load distribution acting on the teeth near the slack side.

Fig. 9 shows the influence of slack side tension on load distribution. In the case of $T(19) = 30$ kg, the loads acting on the 5th to last teeth are equal to nought. This is because when the belt is lightly wound on the pulley, if the ratio of the transmissible load to the slack side tension becomes lower than a certain value, the tight side tooth flanks of the belt which have already been in contact with the pulley tooth flanks pull off to the slack side by the slack side tension $T(19)$ and these teeth shift between the pulley

tooth grooves. From the view point of belt life, it may be best for $Q(K)$ on all teeth to be equal to nought. But as such condition is the same as that of plane belts, a slip between belt and pulley may occur. In the case of $T(19) = 0, 10$, and 20 kg, the change of $Q(1)$ is less than that of $Q(19)$. Namely, when the ratio of the transmissible load to the slack side tension is larger than a certain value, the influence of slack side tension on the load acting on several teeth near the tight side is almost non-existent.

Fig. 10 shows the influence of modulus of elasticity E of the belt on the load distribution. The larger E becomes, the less the elongation of the belt teeth and grooves becomes, and the frictional force at $Q(K)$ between the bottom land of the belt groove and the top land of the pulley tooth increases. Then the transmissible load due to $Q(K)$ decreases; the loads on teeth near the 1st tooth decrease, and the loads on the teeth near the slack side increase.

In the 1st report, we showed that uniform load distribution is obtained for larger E in the case of zero pitch difference. In the case of a plus pitch difference, the same results can be obtained. However in the case of a minus pitch difference, the most uniform load distribution occurs at $E = 9000$ kg/mm² under the conditions shown in Fig. 10. Namely, for $\Delta t < 0$, in order to obtain uniform load distribution the most suitable modulus of elasticity E must be selected according to the service condition such as tension, number of meshing teeth, etc.

Fig. 11 shows the influence of compliance of belt tooth $f(K)$ on load distribution. In the case of $\Delta t < 0$, the load acting on the teeth near both the tight and slack sides decreases and the loads on the teeth near the middle of the meshing increases with an increase in $f(K)$. The softer the tooth is, the more uniform load distribution can be obtained.

Fig. 12 shows the influence of coefficient of friction μ on load distribution. The load $Q(K)$ acting on each tooth decreases uniformly with an increase in μ . This means that the transmissible load due to frictional force will increase.

Fig. 7. Influence of pitch difference on load distribution

Fig. 9. Influence of slack side tension on load distribution

Fig. 8. Influence of tight side tension on load distribution

Fig. 10. Influence of modulus of elasticity on load distribution

From the view point of belt life, the more μ increases, the more advantageous the condition will be. From the view point of the uniformness of the load distribution, the lower μ is, the more uniformly the load distribution becomes.

4. Comparison of Experimental Results with Calculated ones

In order to confirm the effectiveness of the theoretical analysis discussed in section 2, the load distribution of toothed belt drives has been measured. The details of the experimental apparatus and method are given in the 1st report.

Section-L, type 345L075 polychloroprene rubber and polyurethane belts (92teeth, 19 mm width) were used. The numbers of both drive and driven pulley teeth are 36, and the rotational pulley speed is 550 rpm.

Fig. 13 shows examples of the load distribution $W(K)$ acting on the driven pulley under the condition of a tight side tension of $T_0=85$ kg and a slack side tension of $T(19)=20$ kg. Fig. 13-(a) is the result of using a polychloroprene rubber belt and (b) that of using a polyurethane belt. $W(K)$ is shown on the ordinate and the tooth number of the driven pulley is shown on the abscissa. The solid lines represent the experimental results and the dotted lines the calculated ones. The pulse A gives the moving point of the pulley teeth and pulse B the location of the 1st tooth of the belt.

As experimental results agree approximately with the calculated ones, the theoretical analysis discussed in section 2 is considered correct.

Comparing the load distribution of (a) with (b), $W(K)$ on the teeth near the tight side of (a) is found higher than that of (b), and in the case of slack side, the relation is opposite. This is because the pitch difference Δt , the modulus of elasticity E and the coefficient of friction of the polychloroprene rubber belt are lower than those of the polyurethane belt, and conversely the compliance $f(K)$ of the rubber belt is larger than that of the urethane one.

As is obvious from the experimental results, the highest

load acting on the pulley teeth occurs between the 1st and the 2nd tooth. This is because a frictional force does not occur on the top of the 1st pulley tooth but begins to occur between the 1st and the 2nd teeth, and the sum of $Q(K)$ and $R(K)$ becomes maximum at just before the 2nd tooth. Also, because of $\Delta t < 0$, the load acting on several pulley teeth near the slack side increases. The experimental results of load distribution on these teeth are higher than the calculated ones. The reason may be that the point of action of $Q(K)$ shifts somewhat toward outside of the tooth flank.

Also, as belts have pitch errors and scatter of elasticity, compliance, etc., it is difficult to determine the exact values to be used in the calculation. However it is considered that the values in the tables shown in Fig. 13 may be reasonable from the coincidence of the experimental and calculated results.

5. Conclusions

The load distributions in toothed belt drives in which the pitch of the belt and the pulley differ from each other have been discussed analytically and experimentally and the following conclusions have been obtained:

1. The pitch difference exerts an important influence on the load distribution, so the most suitable pitch difference should be determined in consideration of service tension and

Z tooth	Δt mm	T_0 kg	$T(N+1)$ kg	E kg/mm ²	$f(K)$ mm/kg	μ
36	-0.018	85	20	5,000	0.03	0.35

(a) Polychloroprene rubber belt

Z tooth	Δt mm	T_0 kg	$T(N+1)$ kg	E kg/mm ²	$f(K)$ mm/kg	μ
36	-0.022	85	20	11,000	0.018	0.65

(b) Polyurethane belt

Fig. 13. Comparison of experimental results with theoretical ones of load distribution acting on driven pulley

Fig. 11. Influence of compliance of belt on load distribution

Fig. 12. Influence of coefficient of friction on load distribution

number of meshing teeth.

2. As the load distribution is affected significantly by factors such as the modulus of elasticity of belt, the compliance of belt tooth, the coefficient of friction, etc., the pitch difference and these factors have to be considered in the design of toothed belt drives for improvement of belt life.

3. The theoretical results of the load distribution agree almost exactly with the experimental ones.

Acknowledgement

The authors wish to express their appreciation to Dr. A. Kubo of the Kyoto Institute of Technology for his valuable suggestions.

Reference

- (1) Koyama, T., *et al.*, Trans. Japan Soc. Mech. Engrs. (in Japanese), Vol. 44, No. 377(1978-1), p. 312.
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