532.525:534-8

Paper No. 172-11

Atomization of Liquid by Ultrasonic Vibration

(On the Vibrating Circular Disc of a Large Area)*

By Chikashi CHIBA**

For the purpose of clarifying the relationship between the vibration modes of amplitudes and the atomisation, first of all, the amplitudes were precisely measured on the vibrating circular disc of a large area, and the mode of vibration was experimentally and theoretically examined.

In this paper, further investigated were the methods of efficient atomisation, the mean droplet diameter, the droplet size distribution etc.. And by inclining the vibrating surface at an angle of $20^\circ \sim 40^\circ$, and utilizing the mode of vibration, an efficient method of uniform atomisation at a high charge rate was evolved.

1. Introduction

There are many studies as regards the atomisation of liquid by ultrasonics, but any basic study of atomisation which has treated systematically the relation with the mode of vibration, over a wide range of frequencies, is not found. In the former papers^{(1),(5)}, the author made a research of the characteristics of liquid atomisation by ultrasonic (nominal frequencies $f = 200\text{kHz} \sim 2\text{MHz}$) and sonic ($f = 10\text{kHz}$) vibration, obtained the mode of vibration of amplitudes and considered the relation between the mode and the atomisation, in view of the importance of frequency and vibration mode for atomisation, and reported that mechanisms of atomisation were various.

In this paper, the amplitudes are precisely obtained over the whole vibrating surface using a non-contacting electro-optical displacement transducer in the same way as in the former report⁽⁵⁾, and the mode of vibration is obtained and the characteristic and other analyses are performed, in the case in which the vibrating circular disc of a large area (dia. $\phi 156$) is set upwards, the liquid is supplied to the center of vibrating surface and the longitudinal vibration ($f = 20\text{kHz}$) is operated along the central fixed axis (dia. $\phi 30$), for analysing the atomisation on the effect of vibration.

From a wide view point, it will become necessary to produce as many uniform droplets as possible for the combustible relation like the improvement of carburetor etc., and for the chemical and medical sciences like the emulsion dispersion, the inhalation treatment etc..

2. Experimental apparatus and method

Fig.1 shows the experimental apparatus: ① a magneto-strictive vibrator with ferrite of nominal frequency $f = 20\text{kHz}$ and ② a horn. A chip which is made of the same material (SUS27) as the horn and makes the vibrating circular disc of a large area (weight 1.55kg), is fixed upwards to the horn as in Fig.2, and the liquid is supplied downwards to the center of its surface.

Fig.1 The experimental apparatus.

The longitudinal vibration is given to the horn, thence the liquid is atomized. For the purpose of setting the target to measure the displacement on the vibrating surface, 26 pins (dia. 1mm , hardened steel) are exactly built every 3mm to 2mm in depth in the radial direction from the center on the vibrating surface, and are vibrated as one body with the vibrating surface. ③ is a liquid supplying tube, which is set on the vertical line upwards of the center of vibrating surface; ⑦ a cock for adjusting the liquid quantity; ④ a high frequency wattmeter of thermo-couple type which measures the electrical input P W of vibrator; ⑤ an ultrasonic generator; ⑥ an automatic constant voltage system; ⑦ a frequency counter; ⑧ a microflash; ⑨ a flasher (flashing time $1.8\mu\text{s}$) and ⑩ a camera.

When a voltage of alternating current

* Received 6th April, 1976.

** Professor, The Technical College, Yamagata Univ., Yonezawa City, Japan.

is given and the changeable frequency is matched with the frequency of vibration of the normal mode, the vibrator vibrates violently. This frequency is a resonant frequency. This experiment is conducted under the resonant condition of vibrator.

Fig. 2 The chip of vibrator.

There are two methods, viz., the constant electric current circuit and the constant voltage circuit a reliable and simple method for measuring the resonant frequency. The resonant frequency f_r of the magneto-strictive vibrator in this experiment is obtained by the method of constant electric current circuit. This is a method of obtaining the resonant frequency by keeping the electric current which runs into the vibrator constant, because the impedance of vibrator is a function of frequency.

That is, a resistance more than three times as large as the absolute value of impedance of the vibrator is applied between the ultrasonic generator and the vibrator, and the electric current which runs into the vibrator is kept constant. Then, the voltage which is generated in the vibrator becomes proportional to the impedance. At this time, the frequency in the case of minimum voltage V_{min} is the frequency f_r .

The installation which is instantly able to cause a minute displacement of the chip is a non-contacting electro-optical displacement transducer 13~21.

This consists of a camera and its control unit. The camera is constituted of a system of the necessary lenses and an electronic system. The tip of pin on the vibrating surface is selected as the target 13 so as to make clear a contrast of white and black edges. The image of its longitudinal vibration is taken in the photo cathode of the tip of the image converter 14 and the light quantities are converted to the electron quantities. The rectangular wave which yields a phase difference is taken out as an analog voltage by the phase demodulator 15 operating through the electron multiplier. 16 is an amplifier and 17 a generator. The modified electric current proportional to the displacement of target runs into the deflection yoke 18. This is converted into the output voltage by the voltage converter 19. 20 is a low pass filter with a changer of the upper limit frequency. The signal gain of high frequency is damped or cut by selecting the immediate upper frequency of the emissive frequency as the upper limit frequency and adjusting it. Then, the shot noise of various frequencies which is not the original wave, is made as small as possible without lowering the resolution of lens. And the signal, proportional to the displacement, is taken on the synchroscope 21 with an error $\pm 0.2\%$. For measuring the transverse vibration of the chip, the front of camera 14 has only to be rotated at right angles.

The lubricating oil (specific gravity 0.874, viscosity 2220 cSt (17.78°C), 91.2 cSt (37.8°C) and viscosity index 144) is used as the seizing liquid for water droplets. 11 is a slide glass for seizing the liquid droplets. Just after the water droplets are seized, they are photographed through a microscope of $\times 50$ magnification, and their diameters are measured. To prevent the evaporation of liquid by the light and heat from the illuminator, an adiabatic filter is also used. This experiment is conducted at liquid temperatures 13~21°C and relative humidities 50~75% using distilled water.

3. Measurement and analysis of vibration

3.1. Relation between sound intensity and resonant frequency

It is known that the larger the emissive frequency (electric input of vibrator P W, sound intensity $I = P/A$ W/cm², sound intensity level $IL = 10 \times \log I/I_0$, $I_0 = 10^{-16}$ W/cm²), the smaller becomes the resonant frequency f_r kHz, from the experiment of constant current circuit. Thus $f_r = 20.91 \sim 20.878$ kHz in the case of sound intensity level $IL = 147.978 \sim 158.648$ dB. The experiment is conducted in this range.

3.2. Measurement of vibration

3.2.1. Longitudinal vibration mode on the vibrating surface

What governs the droplet diameter in the atomisation of liquid by vibration will be the property of liquid, the frequency and the vibration mode in principle.

Now, the amplitudes on the vibrating surface are obtained with accuracy for the mode of longitudinal vibration. It becomes as small as the sound intensity per unit area of vibrating surface when the vibrating surface becomes large in the case of the same sound output.

It is important to measure the amplitudes with accuracy for clearing up the scale of cavitation strength which has been unknown hitherto and for clarifying the mechanism of atomisation etc.. There are various methods for the measurement of amplitudes, but the traditional methods have draw-backs with regard to the response of frequency and the accuracy in the case of ultrasonics. Now, the measurement is conducted under a response of frequency 0~400 kHz and an error $\pm 0.2\%$. For example, the longitudinal vibration waves are shown in Fig.3 for each output (or resonant frequency) at the center of vibrating surface ($r=0$, pin ①).

(1) Non-vibration

(4) $f_r = 20.899 \text{ kHz}$, $IL = 155.5 \text{ dB}$

(2) $f_r = 20.911 \text{ kHz}$, $IL = 146.03 \text{ dB}$

(5) $f_r = 20.866 \text{ kHz}$, $IL = 157.61 \text{ dB}$

(3) $f_r = 20.91 \text{ kHz}$, $IL = 151.9 \text{ dB}$

(6) $f_r = 20.865 \text{ kHz}$, $IL = 158.61 \text{ dB}$

Fig.3 The longitudinal vibration at the center of f vibrating surface.

Fig.4 The longitudinal vibration mode at the vibrating surface.

Now, the sweep time of its horizontal axis is $50 \mu\text{s}$ ($5 \times 10^{-5} \text{ s}$)/div. and Volt/div. of its vertical axis is 0.05 V/div. (0.05 V corresponds to the displacement $5 \mu\text{m}$). Fig.3 (1) is a non-vibration wave but these little displacement amplitudes are indispensable from the need of selecting the optimum resolution of lens. The amplitude is taken as half of vertical height of wave, by deducing the non-vibration amplitude.

Fig.4 shows the mode of longitudinal vibration on the vibrating surface in the radial direction for IL dB and f , kHz. Namely, three apparent nodal circles (radius $r = 2.05 \text{ cm}$, 4.54 cm and 6.85 cm) exist regardless of f and IL , and the liquid is not atomized on these nodal circles. It becomes as large as the longitudinal amplitude C_0 when IL becomes large. The maximum amplitude $C_{0\text{max}}$ is $8.1 \mu\text{m}$ when $IL_{\text{max}} = 158.6 \text{ dB}$ at the circumference of circular disc ($r = 7.8 \text{ mm}$) in this experiment, and $C_{0\text{max}}$ is $39 \mu\text{m}$ when $IL_{\text{max}} = 175.73 \text{ dB}$ at $r = 5.4 \text{ mm}$ in the sonic vibration ($f = 10 \text{ kHz}$) with the circular disc whose radius is 9 mm . And the ratio of both maximum amplitudes is about $1/4.8$. The liquid is vigorously atomized in the loops in the case of large output.

3.2.2. The transverse vibration of chip

Fig.5 shows the transverse vibration which accompanies the longitudinal vibration of the chip at the periphery of vibrating surface. It is known that a complex vibration is compounded in the lower output as in Fig.5. The scales of horizontal and vertical axes are the same as in Fig.3. The more IL increases, the larger the transverse amplitude C_{t0} becomes, and C_{t0} doesn't increase for $IL \geq 158 \text{ dB}$ as in Fig.6. $C_{t0\text{max}}$ is 65% of $C_{0\text{max}}$ in this ultrasonic vibration, but is 3.6 times $C_{t0\text{max}}$ in the case of sonic vibration (10 kHz)⁽⁵⁾. But $C_{t0\text{max}}$ in this experiment is

a lateral amplitude at the periphery of the vibrating circular disc of a large area. Then, it is inferred that the lateral vibration accompanies the deflection vibration and is negligibly small at the center of circular disc.

(1) Non-vibration

(4) $f = 20.899\text{kHz}$, $IL = 155.5\text{dB}$ (2) $f = 20.91\text{kHz}$, $IL = 146.03\text{dB}$ (5) $f = 20.886\text{kHz}$, $IL = 157.61\text{dB}$ (3) $f = 20.907\text{kHz}$, $IL = 151.9\text{dB}$

Fig.5 The horizontal vibration at the end of vibrating surface.

Fig.6 The sound intensity and the horizontal amplitude.

3.3. Equation of vibration and its solution

Now, the deflecting vibration of the circular disc is analyzed and the frequency coefficient and the vibration mode are obtained in the case of the central fixed axis of the circular disc being longitudinally vibrated. A famous study on the deflecting vibration of the circular disc which was fixed with the central part has been carried out in this connection by R. V. Southwell⁽²⁾.

The equation of vibration which is expressed by cylindrical coordinate (r, θ, z) is given as follows.

$$\nabla^2 z + \frac{Eh^2}{12(1-\sigma^2)} \nabla^4 z = 0 \quad (1)$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$. Now, since the vibration is a symmetrical form around the center of circular disc, thus is unrelated to the central angle θ and it has no nodal diameter, $\frac{\partial z}{\partial \theta} = 0$. ρ is a density of material g/cm^3 , E is Young's modulus g/cm^2 , z a longitudinal displacement cm, t time s, h thickness of the circular disc cm and σ Poisson's

ratio. The solution of equation (1) is expressed as follows.

$$z = z_0(r) \cos pt. \quad (2)$$

By putting $12(1-\sigma^2)\rho p^2 / (Eh^2) = \kappa^2$ (where κ is a frequency coefficient and p is a circular frequency (rad/s), equation (1) is modified as follows.

$$\left\{ \frac{d^2}{dr^2} + \left(\frac{1}{r} \right) \frac{d}{dr} + \kappa^2 \right\} \left\{ \frac{d^2}{dr^2} + \left(\frac{1}{r} \right) \frac{d}{dr} - \kappa^2 \right\} z_0 = 0 \quad (1.a)$$

A perfect solution of this equation is expressed as follows.

$$z_0(r) = \alpha J_0(\kappa r) + \beta Y_0(\kappa r) + \gamma I_0(\kappa r) + \delta K_0(\kappa r) \quad (3)$$

where α, β, γ and δ are arbitrary constants which are obtained from the boundary conditions. The frequency coefficient and the vibration mode are obtained from the next boundary conditions.

(Boundary conditions)

If we put $\beta = 0, \delta = 0$ and omit these terms out of the solution (3), solution (3) becomes as follows, for the reason that both the 2nd kind Bessel functions $Y_0(\kappa r)$ and $K_0(\kappa r)$ become infinite

at $r=0$.

$$z_0(r) = \alpha J_0(\kappa r) + \gamma I_0(\kappa r) \quad (3.a)$$

R. V. Southwell has treated the vibration such that the central part of the circular disc is fixed, $z_0 = 0$ at the center $r=0$, and $z_0 = 0, \frac{\partial z_0}{\partial r} = 0$ at $r=b$. But now, we put $z_0 \neq 0$ at $r=0$ and $r=b$, for the reasons that the vibration is a symmetrical form with respect to the center and the fixed axis ($r=b$, horn) of the circular disc is longitudinally vibrated. For the reasons that the boundary is free at the periphery $r=a$ of the circular disc, $z_0 \neq 0$, and the bending moment and the shear stress are zero, the following boundary conditions are obtained.

$$(z_0)_{r=0} = C_0(r, f), \quad (4.a)$$

$$\left(\frac{dz_0}{dr} \right)_{r=0} = 0, \quad (4.b)$$

$$(z_0)_{r=b} = C_0(r, f), \quad (5)$$

$$(z_0)_{r=a} = C_0(r, f), \quad (6.a)$$

$$\left[Eh^3 / \{ 12(1-\sigma^2) \} \right] \left\{ \frac{d^2 z_0}{dr^2} + \left(\frac{\sigma}{r} \right) \frac{dz_0}{dr} \right\}_{r=a} = 0, \quad (6.b)$$

$$\left[Eh^3 / \{ 12(1-\sigma^2) \} \right] \left\{ \frac{d^3 z_0}{dr^3} + \left(\frac{1}{r} \right) \frac{d^2 z_0}{dr^2} - \left(\frac{1}{r^2} \right) \frac{dz_0}{dr} \right\}_{r=a} = 0. \quad (6.c)$$

Equation (3.a) satisfies equation (4.b).

Equation (3.a) becomes as follows under the conditions (4.a), (5) and (6.a).

$$\alpha + \gamma = C_0(0, f), \quad (4.a)'$$

$$\alpha J_0(\kappa b) + \gamma I_0(\kappa b) = C_0(b, f), \quad (5)'$$

$$\alpha J_0(\kappa a) + \gamma I_0(\kappa a) = C_0(a, f). \quad (6.a)'$$

Using the relation of Bessel function:

$$\frac{dJ_n(x)}{dx} = J_{n-1}(x) - \left(\frac{n}{x} \right) J_n(x),$$

$$\frac{dI_n(x)}{dx} = I_{n-1}(x) - \left(\frac{n}{x} \right) I_n(x).$$

the conditions (6.b) and (6.c) become as follows under the equation (3.a).

$$\alpha \{ -\kappa J_0(\kappa a) + (1-\sigma) J_1(\kappa a) / a \} + \gamma \{ \kappa I_0(\kappa a) - (1-\sigma) I_1(\kappa a) / a \} = 0, \quad (6.b)'$$

$$\alpha J_1(\kappa a) + \gamma I_1(\kappa a) = 0, \quad (6.c)'$$

(Decisive equation of frequency coefficient)

The following decisive equation of the frequency coefficient κ is obtained,

when the constants α and γ are eliminated from equation (6.b)' and (6.c)'.

$$\begin{aligned} J_1(\kappa a) &= -\kappa J_0(\kappa a) + \{(1-\sigma)/a\} J_1(\kappa a) \\ I_1(\kappa a) &= \kappa I_0(\kappa a) - \{(1-\sigma)/a\} I_1(\kappa a) \end{aligned} \quad (7)$$

It is convenient that κ is obtained from the intersection of the graphs of the left side and the right side of the equation (7) as in Fig.7.

Fig.7 The decision of the frequency coefficient κ

The circular frequency p (rad/s) for the value of κ is obtained from $\kappa^2 = 12(1-\sigma^2)p^2 / (Eh^3)$, and the frequency of vibration of the normal mode f_p (Hz) of the chip is obtained from $f_p = 1/l_p = p/(2\pi)$. Now, Table 1 is obtained by calculating p and f_p for κ which can be obtained from Fig.7 in the case of $\sigma=1/4$ and $1/3$, on account of $a=7.8/980=0.007959\text{gs}^2/\text{cm}^4$, $E=21 \times 10^9 \text{g}/\text{cm}^2$ and $h=0.6 \text{ cm}$.

Table 1 Number and dimensions of nodal circles vs. frequency

$\sigma = 1/4$			$\sigma = 1/3$			$\sigma = 1/4, 1/3$	
κ	p rad/s	f_p kHz	κ	p rad/s	f_p kHz	n_n	r cm
0.385	13620	2.167	0.385	13987	2.226	1	5.16
0.794	57932	9.22	0.791	59044	9.397	1	3.00
1.156	122795	19.542	1.203	136569	21.735	3	2.05 (2.05) 4.76 (4.54) 6.70 (6.85)

(Vibration mode)

The location and number n_n of the nodal circles are obtained from $z_0=0$, by calculating α and γ in equation (3.a) from equations (4.a)' and (5)' or (4.a)' and (6.a)'. For instance, the following equations are obtained in the case of $\sigma=1/4$ and $\kappa=1.156$, according to equations (4.a)' and (6.a)'.

$\alpha = \{1095.095 C_0(0, f_p) - C_0(7.8, f_p)\} / 1095.095$, (8)
 $\gamma = \{0.09522 C_0(0, f_p) - C_0(7.8, f_p)\} / 1095.095$, (9)
 $\alpha = 7.70 \times 10^{-4}$ and $\gamma = -6.82 \times 10^{-7}$ are obtained from $C_0(0, 20.865) = 0.00077 \text{ cm}$ and $C_0(7.8, 20.865) = 0.00082 \text{ cm}$, now putting $f_p = 20.865 \text{ kHz}$ from Fig.4, because of $f_p = 19.542 \text{ kHz}$ from Table 1. Therefore, the nodal circles (radius r) are calculated in Table 1 by putting $z_0=0$ from equation (3.a). The numerals in the parentheses are the experimental values, and the calculated values are almost approximate to the experimental values. In the same way, other values

are found. Now, since the vibration is a symmetrical form with respect to the center of circular disc, it is unrelated to the central angle θ and it has no nodal diameter. When $z_0(r)$ is found in this way, a perfect solution of equation (1) is obtained from equation (2). Fig.8(1), (2) show the vibration mode which has been experimentally obtained with water directly.

(1) $f_r = 20.899 \text{ kHz}$, $IL = 155.50 \text{ dB}$

(3) $f_r = 20.893 \text{ kHz}$, $IL = 156.64 \text{ dB}$

(2) $f_r = 20.866 \text{ kHz}$, $IL = 158.41 \text{ dB}$

Fig.8 The mode at vibrating surface on the chip.

The radii of three nodal circles are $r = 1.5 \sim 2.4 \text{ cm}$, $4.3 \sim 5.35 \text{ cm}$ and $6.55 \sim 7.2 \text{ cm}$ according to Fig.8, and the nodal circles in Fig.4 and Table 1 fit in these ranges. These nodal circles are concentric rings during the low emissive output, but as the output increases more and more, the outside of a vibrating surface turns into a regular pentagon and the extreme outside is divided into five parts at regular intervals, but the central part remains nearly unchanged. The liquid is atomized at the parts of loops except the nodes. Fig.8(3) shows a photo which directly reflects the sound field distribution on the vibrating surface of chip onto the printing paper. Namely, this was obtained by immersing the printing paper in water, giving the output without weighting under close contact to the vibrating surface, and immediately developing the print. The image of sound field is direct-

ly obtained as the result of the grains of printing paper being destroyed according to the sound intensity and the corresponding part becoming back when the ultrasonics is radiated to the printing paper. The image consists of a concentric circle, and the black parts show the sites where the sound intensity or amplitude is maximum. These three nodal circles are found in accord with the foregoing result.

4. Spray and droplet size distribution

4.1. On the spray

The larger the vibrating surface is, the smaller the sound output per unit area becomes, and the liquid does not begin to atomize so long as the output does not rise in some extent. The vertical distance between the tip of liquid supply tube and the center of vibrating surface is denoted with H . The falling liquid from the supply mouth makes a transparent smooth flow in case of $H=5\text{mm}$ and 20mm , and is unstable in the direction of the flow and is opaque accompanied with a rugged surface owing to the change of flow with the air resistance, but a change of atomization is not especially recognized in case of $H=100\text{mm}$. The vibrating surface is not too much exposed to high temperature owing to the cooling action of falling liquid. The atomizing part changes on the vibrating surface with the output. The spray arises from the central full circle and three circular rings in case of lower output, but as the output increases more and more, the spray arisen from the outside full circular ring turns into an intense spray in five parts at regular intervals in the circular ring.

As regards the liquid thickness which is supplied to the center, in fig.9(1), in case of $IL=151.9\text{dB}$, the liquid in the part of central loop is not atomized because of a thick liquid layer in the part of central loop circle, but some sprays arise in the outside loop circle because of thin proper liquid layer ($0.8\sim 3\text{mm}$). A pattern of the ramification which consists of a group of air bubbles comes out on the vibrating surface in the loop circle. That the mean amplitude in the loop circle is about $5.2\mu\text{m}$ is known from Fig.4. The liquid in the central part of a vibrating surface becomes a hill in Fig.9(2) and sprays from this place, too.

(1) $f_r=20.907\text{kHz}$, $IL=151.9\text{dB}$

(2) $f_r=20.902\text{kHz}$, $IL=154.05\text{dB}$

(3) $f_r=20.899\text{kHz}$, $IL=155.50\text{dB}$

(4) $f_r=20.866\text{kHz}$, $IL=158.41\text{dB}$

Fig.9 The atomization on the chip, $H=20\text{mm}$.

The mean amplitude is about $6.3\mu\text{m}$ in this case. The extended spray particles which are atomized in the central part are successively atomized in the outside loop when the output still increases, where the mean longitudinal amplitude is about $7.8\mu\text{m}$. The extent of spray is small, unlike the chip of sonic vibration and the height of spray decreases toward the periphery in spite of the comparatively large lateral amplitude (the maximum lateral amplitude is 65% of the maximum longitudinal amplitude). This is the reason why the liquid supplied to the center of vibrating surface is atomized at the central part of loop where the lateral vibration is really very small, mainly by the longitudinal vibration in this place. The treatment of spray is insufficient and the thickness of liquid layer becomes large in case of such a horizontal vibrating surface, then the atomisation tends to be prevented.

So, the instantaneous photos (flashing time $1.8\mu\text{s}$) shown in Fig.10 were obtained by increasing the efficiency of atomisation as follows. That is, the vibrating surface was inclined to an angle of $20^\circ\sim 40^\circ$; the liquid layer thickness was adjusted to the proper values ($0.8\sim 3\text{mm}$) and the additional effect of the displace-

ment of radial direction on the periphery of disc was utilized. Then, the following might be considered.

(1) $f_r = 20.913\text{kHz}$, $IL = 142.23\text{dB}$

(2) $f_r = 20.902\text{kHz}$, $IL = 154.05\text{dB}$

(3) $f_r = 20.899\text{kHz}$, $IL = 155.50\text{dB}$

(4) $f_r = 20.866\text{kHz}$, $IL = 158.41\text{dB}$

Fig.10 The atomisation on the inclined chip, $H=20\text{mm}$.

That is, the spray in the central loop is almost vertical to the vibrating surface for the reason that the amplitude in the radial direction is small, but the extent of spray becomes large for reasons that the displacement in radial direction due to the deflecting vibration becomes larger toward the periphery, then the extent of impact-interference and the impact force increase. Fig.10(1) shows that the flowing liquid becomes unstable and undulates according to the mode in Fig.4 in case of lower output, and Fig.10(2) shows an apparent mode pattern and commencement of ato-

misation in each loop. The amplitude z is about $6.3\mu\text{m}$ in this case. Fig.10(3) and (4) show the patterns of atomisation in the case in which the mean amplitudes in loops are respectively about $z=7.6\mu\text{m}$ and $7.8\mu\text{m}$. This is the most desirable condition under which the liquid is successively atomized from the central part to the outside in each loop circle without atomizing the liquid in three nodal circles and the liquid does not droop. When the liquid layer thickness is more suitable, the liquid of a large discharge rate is almost uniformly atomized efficiently. It is the most pertinent method that the liquid is supplied on the radius which connects with the center and the summit of inclined circular disc, so as to pass the center of vibrating surface.

4.2. Mean droplet diameter

Using Sauter's mean droplet diameter which is expressed with the surface area and the volume of droplet, the mean droplet size was expressed by

$$\bar{d} = \sum x_i^3 \Delta n_i / \sum x_i^2 \Delta n_i \quad (10)$$

where n_i is the number of the droplets of the diameter x_i . The droplet size has not a qualitative relation with the vertical distance between the tip of liquid supply tube and the center of vibrating surface $H=5\sim 100\text{mm}$. when the vibrating surface is inclined to an angle of $20^\circ\sim 30^\circ$ in case of the horizontal distance between the periphery of vibrating surface and the catching position of droplets $L=20\sim 120\text{mm}$, the larger L is, the smaller the droplet size becomes⁽⁴⁾. It seems to be due to the effect that the liquid is successively atomized from the center to the outside loop circle of disc. Now, the droplet size of spray was obtained representatively at $L=20\text{mm}$ which was supposed to represent sufficient atomization. In general, the maximum droplet diameter d_{max} is related to the time of combustion, the volume of combustion chamber etc., and poses a problem together with the mean droplet diameter $\bar{d}$. In this experiment,

$$d_{\text{max}} = (1.4 \sim 2.5) \bar{d} \quad (11)$$

This is nearly the same value as with the other general method of atomisation.

4.3. Droplet size distribution

To express the non-uniformity of droplets, the droplet size distribution is obtained. The droplet size distribution is given in Fig.11(a) for $f_r = 20.865\text{kHz}$, discharge ratio $w=1.1\text{cm}^3/\text{s}$ and $n=1166$. The following Nukiyama-Tanasawa's equation⁽¹⁾ was applied as the equation of droplet size distribution.

$$dn/dx = ax^u \exp(-bx^v), \quad (12)$$

$$x_{\text{max}} = \{u/(bv)\}^{1/v}, \quad (13)$$

$$\bar{d}_{th} = b^{1/v} \Gamma\{(u+4)/v\} / \Gamma\{(u+3)/v\}, \quad (14)$$

$$n_{th} = [a/\{v \cdot b^{(u+1)/v}\} \Gamma\{(u+1)/v\}]. \quad (15)$$

where a, b, u and v are the experimental constants. x_{max} is the droplet diameter at which the function dn/dx becomes maximum, $\bar{d}_{th}$ is the theoretical mean droplet diameter and n_{th} is the theoretical total number of droplets. Now, the droplet size distribution becomes as in Fig.11(b) when plotted on a semi-logarithmic section

paper for $u=3$, $v=1$.

Fig.11 The droplet size distribution with chip

It is found that the distribution approximately follows a straight line in the whole range of the droplet sizes and equation (12) is almost suitable as the equation of droplet size distribution in this experiment.

5. Conclusions

On the whole vibrating surface, the amplitudes were precisely measured, and the vibration wave, mode, spray, the droplet size distribution etc. were analyzed, yielding the following conclusions:

- (1) The vibration mode of amplitudes was obtained experimentally and theoretically, then it was shown how experiment and theory agree, and the relation between the mode and the atomisation was made clear.
- (2) It was shown that the mode effect could be utilized by inclining the vibrating circular disc of a large area to an angle of $20^\circ \sim 40^\circ$ to atomize the liquid at a high charge rate, efficiently and uniformly.
- (3) The droplet size distribution is approximately expressed by Nukiyama-Tanasawa's equation in case of $u=3$ and $v=1$.

Finally, author's heartiest thanks are due to Professor Kiyoshi KOBAYASHI, Faculty of Engineering, Shizuoka University for his helpful guidance as well as to Mr. Shōzō TOYAMA to the Honda Giken Motor LTD, Moritani Company and Zimmer OHG Corporation Japan LTD for their co-operations.

References

- (1). Nukiyama, S., and Tanasawa, Y., Trans. Japan Soc. Mech. Engrs. (in Japanese), Vol.5, No.18 (1939-2), p.68.
- (2). Southwell, R.V., Proc. Roy. Soc. London, Vol.101, No.A709 (1922-5), p.133.
- (3). Chiba, C., Bulletin of the JSME, Vol.18, No.118 (1975-4), p.376.
- (4). Chiba, C., and Toyama, S., 2nd Conference on Liquid Atomisation and Spray Systems, (1973-4), p.45.
- (5). Chiba, C., 53rd term Japan Soc. Mech. Engrs., No.760-4 (1976-4), p.25.