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Balancing of a Flexible Rotor*

(The Sixth Report, Some Experiments of Balancing

Irrespective of Characteristics of Supporting Structure)

by SHIMADA Kiyoshi** MIWA Shuzo***

Unbalanced vibration and balancing of a flexible rotor/bearing system have been studied analytically in the preceding paper by the same authors for the case of elastic support with viscous damping. Based on the theoretical results obtained, some experiments are carried out and the analysis is verified experimentally with the following conclusions:

- (1) The vibration of the system due to unbalances is no longer a linear combination of undamped eigen-modes of an elastically supported rotor but that of forced modes of a damped system.
- (2) The bearing- and shaft-vibrations are minimized within the whole speed range up to the operating speed, if the rotor is balanced by means of dynamic- and modal-balancing for a simply supported rotor.
- (3) The rotor can be successfully balanced if bearing- and shaft-vibration is measured at high speed, even though the dynamic characteristics of the supports are not known.

1. Introduction

Unbalanced vibration and balancing of a flexible rotor have been studied in recent years by many scholars and engineers. Because of difficulties of analysis, most of them have paid little attention to the effect of damping and stiffness of the supports upon vibration of the rotor. The authors tried to solve the problem and given an exact solution of vibration response for the rotor/bearing system under influence of stiffness and viscous damping of supports. Results of analysis are presented in the previous paper⁽¹⁾. It tells that the steady state response of a flexible rotor is expressed as a sum of two parts, one corresponding to the response of a flexible rotor under simply-supported end condition and the other being a part influenced by the stiffness and damping in the bearings. Furthermore, an appropriate procedure of balancing such a rotor has been proposed.

This paper deals with some experiments which verify the theoretical results obtained in the preceding paper.

2. Steady state response and balancing of the rotor

2.1 Specification of the system

Fig.1 shows a rotor/bearing system applied in the experiments. The flexible rotor consists of a uniform shaft carrying seven disks of the same size at unequally

spaced axial stations as shown in the figure. Both ends of the shaft are supported by ball bearings of self-aligning type, so that the end condition of the shaft can be regarded as simply-supported on a bearing box. The bearing box is suspended by two parallel leaf springs from the bed, and it is free to vibrate in horizontal plane. This bearing box is furthermore connected to an oil-damper. The stiffness of the supports can be varied by changing the effective length of leaf springs which sustain the bearing box. The magnitude of damping in the oil-damper is also adjustable by design. To perform experiments on an undamped rotor/bearing system, the damper is disconnected from the bearing box.

As seen in Fig.1, the end condition of the rotor considered is simply-supported in horizontal plane. But, there being no coupled motion between the two components of shaft vibration in vertical and horizontal planes, only the latter, i.e. vibration in horizontal plane, is considered in the following experiments as representative of vibration of the rotor on flexible supports with viscous damping. One end of the rotor is connected to a small universal joint shaft for drive, but the overhung part of the rotor is designed as small as possible. Further, the following assumptions are practically acceptable in the rotor/bearing system.

- (i) The influences of gravitational force and gyroscopic effect acting on the disks are neglected.
- (ii) Flexural vibration of the shaft is small compared to shaft length, so that the system is considered to be linear.
- (iii) Mass of the slender shaft itself is divided and considered as excess masses of disks and bearing boxes. Thus, the rotor/bearing system constitutes a discrete system of nine degrees of freedom (concentrated mass

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** Research Engineer, Central Engineering Laboratories,
Nissan Motor Co., Ltd., Yokosuka

*** Professor, Aoyama-Gakuin University.

of seven disks and two bearings).

(iv) Damping exists only in the bearings at both ends of the shaft, and behaves as viscous damping within the operating range of the experiments.

For the system of Fig.1, shaft vibration is easily analysed as shown in the preceding paper.

Nomenclature used in this paper are as follows:

- O-xyz : Co-ordinate system fixed in the space, where z-axis coincides with the shaft axis in static equilibrium.
- M** : Mass matrix, a square matrix of order 9.
- K** : Stiffness matrix, a square matrix of order 9.
- C** : Damping matrix, a square matrix of order 9.
- Φ_r, ω_r : The r-th principal mode of vibration, a column matrix of order 9 and r-th natural frequency of the undamped system where damping is absent ($C_L=C_R=0$) in Fig.1 ($r=1, 2, \dots, 9$)
- Ψ, σ_r : The r-th principal mode of vibration, a column matrix of order 9 and r-th natural frequency to the rotor of Fig.1 under simply-supported end condition ($K_L, K_R \rightarrow \infty$), ($r=1, 2, \dots, 7$)
- κ_a, κ_b : Forced mode, a column matrix of order 9, which is affected by characteristics of the bearings.
- s** : Mass eccentricity, a column matrix of order 9.
- u** : Mass unbalance, a column matrix of order 9.
- X** : Steady state response of each mass, a column matrix of order 9.
- $\epsilon^{(i)}$: The i-th modal mass eccentricity of simply-supported rotor. ($i=1, 2, \dots, 7$)
- $\epsilon^{(a)}, \epsilon^{(b)}$: Modal masses eccentricity for forced modes κ_a and κ_b , respectively.
- M_r** : The r-th modal mass for eigen-mode. Ψ_r ($r=1, 2, \dots, 7$)
- l_i** : Distance of each mass from the left bearing. ($i=1, 2, \dots, 9$)
- T** : The transpose of matrix.

2.2 Steady state response

As shown in the preceding paper, the steady state response of the masses caused by unbalances is described as a linear combination of forced modes for the case of a rotor supported on two flexible bearings with viscous damping,

$$\begin{aligned} X = & \sum_{i=a,b} \frac{(\omega/\tau_i)^2}{1-(\omega/\tau_i)^2+2j\zeta_i(\omega/\tau_i)} \epsilon^{(i)} \cdot \kappa_i \\ & + \sum_{i=1}^7 \frac{(\omega/\sigma_i)^2}{1-(\omega/\sigma_i)^2} \epsilon^{(i)} \cdot \Psi_i \dots\dots\dots (1) \\ = & (\text{magnification factor}) \times (\text{modal eccentricity}) \times (\text{mode of vibration}). \end{aligned}$$

In right hand side of eq.1, the first two terms with subscripts $i=a$ and b are responses which have similar form to two forced modes κ_a and κ_b , where

$$\begin{aligned} \tau_i^2 &= \kappa_i^T K \kappa_i / \kappa_i^T M \kappa_i \quad \epsilon^{(i)} = \kappa_i^T M s / \kappa_i^T M \kappa_i \\ \zeta_i &= \kappa_i^T C \kappa_i / 2(\kappa_i^T M \kappa_i \cdot \kappa_i^T K \kappa_i)^{1/2} \end{aligned}$$

The second terms with subscripts $i=1, 2, \dots, 7$, are to responses with similar form to eigen modes Ψ_i ($i=1-7$) which suffer no influence from the characteristics of the supports. This comes from the fact that, in the present case, damping exists only in the bearing at both ends. It is easily seen that the modal eccentricities for these eigen-modes are $\epsilon^{(i)} = \Psi_i^T M s / \Psi_i^T M \Psi_i$.

The interesting nature and behavior of forced modes are already given in the previous paper. For the purpose of this paper, it is enough to point out that κ_a and κ_b change their form depending on rotational speed ω .

2.3 The condition of balance

Let the maximum operating speed of the rotor be Ω . As stated in the preceding paper, the state of balance of a flexible rotor is practically guaranteed within the whole speed range up to Ω , provided that the rotor is balanced by two-plane-balancing at low speed and balanced by modal balancing up to k-th order of shaft deflection under simply-supported end condition, where k is the lowest number of i that satisfies the fol-

Fig. 1 Test rotor/bearing system

lowing equation $A_i(\omega) = (\omega/\sigma_i)^2 / [1 - (\omega/\sigma_i)^2]$. Residual vibration of the rotor after balancing can be estimated by substituting numerical values of $\epsilon^{(a)}$, $\epsilon^{(b)}$, $\epsilon^{(k+1)} \sim \epsilon^{(7)}$ of residual modal mass eccentricities after balancing, into Eq.1.

The $(k+1)$ -th modal mass eccentricity and magnification factor $A_{k+1}(\omega)$ have an important effect upon the residual vibration. To what extent the residual vibration shall be suppressed depends upon the operating condition of the machine, but, generally speaking, the value of k is determined so as to satisfy the condition of $A_{k+1}(\omega) < 0.2$. For more precise balancing, the condition $A_{k+1}(\omega) < 0.1$ will be preferable.

2.4 Balancing procedure

According to the preceding section the rotor should be balanced by modal balancing for simply-supported eigen-mode Ψ_i . This can be achieved successfully for a rotor supported on a flexible bearing with viscous damping, as stated later. It should be emphasized that balance of a flexible rotor for simply-supported modes can be obtained easily under flexible supporting condition without knowing any dynamic characteristics of the support. On the numerical values of natural frequency σ_i , eigen-mode Ψ_i , under simply-supported condition should be known preliminarily.

Correction planes as many as $(k+2)$ are needed to balance a rotor for dynamic balancing together with modal balancing up to k -th order of shaft deflection for simply-supported eigen-modes. For example, if the rotor is balanced up to second order of shaft deflection, i.e. $k=2$, $k+2=2+2=4$ planes are necessary and sufficient. Then unbalance compensation $U = (U_I U_{II} U_{III} U_{IV})^T$ to be attached to four correction planes I, II, III, IV is determined from the following conditional equation of balance, if the initial unbalance of the rotor $u = Me = (0u_1 u_2 \dots u_7 0)^T$ is known.

$$\begin{pmatrix} U_I \\ U_{II} \\ U_{III} \\ U_{IV} \end{pmatrix} = - \begin{pmatrix} 1 & 1 & 1 & 1 \\ l_I/l & l_{II}/l & l_{III}/l & l_{IV}/l \\ \Psi_{I1} & \Psi_{II1} & \Psi_{III1} & \Psi_{IV1} \\ \Psi_{I2} & \Psi_{II2} & \Psi_{III2} & \Psi_{IV2} \end{pmatrix}^{-1} \begin{pmatrix} \sum_{i=1}^7 u_i \\ \sum_{i=1}^7 u_i l_i / l \\ \epsilon^{(1)} M_1 \\ \epsilon^{(2)} M_2 \end{pmatrix} \dots (2)$$

Modal eccentricities $\epsilon^{(1)}$ and $\epsilon^{(2)}$ in the right hand side of Eq.2 can be obtained from Eqs.49,50 in the literature (1), if left and right bearing vibration X_L and X_R and shaft deflection at two axially different stations $X_{(1)}$ and $X_{(2)}$ are measured. Let ω be a specific testing speed, then $\epsilon^{(r)}$ ($r=1,2$) is calculated as

$$\epsilon^{(r)} = \frac{b_r}{A_r(\omega)} - \frac{1}{M_r} \left[X_L \sum_{i=1}^7 m_i \Psi_{ir} + (X_R - X_L) \sum_{i=1}^7 m_i \Psi_{ir} l_i / l \right] \dots (3)$$

where, coefficients b_1 and b_2 are obtained from the following simultaneous equation

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \Psi_{(1)1} & \Psi_{(1)2} \\ \Psi_{(2)1} & \Psi_{(2)2} \end{bmatrix}^{-1} \times \begin{bmatrix} X_{(1)} - (1 - l_{(1)}/l) X_L - (l_{(1)}/l) X_R \\ X_{(2)} - (1 - l_{(2)}/l) X_L - (l_{(2)}/l) X_R \end{bmatrix} \dots (4)$$

Thus, modal eccentricities $\epsilon^{(1)}$ and $\epsilon^{(2)}$ are estimated from Eqs.3,4, if four data of bearing- and shaft-vibrations X_L , X_R , $X_{(1)}$, $X_{(2)}$ at frequency ω are measured.

Balancing procedure of the flexible rotor on flexible support up to the second mode of eigen-mode consists of three steps: (i) Firstly balance to rotor at low speed, (ii) measure left and right bearing vibrations X_L , X_R and shaft deflections $X_{(1)}$, $X_{(2)}$ at two different axial stations at high speed, (iii) determine unbalance compensation U by Eqs.2,3 and 4. In this way, so-called "One-shot balance" can be achieved, provided that bearing- and shaft-vibrations are measured at a sufficiently high speed over some resonances where the higher components of shaft deflection affect the vibration. If the rotor cannot be accelerated up to a high testing speed owing to, e.g., excess unbalances, balance a rotor at a lower speed for eliminating the lower modal components only, and then accelerate the rotor up to high speed and repeat the preceding steps (ii) and (iii), until the shaft deflection of higher order is eliminated.

Balancing speed ω should be so high that the condition $A_i(\omega) > 0.1 \sim 0.2$ is satisfied, if i -th modal component is to be eliminated. Shaft deflection of higher order does not appear in a low speed range. Further, it should be noted that measurement at or near natural frequency σ_i of a simply-supported rotor is avoided. Because, magnification factor $A_i(\omega)$ being very sensitive to the variation of ω in this speed range, the numerical value of $\epsilon^{(i)}$ calculated by Eq. 3 may induce a large error.

3. Experiment

To verify the theoretical results obtained in the previous paper, some experiments are carried out as for three points, i.e., (1) Does the steady state vibration analysed by using "forced mode" coincide with that of experiment? (2) Can the rotor be balanced irrespective of supporting condition, the possibility of which has been presented by the authors? (3) Even when a rotor is mounted on flexible supports including damping, can the rotor be balanced for simply-supported modes?

3.1 Apparatus

The rotor/bearing system applied in the experiments has already been illustrated in Fig.1. A block diagram of the whole apparatus is shown in Fig.2. One end of the rotor is connected to the drive shaft and can be rotated at any speed between 450 and 4,500 rpm. The vibrations of left and right bearings are detected by electro accelerometer (Endevco, 213E) and Tele-vibrometer (Akashi, AVZ-75), while shaft vibrations are detected by a non-contact capacity type displacement meter (Iwasaki-Tsushinki, MD-31U, MPG-643). Signals from these detectors are analyzed by a frequency analyzer (Akashi, AVA-8) and the synchronous component to rotating speed is separated from the noise which may appear from drive or bearings. Finally, the amplitude and phase lag of vi-

brations due to unbalances are read from a digital indicator. Their vibration to rotational speed ω is plotted by an X-Y recorder (Yokogawa Electric Ltd., M-3078).

In case of measuring shaft vibrations, an equipment specially designed by ourselves is used to eliminate the influence of initial shaft deflection, because a perfectly straight slender shaft is quite difficult to obtain. The linearity of the amplitude for shaft vibration is kept at less than $\pm 3\%$ within the amplitude of 1 mm p-p for all speeds of 450-4,500 rpm, by reforming the detectors to fit our system. Furthermore, these detectors can be slid axially and set at any axial position. Thus the shaft vibration is measured at many axial stations.

Natural frequency and natural mode of vibration are measured by a shaking test using a small vibration-exciter (LING CO. LTD., M201). Characteristics of the oil-damper, i.e. frequency and input-output responses, are also examined and proved to show the feature of viscous damping, within the range of our examination.

Dynamic properties of the bearing, i.e. stiffness and damping, may vary greatly according to the condition of the foundation and installation of the machine. Since it is impossible to perform the experiments for all values of bearing properties, experiments are carried out for a variety of stiffnesses and dampings as shown in Table 1 and Table 2, respectively. Condition of experiment is shown as "CASE I-UD" for example. CASE IV is the simply-supported end condition of the rotor. Stiffness of bearings is so chosen that two or three critical speeds appear within the whole speed range up to 4,500 rpm. In every case, left and right bearing stiffnesses have different values for generality. The magnitude of damping is so chosen that peaks of amplitude appear at the resonance under the test condition of CASE I-D. Experiments are also performed for a special case of undamped system (symbol UD). In undamped condition, residual vibration is exaggerated, so that it is easy to observe the effect of balancing.

Initial unbalances, number of correction planes and their selection are stated as follows. Let the state of balance be guaranteed through the whole speed range up to 4,000 rpm. The values of magnification factor for a simply-supported rotor are calculated as $A_2(Q) = 2.68$, $A_3(Q) = 0.169, \dots$ for $\sigma_1 = 1225$, $\sigma_2 = 4687$, $\sigma_3 = 10,530$ rpm and maximum operating speed $Q = 4,000$ rpm. If the criterion $A_{k+1}(Q) < 0.2$ stated in section 2.3, is applied, shaft deflection shall be eliminated up to mode of $k=2$. Correction planes as many as $k+2=2+2=4$ are thus required. We choose stations of these planes as #1, #3, #5 and

#7 disks, since they are chosen freely. Initial unbalances are attached at #2, #4 and #6 disks, magnitudes of which are so chosen that they affect each of the modal component of vibration in the similar order. For simplicity, unbalances are placed at the same angular position of each disk.

Initial unbalances and their corrections (calculated by Eq.2) used in experiments are shown in Table 3. Modal mass eccentricities of initial unbalances and residual unbalances after balancing are also shown in Table 4, where modal mass eccentricities are for eigen-mode, $\epsilon^{(i)} = \Psi_i^T M e_i / \Psi_i^T M \Psi_i$. As seen in Table 4, the fourth and fifth modal eccentricities become larger after balancing, but they give little influence on vibration response through the whole speed range up to 4,000 rpm, as stated before.

Table 1 Stiffness coefficient (kgf/mm)

	K_L	K_R
CASE I	6	10
CASE II	20	10
CASE III	20	50
CASE IV	∞^*	∞^*

Table 2 Damping coefficient (kgf.s/mm)

Symbol	C_L	C_R
D	9.3×10^{-3}	5.6×10^{-3}
UD	0.0	0.0

Fig. 2 Schematic diagram of the whole system

* The actual value being more than 1,000 kgf/mm, the support condition can be regarded as 'simply-supported'.

3.2 Natural frequency and natural mode

Undamped natural frequencies and natural modes of the rotor on flexible bearings have great influence on the response of the rotor due to unbalances, even when there exists a damping in bearings as stated in the latest paper. Natural frequencies and natural modes are obtained experimentally and analytically for four variations of supporting conditions. Results are expressed in Fig. 3.

In the figure, distance of the neutral line of each modal shape and the bottom line (abscissa) indicate the value of natural frequency. Error of natural frequency against calculated value is within $\pm 5\%$ and good results are obtained for mode of vibration.

3.3 Unbalanced vibration of the system

The vibration of the rotor for the excitation of the initial unbalances is shown in Fig. 4 for the condition of CASE III-D, i. e. relatively hard bearing with damping. Vibration response has been measured at eight different axial positions, i. e. left

and right bearings and six positions on shaft. Since the responses of all masses are not in phase to each other owing to the effect of damping, responses are indicated separately into two components, i. e. in phase component in the left and 90° -shifted component in the right of the figure.

Good coincidence is recognized for the response between measured and calculated values except at the frequency $\omega = 3,560$ rpm, which is very close to the second critical ω_2 for the whole system. Near the critical speed, a slight variation of test frequency or estimation of damping coefficient may result in a large difference between measurement and calculation. First modal component Ψ_1 is predominant in shaft vibration until about 2,500 rpm and then there increases the second modal component gradually with a rising frequency ω . Response near or at the second critical speed ω_2 , see the shape of right row at 4,200 rpm in the figure, almost coincides with the natural mode for simply-supported rotor. This fact is easily anticipated from Eq. 1, which has been derived by using "forced mode" technique of analysis. Thus the response of the flexible rotor on flexible bearing with viscous damping can be described no longer as a linear combination of undamped natural mode but as a forced mode of damped system. In this particular system as shown in Fig. 1, where damping exists only at both ends of the rotor, seven modes in nine forced modes coincide with natural mode of a simply-supported rotor. Hence, it is necessary to consider the state of balance and/or balancing procedure on the basis of "forced mode" expressed in Eq. 1.

Fig. 3 Natural frequency and mode of vibration

3.4 Experiments for the state of balance

This experiment is to verify the state of balance proposed theoretically in the preceding paper. After the initial unbalances of certain known magnitude are attached to the rotor, the bearing- and

Table 3 Initial unbalance and unbalance correction** unit: gram

Disk No.	1	2	3	4	5	6	7
Initial unbalance	—	1.0	—	2.0	—	-1.0	—
Unbalance correction (A)	-0.9	—	-0.5	—	-1.8	—	1.2
Unbalance correction (B)	-0.1	—	-2.3	—	-0.1	—	0.5

Table 4 Modal mass eccentricities*** existing in the unbalances unit: μm

Order of mode	1	2	3	4	5	6	7
Before balancing	53.4	-57.8	52.2	10.1	65.8	-21.8	-57.7
After balancing	0.0	0.0	40.0	45.1	136.2	-33.8	116.0

** Four-plane-balancing (balancing up to second mode) is considered. Correction weights are calculated value. Unbalance mass is attached at the radius of 32mm.

***These are for simply-supported mode.

shaft-vibrations caused by these unbalances are measured. Next, unbalance corrections calculated by the conditional equation of balance, Eq.2, are attached to the rotor, and the bearing- and shaft-vibrations after balancing are again measured. For the four variation of support condition, CASE I to CASE IV, how the state of balance changes is observed.

Initial unbalances and the unbalance corrections have been shown in Table 3. Unbalance correction(A) is for dynamic balancing and modal balancing only for the first modal component of a simply-supported rotor. Here, four correction planes are adopted in this balancing, so that this way of balancing does not affect the second modal component. Unbalance correction(A) is obtained by solving Eq.2, where the second modal mass eccentricity $\epsilon^{(2)}$ is regarded as zero. Observing the residual vibration after balancing for this unbalance correction(A), it is clearly seen that the first modal component of a simply-supported rotor is extremely decreased. If modal balancing up to the second modal component under simply-supported condition is to be performed, unbalance correction(B), for example, is recommendably attached to the rotor. Modal mass eccentricities after this balancing are shown in the bottom line of Table 4.

Experiment is performed in the following manner.

- (i) Balance a rotor previously.
- (ii) Compensate the effect of the initial deflection of the shaft electrically by 'initial deflection canceller'. Compensation is perfect within $3\mu\text{m}$ of amplitude of shaft deflection for speed range of 600-4,500 rpm.

Fig. 4 Response of the system due to unbalance (CASE I-D)

(iii) Measure the bearing- and shaft-vibration for the initial unbalances. Amplitude and phase lag of vibration are plotted by an X-Y recorder for rotating speeds from 600 rpm to 4,500 rpm continuously. In the vicinity of critical speeds, rotational speed is raised rapidly to pass through the critical one whereupon plotting of curves is given up.

(iv) Attach unbalance correction(A) to the rotor and repeat vibration measurement in the same manner as (iii).

(v) Change unbalance correction(A) to (B), and repeat vibration measurement.

(vi) Alter the supporting condition, i.e. stiffness and damping coefficient, as shown in Table 1 and Table 2, then repeat the steps (iii), (iv), (v).

Fig. 5 shows the vibration amplitude before and after balancing at the station of #6 disk for bearing stiffness of CASE I to CASE IV. In plotting these curves, damping is removed (condition UD, i.e. "undamped"), in order to exaggerate residual vibration. There exists a ripple near the critical speed in the figure, which seems to be a phenomenon occurring in a rotor passing through the critical speed range. This ripple is very small compared with the vibration due to unbalances and is out of the scope of our experiment.

At a glance of the figure, good results are recognized as for the balance effect though a small residual vibration still remains especially near the critical speed(s).

Observing the balance effect for the unbalance

Fig. 5 Responses before and after balancing (#6 disk)

correction(A), chain line in the figure, the response for the first modal component of the whole system Φ_1 is found to be almost balanced, though the response for the components of the second and higher order still remain. It may be said that the rotor has been well balanced in this case below the rotational speed of about 1,800 rpm. Calculation of the value of magnification factor $A_2(Q)$ for the simply-supported rotor gives 0.17 for $Q=1,850$ rpm. The response after balancing by unbalance correction(B), thin solid-line in the figure, holds better state of balance over a range of maximum operating speed. From these results, the order k of component of shaft deflection to be eliminated can be decided as a minimum of k to satisfy the condition $A_{k+1}(Q) < 0.2$ as stated previously in section 2.3.

The responses to the damped system are observed in the same manner as described above, and similar results are obtained for the balance effect. Thus it is concluded that a rotor on flexible bearings with damping is well balanced through dynamic balancing followed by modal balancing for the natural modes of a simply-supported rotor. The state of balance thus obtained is guaranteed regardless of the change of support conditions.

3.5 Experiments of balancing for unknown initial unbalances

This experiment is done to verify the possibility of balancing a rotor by measuring the bearing and shaft vibration. The principle of the procedure has been described precisely in the literature(1), the main part of which is mentioned in section 2.4 in this paper. To balance a rotor, dynamic balancing and modal balancing for the

Fig. 6 Balancing effect for unknown unbalance (#6 disk)

Fig. 7 Deviation of the state of balance, depending on the bearing condition (#6 disk)

simply supported mode are necessary. Initial unbalances given in Table 3 are attached to the rotor, but we consider them to be unknown. The condition of experiments is set as CASE I-D. Firstly dynamic balancing is performed at low speed by conventional method, and the rotor is balanced at two planes, #1 and #7 disks. Then high-speed-balancing is tried.

Fig.6 shows an example for balance effect at the station of #6 disk. In the first trial, vibration is measured at two bearings and #2 and #4 disks at 2,000 rpm. in order to determine the unbalance correction from Eq.2,3 and 4. The residual response after balancing, chain line in the figure, illustrates good result and the rotor is well balanced over the whole speed range. It should be stressed that only one trial run of balancing has brought a good state of balance to the rotor over the whole speed range up to 3,500 rpm including three critical speeds ω_1 , ω_2 and ω_3 . The second trial for more precise balancing is done in a similar manner as mentioned above at 4,000 rpm, the result of which is shown with a thin solid-line in the figure. Better state of balance is obtained and the system is completely balanced. Since measuring speed is selected high enough where the higher modal component of shaft deflection is included largely than the first run, better result is obtained.

Fig. 7 shows the stability of the state of balance regardless of the deviation of the bearing condition from initial state CASE I-D to CASE I-UD, CASE II-UD. In the figure, the initial unbalances and/or unbalance correction remain unchanged. Since the damper is removed to plot these curves, residual vibration is exaggerated near critical speeds, but little change is observed in the state of balance over a wide speed range. This is verified our prediction stated before.

4. Conclusion

From experiments on a seven-disked flexible rotor on two flexible bearings with viscous damping the following conclusions are obtained.

(1) The vibration of a damped system due to unbalances is no longer a linear combination of undamped eigen-modes of an elastically supported rotor but that of forced modes of the damped system. Hence, the state of balance of such a rotor system should be determined on the basis of forced modes of the damped system instead of natural mode of undamped system.

(2) The rotor is balanced satisfactorily, if the dynamic balancing is followed by a modal balancing for the modes of simply-supported rotor.

(3) The highest order k of component of shaft deflection to be eliminated within the speed range up to the maximum operating speed Q is determined as the minimum number k satisfying the condition $A_{k+1}(Q) < 0.2$, where $A_i(Q)$ is i -th magnification factor for simply-supported rotor.

(4) $(k+2)$ correction planes are necessary and sufficient to the flexible-rotor-balancing to maintain the state of balance unchanged, irrespective of variation of bearing condition, stiffness and/or damping.

(5) The number of correction planes does not depend on the numbers of critical speeds of the actual system which falls within the operating speed range but it depends on those of simply-supported system.

(6) The rotor can be successfully balanced when simply-supported, even when it is a rotor on flexible bearings with damping, or even when the dynamic characteristics of bearings are not known. For this purpose, it is necessary to measure both bearings and shaft vibrations at axially different k stations.

(7) In this way, so-called "one-shot-balance" can be attained as far as the measuring speed ω keeps the condition $A_k(\omega) > 0.1-0.2$ where k is the order of the maximum modal component of shaft deflection to be eliminated.

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