On the Relation between the Fluctuation of the Rotational Speed of the Workpiece and the Fluctuation of the Frequency of the Self-Excited Machine Tool Vibration*

By Shinichi OHNO** and Toshihiro ARAI***

The instantaneous rotational speed of the workpiece and the instantaneous frequency of the self-excited machine tool vibration are measured by an apparatus developed by the authors based on the demodulation of frequency modulated signals. It is experimentally shown that the rotational speed fluctuates periodically because of the slippage of a belt in the driving train and the frequency fluctuates with similar time-history to that of the rotational speed to maintain a constant cycle per rotation of the workpiece. It is concluded that the similarity of the time-history is brought about by the fact that the frequency follows the fluctuation of the rotational speed. It is finally pointed out that making clear the cause that makes the frequency follow the rotational speed is necessary to get full understanding of the self-excited machine tool vibration.

1. Introduction

The regenerative effect theory has been accepted as a fundamental theory of the self-excited machine tool vibration.¹⁾⁻³⁾ The basic idea of the theory is that the phase difference between the vibratory motion and the cutting force variation that supplies the energy to sustain the self-excited vibration is caused by cutting the waviness on the workpiece surface produced by the vibration at the preceding cutting. The phase difference or in other words the delay in the cutting force variation relative to the vibratory motion was first found by S. Doi.⁴⁾ He developed a theory based on the delay he found, but the cause of the delay he assumed is different from that assumed in the regenerative effect theory. The phase difference in the regenerative effect theory can either excite or damp the vibration. Take the turning operation for example. The phase difference supplies energy to the vibratory system when the cycle of the vibration per rotation of the workpiece takes a value between $m-1/2$ and m , where m is a positive integer. If the energy thus supplied overcomes dampings in the vibratory system, a self-excited vibration will arise. When on the contrary the cycle takes a value between m and $m+1/2$, the energy is consumed and no vibration occurs. The cycle is obviously the number of the wavinesses counted around the periphery of the workpiece. The cycle or the integer m needs to be kept almost the same once the self-excited vibration arises because if the cycle changes from a value between $m-1/2$ and m to for example

a value between $(m+1)-1/2$ and $(m+1)$ it needs to pass through the transitional range between m and $m+1/2$ and the energy will be dissipated there. The regular chatter marks produced on the workpiece by the self-excited vibration evidently show the invariability of the cycle per rotation of the workpiece.

There will then naturally arise the question how the cycle is kept almost the same. It seems that papers based on the regenerative effect theory have made no comment on this question. Some may expect that both the rotational speed of the workpiece and the frequency of the vibration are extremely stable and the quotient of the rotational period divided by the vibration period accidentally takes a value between $m-1/2$ and m . But this idea is mistaken. To keep a constant cycle, the stability of the rotational speed and the frequency needs to be unrealistically high as shown in the appendix. On top of this, if the rotational speed and the frequency are stable enough, the quotient can take with equal probability a value between $m-1/2$ and m and a value between m and $m+1/2$. This leads to the conclusion that the self-excited machine tool vibration is an accidental phenomenon. This conclusion is not supported by authors' general experiences on the self-excited machine tool vibration. The case will be such that the frequency follows the fluctuation of the rotational speed of the workpiece to maintain the same cycle per rotation.

The authors developed an apparatus to measure the instantaneous rotational speed and the instantaneous frequency, and obtained experimental results to support authors' expectation.

2. Measuring Apparatus

2.1 Basis of measurement

The demodulation of frequency modulated signals is the basis of the measurement of the instantaneous rotational speed and the instantaneous frequency.

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** Professor
Institute of Industrial Science, University of
Tokyo, Minato-ku, Tokyo

*** Research Fellow
Japan Automobile Research Institute

The fluctuation of the rotational speed can be converted into a frequency modulated signal in the following way. Assuming that the fluctuation of the rotational speed of the workpiece is sinusoidal, the instantaneous angular speed of rotation can be expressed as

$$\omega(t) = \omega_0 + \Delta\omega \cos pt \quad (1)$$

where $\omega(t)$ is the instantaneous angular speed, and ω_0 is the average angular speed, $\Delta\omega$ is the amplitude of the fluctuation and $p/(2\pi)$ is the frequency of the fluctuation. The angle of rotation is given by integrating Eq. (1) as

$$\phi(t) = \omega_0 t + \frac{\Delta\omega}{p} \sin pt \quad (2)$$

where $\phi(0) = 0$ is assumed. A frequency modulated signal can be produced by a couple of the same spur gears positioned as shown in Fig. 1. One gear is fixed to the workpiece to rotate with it, and another is fixed to the bed of the machine tool with an electric insulating material. If the number of the teeth of the gears is z , the electrostatic capacity of the air-gap between the gears varies z times while the workpiece makes a full rotation. The fundamental component of the capacitance variation is then given by

$$C = C_0 (z\omega_0 t + \frac{\Delta\omega}{p} \sin pt) \quad (3)$$

Equation (3) shows that the capacitance variation is a frequency modulated signal. Therefore, the instantaneous rotational speed of the workpiece $60\omega(t)/(2\pi)$ rpm can be measured by picking up the capacitance variation and then demodulating it. The number of teeth needs to be large to detect high frequency components of the fluctuation. The gears used in authors' experiment have 400 teeth, and the module is 0.3.

Fig. 1

The two gear set-up shown in Fig. 1 is advantageous to reduce the effect of mechanical errors such as pitch error, but the drawback of it is that it needs fine adjustment of the relative position of the gears. If mechanical errors of the gears are negligible, an electromagnetic pick-up positioned near the periphery of the rotating gear can generate an electromagnetic output expressed by Eq. (3), and it needs no fine adjustment of the relative position. As no difference was found between the electrostatic output and the electromagnetic output that were taken at the same time, the two gear set-up shown in Fig. 1 was used only at the developing stage of the measuring apparatus.

The instantaneous frequency can be measured by demodulating the output of the vibration pick-up because the signal whose frequency varies with time is a frequency modulated signal.

2.2 Characteristics of demodulator The block diagram of the demodulator used in the experiment is shown in Fig. 2. The frequency modulated signal is first fed to a high pass filter to screen out very low frequency components produced by the eccentricity of the rotating gear and the drift of the vibrometer. The amplifier and the amplitude limiter are combined to generate a square wave train. It is then differentiated and the positive pulse train thus obtained is used to trigger the monostable multivibrator. The vibrator generates a square wave train with constant height and width and with the same frequency as that of the input signal. The final square wave train is fed to the low pass filter to give an output that is proportional to the frequency of the input signal. The bias is given at the final stage to cancel the direct current component corresponding to the average rotational speed and to amplify the fluctuation itself.

Figure 3 shows the static characteristics of the demodulator that is obtained when $p = 0$ and $z(\omega_0 + \Delta\omega)$ is varied. The width of the final square waves generated by the multivibrator is selectable according to the average rotational speed and the average frequency so as to obtain as high as possible output. Three numerals written near the curves in Fig. 3 denote the upper limit frequencies the curves are applicable to.

Fig. 2

Fig. 3 The static characteristics of the demodulator

Fig. 4 The dynamic characteristics of the demodulator

Figure 4 shows the dynamic characteristics to be obtained when $z\omega_0$ and $z\Delta\omega$ are kept constant and p is varied. A couple of numerals near the curves in Fig. 4 are the cut-off frequencies of the low pass filter shown in Fig. 2.

3. Results of measurement

3.1 Oscillograms Fluctuations were measured in turning and cylindrical grinding. In addition to the rotational speed of the workpiece and the frequency of the self-excited vibration, the rotational speed of the grinding wheel was measured in cylindrical grinding.

Oscillograms shown in Figs. 5 to 8 are typical examples. Self-excited vibrations with substantial amplitude were observed in the experiments of Figs. 5 to 8, and regular chatter marks were generated on the workpiece surface. It is obvious in Figs. 5 to 7 that the rotational speed of the workpiece and the frequency have similar time-histories to each other. This implies that both have similar frequency components. The similarity of the time-history is not so clear in Fig. 8, but the frequency analysis detailed later in this paper will show that fluctuations of the rotational speed of the workpiece and the frequency have similar frequency components even in the case of Fig. 8. Therefore it can be concluded that the condition of the same cycle of vibration per rotation of the workpiece is satisfied (see appendix). It should be noted in Figs. 7 and 8 that the workpiece speed is higher than the practical speed. The self-excited vibration with substantial amplitude arises in cylindrical grinding when the workpiece speed is high and the feed is small.⁵⁾

It should be pointed out that the fluctuation of the rotational speed of the workpiece during cutting operation shows the same time-history as that during idling. Figure 9 is the record of the rotational speed of the workpiece taken in the same experiment as Fig. 8 before the grinding wheel came in contact with the workpiece. Although there is a decrease in the average rotational speed by about 5% due to the grinding force, no essential difference is observed between the waveforms shown in Figs. 8 and 9. This leads to an important conclusion that the fluctuation of the rotational speed is a phenomenon independent of the self-excited vibration and the similarity of the waveforms between the rotational speed and the frequency is brought about by the fact that the frequency follows the fluctuation of the rotational speed.

Fig. 5 Turning

Fig. 6 Turning

Fig. 7 Grinding (Cylindrical, Traversing)

Fig. 8 Grinding (Cylindrical, Plunging)

Fig. 9

Fig. 10

Fig. 11

In authors' experiments the fluctuation of the rotational speed is due to the slippage of a belt in the driving train because the fundamental period of the fluctuation of the rotational speed of the workpiece is the same as the period of circulation of the belt. This is also the case with the fluctuation of the rotational speed of the grinding wheel.

The frequency of the self-excited vibration follows intentional changes in the rotational speed too. Figure 10 shows an example. When the average rotational speed was increased by about 11% during the grinding operation, the frequency of the self-excited vibration increased by about 11%, and when the speed was decreased to the initial value the frequency returned to its initial value. But this is not always the case. The frequency sometimes failed to follow. In Fig. 11 the frequency once increased returned to the initial value while the rotational speed was kept raised. It should be noted in Fig. 11 that the frequency decreased at the end of the oscillogram following a decrease in the rotational speed. The result of Fig. 11 suggests that although the frequency tends to follow the rotational speed large fluctuations of the latter cannot always be followed. Overshootings in the oscillogram of the frequency in Fig. 11 reflect large fluctuations of the frequency.

3.2 Frequency analysis Frequency analysis of the time-histories of the fluctuations shown in Figs. 5 to 8 were carried out to make quantitative estimation of the similarity of the fluctuation. The outputs of the demodulator were recorded by a magnetic tape recorder. Loops of the magnetic tape that bore the signal were made and the outputs of the endless playing of the loops were analysed by a narrow band filter. The endless playing was made at increased tape speeds to achieve better separation of the frequency components. Table 1 gives the results of the frequency analysis of the time-histories. It

Table 1 Results of frequency analysis

	Frequency component Hz	Rate of fluctuation %		Note
		Rotational speed	Frequency	
Fig. 5	7.0	1.95	1.83	f_{BW}
	14.8	0.94	0.88	$2f_{BW}$
Fig. 6	7.3	1.99	1.77	f_{BW}
	14.8	0.96	0.75	$2f_{BW}$
Fig. 7	3.64	0.26	0.20	f_{BW}
	7.3	0.69	0.48	$2f_{BW}$
	10.7	1.59	1.30	$3f_{BW}$
	14.8	2.09	1.47	$4f_{BW}$
	18.8	0.31	0.20	$5f_{BW}$
Fig. 8	2.50	0.29	0.21	f_{BW}
	4.5	0.54	2.09	f_W
	7.5	0.88	0.81	$3f_{BW}$
	10.0	1.21	2.09	$4f_{BW}, f_{BG}$
	14.7	0.80	0.88	$6f_{BW}$
	19.5	0.54	0.98	$8f_{BW}$

f_{BW} : 1/circulation period of workpiece driving belt

f_{BG} : 1/circulation period of grinding wheel driving belt

f_W : Rotational speed of workpiece rps

$2f_{BW}, 3f_{BW} \dots$: 2, 3 ... times of f_{BW}

shows that the rotational speed and the frequency have the same frequency components with similar fluctuation ratio (amplitude of fluctuation/average) to each other. This means that the condition of $\delta f/f = \delta N/N$ is satisfied (see appendix).

4. Conclusions

An apparatus to detect the instantaneous rota-

tional speed and the instantaneous frequency was developed, and measurements of these were carried out in turning and cylindrical grinding by making use of the apparatus. The results obtained experimentally are summarized as follows. (1) The rotational speed of the workpiece fluctuated periodically with a substantial amplitude. (2) The fluctuation of the rotational speed measured during cutting operation was almost the same as that taken during idling although the average speed was decreased because of the cutting force. (3) In authors' experiments the fluctuation of the rotational speed was brought about by the periodic slippage of a belt in the driving mechanism. (4) The time-history of the fluctuation of the frequency observed when the self-excited vibration arose was very similar to the time-history of the fluctuation of the rotational speed of the workpiece. (5) An intentional increase in the average rotational speed by about 11% was followed by the same increase in the average frequency. In some cases the frequency was kept increased, but in others the frequency returned to its initial value while the rotational speed was kept increased.

The conclusions derived from these experimental results are as follows. The fluctuation of the rotational speed is a phenomenon independent of the self-excited machine tool vibration. The frequency of the self-excited vibration follows the fluctuation of the rotational speed, and thus the similarity of the time-histories between the rotational speed and the frequency is brought about.

As shown in the appendix, if there is no difference between the time-history of the fluctuation of the rotational speed of the workpiece and that of the frequency, the cycle of vibration per rotation of the workpiece can be maintained constant, and therefore the phase difference is kept constant between the vibratory motion and the cutting force variation that results from cutting the chatter marks on the workpiece surface generated by the vibration at the preceding cutting. This phase difference must be such as supplies the energy to the vibratory system because the self-excited vibration arises.

From the discussions above it can be concluded now that the phase difference that supplies the energy to sustain the self-excited vibration is not caused by an accidental combination of the rotational speed with the frequency, but is brought about by the fact that the frequency follows the rotational

speed. This phenomenon has not been taken into account by the regenerative effect theory yet. The mechanism to make the frequency follow the rotational speed is not known. Making clear this mechanism will be the right way of getting full understanding of the self-excited machine tool vibration.

Appendix

The cycle n of the vibration per rotation of the workpiece is given by

$$n = 60f/N \quad (4)$$

where N is the rotational speed of the workpiece in rpm and f is the frequency of the self-excited vibration in Hz. When the rotational speed varies by δN and the frequency varies by δf , the fluctuation of the cycle δn is given by

$$\delta n = 60f/N(\delta f/f - \delta N/N) = n(\delta f/f - \delta N/N) \quad (5)$$

Putting into Eq. (5) $N = 132$ rpm and $f = 682$ Hz of Fig. 5 gives

$$\delta n = 0.455 \delta f - 2.35 \delta N$$

This numerical example shows that if the rotational speed N and the frequency f vary independently of each other both fluctuations δN and δf must be kept very small to maintain the cycle n between $m-1/2$ and m or in other words to maintain δn less than $1/2$. But it is seen from Eq. (5) that if the fluctuation rate of the rotational speed $\delta N/N$ is the same as the fluctuation rate of the frequency $\delta f/f$ the cycle of vibration per rotation of the workpiece can be kept constant however large they may fluctuate.

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