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On the Control of Attitude of Biped Locomotion by Linear-Motion Balancer*

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A biped locomotion has been developed for the purpose of its application to transport machines. A leg-mechanism has been designed to realize a horizontal walking at a constant speed. A new control system has been also composed of a linear-motion balancer in order to avoid the vertical fluctuations of the gravitational center of the body and to decrease the space occupied by the balancer. The conditions of dynamic stability and the determination of parameters of the system have been discussed both theoretically and experimentally.

1. Introduction

Most of recent studies on the stabilities and control systems of biped locomotion are concerned with machines or mechanisms based on the human walking or lower limb prostheses, and they contribute little to the legged transport machines. For example, the stable gaits of the traditional biped locomotion are mainly secured by the controls⁽¹⁾ of the angular motions of the bodies about horizontal axes or by inverted pendulums^{(2), (3)}.

For the transport machines, angular motions about horizontal axes must be avoided and inverted pendulums are undesirable because they occupy large spaces. Thus, we have developed a leg-mechanism which supports the body upright and a control system which occupies far less space than the traditional inverted pendulum.

2. Leg-mechanism

The leg-mechanism must satisfy at least the next three conditions to realize a walk on a flat surface.

- The locus of the foot traced on the body is a closed curve.
- The part of the above locus where the foot is in contact with the ground must be located outside the domains occupied by moving links of the mechanism.
- The time interval corresponding to the above part of the locus must be greater than a half period of walking.

For transport machines, the next condition is further desirable to realize a constant speed gait and to avoid vertical motions of the body which causes fluctuations of input torques.

- The part of the locus of the foot traced on the body where the foot is in contact with the ground is a straight line whose derivative with respect to the displacement

of the driving link is constant.

The oscillating block slider crank mechanism for conchoidal generation of a straight line may be used as a mechanism satisfying the above conditions.

A point E on the coupler BD of the oscillating block slider crank mechanism ABD shown in Fig.1 traces a closed path a part of which is approximately straight when the crank AB makes a complete revolution with respect to the fixed link DA. Let us fix a co-ordinate system o-xy on the plane where the mechanism is located such that the y-axis coincides with the straight line AD. We shall put $dx/d\phi = 2a$, $dy/d\phi = 0$, $d^2x/d\phi^2 = d^2y/d\phi^2 = 0$ ($k=2, 3, 4$) at $x=y=0$ in order that the path E(x,y) of the coupler point E may become an approximately straight-line in the vicinity of the origin o. Where ϕ is the displacement angle of the crank AB measured from the y-axis (briefly, the crank angle) and a is the length of the crank.

By modifying the fifth-order approximation⁽⁴⁾ developed for the synthesis of the plane four-bar path-generating mechanism, we may obtain the mechanism shown in

Fig.1 Leg-mechanism

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Fig.1, whose coupler point E is expressed as follows.

$$x(\phi) = a \left(\frac{9}{\sqrt{5}} \frac{\sin \phi}{\sqrt{1-0.8 \cos \phi}} - \sin \phi \right) \dots (1)$$

$$y(\phi) = a \left(\frac{9}{\sqrt{5}} \frac{2 - \cos \phi}{\sqrt{1-0.8 \cos \phi}} + \cos \phi - 8 \right) \dots (2)$$

This path is symmetric with respect to the y -axis and the half part of it is shown in Fig.2. The marks O on the path indicate the positions of the point E at every 10° of ϕ in the first half cycle. This figure implies that the mechanism of Fig.1 satisfies the above conditions (a), (c) and (d). Then a congruent mechanism A'B'D' is added to the mechanism ABD and both of them are driven synchronously so that the condition (b) may be satisfied.

A biped locomotion may be obtained by attaching two leg-mechanisms at points A, A', D and D' to both sides of the body which carries the power source and the control system. When the leg-mechanisms are driven with a phase angle of 180° , the locomotion may take place with two feet touching alternately the ground surface, and the body may move horizontally at almost constant speeds. When the leg-mechanisms are driven without the phase angle, the locomotion may take place with the feet and the bottom of the body touching alternately the ground. In the following discussion, we choose the latter case because we are mainly interested in revealing the dynamic behaviour of a new control mechanism of attitudes of locomotion.

3. Servomechanism with linear-motion balancer

3.1 Linear-motion balancer

We shall control the moment about the edge line of the foot being in contact with the ground by a balancer⁽⁵⁾ moving on a linear guide rail fixed horizontally on the body of the biped machine. Figure 3 shows the structure of the balancer. A wire is wound around the pulley fixed on the output shaft of the servomotor with a reduction-gear train which is installed on a truck and the wire is stretched parallel to the guide rail so that the truck may perform a linear motion with the rotation of the servomotor. In this mechanism, the truck itself and other elements set up on the truck function as the linear-motion balancer and its center of gravity and velocity are detected by a potentiometer and a tachometer generator fixed directly to the shafts of the pulley and the servomotor respectively.

Meanwhile, there exists the following

Fig.2 Coupler curve of the oscillating block slider crank mechanism ABD

dilemma in the control system with an inverted pendulum. Adopting a larger swing angle of the pendulum, we can make the space occupied by the pendulum smaller but the vertical displacement of the gravitational center becomes greater. Adopting a smaller swing angle, we can make the vertical displacement of the gravitational center smaller but the space of the pendulum becomes greater and the gravitational center becomes higher.

As for the control system having a linear-motion balancer, the space occupied by the compensating weight and its guide rail is far less than the one with an inverted pendulum and the center of gravity of the compensating weight moves only in a horizontal direction. Further, the weights of the servomotor and other elements function as the compensating weight. Therefore, we can obtain easily a large ratio of the compensating weight to the total weight of the locomotion mechanism.

3.2 Equation of motion

[Nomenclature] D_M : Damping coefficient of the servomotor. e_i : Input voltage of the servomechanism for driving the balancer. e_o : Output voltage of the potentiometer for detecting the relative displacement of the balancer. J_M : Total moment of inertia of the servomotor, tachometer generator, reduction gears and pulley about the shaft of the servomotor. K_A : Gain of the servoamplifier. K_M : Torque coefficient of the servomotor. K_F : Coefficient of the potentiometer in the tachometer feedback circuit. K_T : Gain of the tachometer generator.

- ① Guide rail, ② Truck, ③ Servomotor,
- ④ Reduction gear, ⑤ Pulley, ⑥ Wire,
- ⑦ Potentiometer, ⑧ Tachometer generator

Fig.3 Structure of the linear motion balancer

K_s : Ratio of θ to s . K_θ : Ratio of θ to e_0 .
 s : Displacement of the balancer measured from a reference position on the guide rail.
 W_c : Weight of the balancer. θ : Angular displacement of the rotor of the servomotor.

Using the above nomenclature, we can describe the motion of the balancer by the next equation.

$$s + 2\zeta\omega_n s + (\omega_n^2 - K_f\dot{\psi}^2)s = K_s e_0 + K_f \dot{\psi} \quad \dots (3)$$

Where

$$\zeta = \frac{1}{2} (D_M + K_A K_M K_T K_P) / \sqrt{K_A K_M K_\theta (J_M + K_s^2 \frac{W_c}{g})}$$

$$\omega_n = \sqrt{K_A K_M K_\theta / (J_M + K_s^2 \frac{W_c}{g})}$$

$$K_s = K_A K_M K_\theta / (J_M + K_s^2 \frac{W_c}{g})$$

$$K_f = K_s^2 \frac{W_c}{g} / (J_M + K_s^2 \frac{W_c}{g})$$

The variable $\dot{\psi}$ in the right side of Eq.(3) expresses the magnitude of the acceleration of the balancer relative to the guide rail.

3.3 Input voltage

We shall divide a cycle of walking into five periods; (i) the first is the period which starts from the instant of the foot coming in contact with the ground and ends at the instant of the body leaving the ground, (ii) the second is the period while the body performs a uniformly accelerated motion, (iii) the third is the period while the body performs a uniform motion, (iv) the fourth is the period which starts from the instant of the body coming in contact with the ground and ends at the instant of the foot leaving the ground, and (v) the fifth is the period while the foot returns to the initial position.

About the leg-mechanism obtained in the section 2, the voltage for static balance, which is proportional to the displacement of the linear-motion balancer, varies with respect to the crank angle ϕ as shown in Fig.4 while the leg-link is in contact with the ground. Therefore we shall give the input voltage such that the balancer may dwell ahead of the body for the first and second periods mentioned just above, move uniformly for the third period, dwell behind the body for the fourth period and stand still for the fifth period. Then, the moment about the center line of the foot may be kept within a small value while the foot is in contact with the ground, and the locomotion may progress on flat surfaces in the state of the upright attitude. Thus, we have

$$e_i = \begin{cases} K_\phi(\phi_z - \phi_s) & (\phi_0 \leq \phi < \phi_R) \\ K_\phi(\phi_z - \phi_s) & (\phi_R \leq \phi < \phi_s) \\ K_\phi(\phi_z - \phi) & (\phi_s \leq \phi < \phi_T) \\ K_\phi(\phi_z - \phi_T) & (\phi_T \leq \phi < \phi_U) \\ 0 & (\phi_U \leq \phi < \phi_0 + 2\pi) \end{cases} \quad \dots (4)$$

Where K_ϕ is a proportional constant to convert ϕ into e_i , and ϕ_0 , ϕ_R , ϕ_s , ϕ_T , and ϕ_U indicate the values of ϕ at the instants when each period of a walk mentioned at the beginning of this section starts, and ϕ_z is the value of ϕ at which e_i vanishes in the third period [ϕ_s , ϕ_T].

In the case where the initial and final values of ϕ of each period and other parameters are determined by the dynamic conditions described in the next section, the input voltage may be given by a polygonal-line function which cancels the steady-state error of the velocity. An example of the input voltage is shown in Fig.4, which shall be used in experiments of the section 5. ϕ_R' in Fig.4 is a value determined from Eq.(5) derived in the next section, and ϕ_s' and ϕ_T' are values of ϕ at which an actual uniform motion of the body starts and ends respectively.

4. Conditions for dynamic stability

4.1 Relative positions of feet

While the leg-link is located ahead of the body as shown in Fig.5, the relative positions of the leg-link and the body with respect to the ground may be classified into eight kinds of states as shown in Fig.6, where L_T , L_H ; B_T , B_H indicate the front and rear edges, i.e. the tiptoes and the heels, of the bottoms of the leg-link and the body which are here considered to be two parallel feet.

The relative positions of the leg-link and the body with respect to the ground while the leg-link is located behind the body may be similarly classified into eight kinds of states $P1^* \sim P8^*$, corresponding to $P1 \sim P8$ of Fig.6. Thus, we can express a full cycle of a gait by combining some of the sixteen states. A typical combination for a stable gait of the biped locomotion may be described as

$$P1 \rightarrow P2 \rightarrow P8 \rightarrow P5 \rightarrow P5^* \rightarrow P7^* \rightarrow P8^* \rightarrow P1^* \rightarrow P1$$

4.2 Conditions for maintaining upright attitude

Let ϕ_R denote the value of the crank angle at the instant when the perpendicular distance between the bottoms of the body and the leg-link vanishes. Moreover, let K_x denote the proportional constant between the crank angle and the displacement of the body while the body makes a uniform motion. Then the crank angle ϕ_R' when the uniform motion begins may be given by the following equation.

Fig.4 Example of the input voltage for the servomechanism with a linear motion balancer

$$\phi_R' = \pi - (x(\phi_R) + \mu b) / Kx, \quad -0.5 < \mu < 0.5 \quad (5)$$

Where b is the length $L_T L_H$ of the bottom of the leg-link which is bisected perpendicularly by EE' .

Then the position of the compensating weight which balances statically the body with respect to the fulcrum which is deviated by μb from the center of the bottom of the leg-link becomes as follows.

$$s_R = \frac{W_B + W_C}{W_C} Kx(\pi - \phi_R')$$

Where W_B is the weight of the body. We shall call s_R the objective position of the balancer.

The values of $K\phi$ and $\phi_z - \phi_s$ in Eq. (4) are given such that

$$K\phi(\phi_z - \phi_s) = \frac{K_z}{K_t} s_R \quad (6)$$

Now, we introduce a stationary rectangular coordinate system $O.XY$ whose X -axis is a parallel line to the locomotive direction, and Y -axis is the center line of the leg-link EE' . Denoting the vertical force and the moment about Z -axis by F_Y and M_Z respectively, the condition where the locomotion model may perform a uniformly accelerated motion and a uniform motion in the state of upright attitude becomes as follows.

$$-0.5b|F_Y| < M_Z < 0.5b|F_Y| \quad (7)$$

4.2.1 Accelerated motion Denoting the crank speed by ω , the acceleration of the body becomes equal to a constant value of $Kx\omega^2/(\phi_s' - \phi_R)$. Because the period while the body is accelerated is very short, we may describe the X - and Y -coordinates of the gravitational center G_B of the body by the next constants.

$$-Kx(\pi - \phi_R') \quad \text{and} \quad f + l - 8a$$

Then, F_Y is a constant and M_Z is the sum of a constant term and a damped oscillation term.

Fig. 5 Skeleton diagram of the biped mechanism

lation term. Therefore the condition for maintaining the upright attitude is secured by the condition that the inequality of Eq. (7) into which F_Y and M_Z are substituted may hold for any value of the time t . If the body begins to make a uniformly accelerated motion at the instant when the balancer arrives at the objective position, this condition is described as

$$\sqrt{(\lambda\omega_n^2 + 1)^2 - 4\lambda\zeta^2\omega_n^2} < \frac{\sqrt{1 - \zeta^2}(\eta_1\omega_n^2 - 1)}{\sqrt{(\zeta\eta_2)^2\omega_n^4 + 2\zeta\eta_2\omega_n^2 + 1}} \quad (8)$$

Where

$$W = W_B + W_C + W_L, \quad \zeta = \exp\left\{-\frac{\zeta}{\sqrt{1 - \zeta^2}}\left(\frac{\pi}{2} + \tan^{-1}\frac{\zeta}{\sqrt{1 - \zeta^2}}\right)\right\}$$

$$\lambda = (f + h + l - 8a)/g, \quad \lambda^* = \left(f + \frac{W_C}{W_B + W_C}h + l - 8a\right)/g$$

$$\eta_1 = \frac{1}{K_f} \frac{W_B + W_C}{W_C} \left\{ \left(2\mu + \frac{W}{W_B + W_C}\right) \frac{b}{2} \frac{\phi_s' - \phi_R}{Kx\omega^2} - \lambda^* \right\},$$

$$\eta_2 = \frac{1}{K_f} \frac{W_B + W_C}{W_C} \frac{(\phi_s' - \phi_R)(\pi - \phi_R')}{\omega^2}$$

Here, W_L is the weight of the leg-link and h is the perpendicular distance between the center of gravity G_B of the body and the trajectory of the center of gravity G_C of the balancer.

Denoting the left side of Eq. (8) by $U_1(\omega_n)$, $U_1(\omega_n)$ comes to monotonously increase when $\zeta \leq 1/\sqrt{2}$ and it becomes a function having the minimum $2\zeta\sqrt{1 - \zeta^2}$ at $\omega_n = \sqrt{(2\zeta^2 - 1)/\lambda}$ when $\zeta > 1/\sqrt{2}$ as shown in Fig. 7.

Denoting the right side of Eq. (8) by $U_2(\omega_n)$, it becomes a monotonous function when $\eta_1 > 0$ as shown in Fig. 8.

4.2.2 Uniform motion When the body moves at a constant velocity $Kx\omega$, the X - and Y -coordinates of its center of gravity G_B becomes as follows.

$$Kx(\omega t - \pi) \quad \text{and} \quad f + l - 8a$$

Then F_Y is expressed by the sum of a uniform variation term and a constant term, and M_Z is expressed by the sum of a damped oscillation term, a uniform variation term and a constant term. Therefore the condition for maintaining the upright attitude is also secured by the condition that the inequality of Eq. (7) into which F_Y and M_Z

Fig. 6 Relative positions of the bottoms of the body and leg-link with respect to the ground (in the case where the leg-link is located ahead of the body)

are substituted may hold for any value of the time t . Putting

$$\phi_z - \phi_s = \pi - \phi_s', \dots\dots\dots (9)$$

the above condition may become as follows.

$$\sqrt{(\lambda\omega_n^2+1)^2-4\lambda^2\zeta^2\omega_n^2} < \frac{\sqrt{1-\zeta^2}}{u(\omega_n)} (\gamma_1\omega_n+2\zeta) \dots\dots (10)$$

Where

$$u(\omega_n) = \sqrt{\gamma_1^2\omega_n^2+2\zeta\gamma_1(\gamma_2-1)\omega_n+\gamma_2^2+2(1-2\zeta^2)\gamma_2+1}$$

$$\gamma_1 = \left\{ \frac{Wc}{W_B+Wc} \frac{s_s}{Kx} - (\phi_z - \phi_s') \right\} / \omega, \quad \gamma_2 = \frac{Wc}{W_B+Wc} \frac{s_s}{Kx\omega}$$

$$\gamma_3 = \left\{ \frac{W}{W_B+Wc} \frac{b}{2Kx} \pm (\phi_s' - \phi_s) \right\} / \omega.$$

In the above equations, s_s and s_s' are respectively the displacement and velocity of the balancer at the instant when ϕ takes the value of ϕ_s , and the upper and lower signs of γ_3 are adopted respectively in accordance with $1/\omega_n > (\phi_s' - \phi_s)/2\zeta\omega$ and $1/\omega_n < (\phi_s' - \phi_s)/2\zeta\omega$.

Putting the right side of Eq.(10) as $U_1(\omega_n)$, $U_1(\omega_n)$ has a maximum value only in the case where

$$(\gamma_2-1)\gamma_1/\gamma_2+2 < 0. \dots\dots\dots (11)$$

Moreover $U_1(\omega_n)$ becomes a straight line when $\gamma_1=0$. The curves of $U_1(\omega_n)$ are shown in Fig.9 for the cases where ζ takes 0.6 or 0.8 and $(\gamma_1, \gamma_2, \gamma_3)$ takes $(-0.2, -1.0, 1.0)$, $(-0.2, 0.0, 1.0)$, $(0.0, -1.0, 1.0)$, $(0.0, 0.0, 1.0)$, $(0.2, -1.0, 1.0)$ or $(0.2, 0.0, 1.0)$.

4.2.3 Determination of parameters of servomechanism The shape of $U_1(\omega_n)$ changes considerably with λ , but it is scarcely affected by ζ as is observed in Fig.7. $U_2(\omega_n)$ approaches a constant of $\sqrt{1-\zeta^2} \times \eta_1/\xi\eta_2$ when $\omega_n > 1/\sqrt{\eta_1}$ and this value changes considerably with ζ when η_1/η_2 is a constant. As far as Eq.(11) holds, $U_1(\omega_n)$ becomes maximum at $\omega_n = \omega_n^*$ and it takes a larger value than $\sqrt{1-\zeta^2} \gamma_1/|\gamma_1|$ when $\omega > \omega_n^*$.

Fig.7 Variation of $U_1(\omega_n)$ with respect to ζ and λ

$$\omega_n^* = -\frac{\zeta(\gamma_2-1)}{\gamma_1} - \frac{(1-\zeta^2)(\gamma_2+1)^2\gamma_3}{\zeta\gamma_1(2\gamma_1+(\gamma_2-1)\gamma_3)}$$

Therefore the procedure to determine the parameters of the servomechanism becomes as follows.

(1) Determine the values of ϕ_s and ϕ_z such that Eqs.(9) and (11) may hold and the value of $|\gamma_1|$ may become as small as possible and determine the value of $K\phi$ according to Eq.(6).

(2) Determine the value of ζ such that $\xi(\zeta) = \eta_1|\gamma_1|/\gamma_2\gamma_3$ may hold.

(3) Determine the value of ω_n between the two values of ω_n with which $U_1(\omega_n)$ becomes 1 when $\gamma_1=0$ and $U_1(\omega_n)$ intersects with the asymptote of $U_2(\omega_n)$.

4.3 Condition for returning to upright attitude

When the body leaves the ground or the leg-link touches the ground, the state-change of the biped locomotion may be expressed as

$$P1 \rightarrow P2 \rightarrow P8 \rightarrow P5 \quad \text{or} \quad P1 \rightarrow P4 \rightarrow P6 \rightarrow P5.$$

If the balancer arrives at the objective position too early, P7, in place of P5, may follow P8 and the mechanism may fall down forward.

When the leg-link leaves the ground or the body touches the ground, the state-change may be expressed as

$$P5^* \rightarrow P7^* \rightarrow P8^* \rightarrow P1^* \quad \text{or} \quad P5^* \rightarrow P4^* \rightarrow P3^* \rightarrow P1^*.$$

In this case, if the balancer returns to the initial position too late, the biped mechanism may fall down backward after the state of P3*.

Thus, we shall investigate the conditions where the states P5 and P1* may follow the states P7 and P3* respectively.

4.3.1 Approximate Equation of motion

We shall denote the length of the bottom of the body, the distance from the bot-

Fig.8 Variation of $U_2(\omega_n)$ with respect to ζ , γ_1 and γ_2

tom of the body to the driving axis and the distance from the coupler point E to the center of gravity of the leg-link by c , e and p respectively as shown in Fig.5. Moreover we shall denote the moments of inertia of the body, the balancer and the leg-link about their centers of gravity by J_B , J_C and J_L respectively.

Then, assuming that $|\dot{\psi}| \ll 1$ at P7 and P3*, we can approximate the equation of motion of the body as follows.

$$\ddot{\psi} - \omega_p^2 \psi = e^{-\zeta \omega_n t} (q_1 \cos \omega_n t + q_2 \sin \omega_n t) + q_3 \dots (12)$$

Where

$$q_1 = -W_C \left\{ (1 + \lambda \omega_n^2) \left(s_p - \frac{C_p}{\omega_n^2} \right) + 2\lambda \zeta \omega_n s_p \right\} / J_p,$$

$$q_2 = -\frac{W_C}{\sqrt{1-\zeta^2} J_p} \left[\zeta (1 - \lambda \omega_n^2) \left(s_p - \frac{C_p}{\omega_n^2} \right) + (1 - (2\zeta^2 - 1) \lambda \omega_n^2) \frac{s_p}{\omega_n} \right],$$

$$\omega_p = \sqrt{(W_B(e+f) + W_C(e+f+h) + W_L(l-p)) / J_p},$$

$$\omega_n = \sqrt{1 - \zeta^2} \omega_n.$$

For the state P7, we have

$$q_3 = \left\{ (W_B + W_C + W_L) \frac{b}{2} + (W_B + W_C) x_p - W_C \frac{C_p}{\omega_n^2} \right\} / J_p,$$

$$J_p = J_B + \frac{W_B}{g} \left\{ \left(x_p + \frac{b}{2} \right)^2 + (e+f)^2 \right\} + J_C + \frac{W_C}{g} \left\{ \left(x_p + \frac{b}{2} \right)^2 + (e+f+h)^2 \right\} + J_L + \frac{W_L}{g} \left\{ \frac{b^2}{4} + (l-p)^2 \right\}.$$

For the state P3*, we have

$$q_3 = - \left\{ (W_B + W_C + W_L) \frac{c}{2} + W_L x_p + W_C \frac{C_p}{\omega_n^2} \right\} / J_p,$$

$$J_p = J_B + \frac{W_B}{g} \left\{ \frac{c^2}{4} + (e+f)^2 \right\} + J_C + \frac{W_C}{g} \left\{ \frac{c^2}{4} + (e+f+h)^2 \right\} + J_L + \frac{W_L}{g} \left\{ \left(x_p + \frac{c}{2} \right)^2 + (l-p)^2 \right\}.$$

In the above equations, x_p , s_p and s_p are the values of x , s and s at the instant when the body begins to rotate about the edge line of the bottom of the leg-link. C_p is the forced input of the balancer and is given as the product of K_r and the input voltage at this instant.

4.3.2 Condition for returning to upright attitude Putting $\psi = \psi_p$ and $\dot{\psi} = \dot{\psi}_p$ at $t=0$, we may have the solution of Eq.(12) as follows.

low.

$$\psi = d_1 \cosh \omega_p t + d_2 \sinh \omega_p t + e^{-\zeta \omega_n t} (d_3 \cos \omega_n t + d_4 \sin \omega_n t) + d_5 \dots (13)$$

Where

$$d_1 = \dot{\psi}_p + q_3 / \omega_p^2 + W_C \left[(1 - \lambda \omega_p^2) \omega_n^2 - (\omega_n^2 + \omega_p^2) \lambda \omega_n^2 \right] s_p^* + 2\zeta \omega_n (1 - \lambda \omega_p^2) s_p / (J_p U),$$

$$d_2 = \dot{\psi}_p / \omega_p - W_C \left[2\zeta \omega_n^3 (1 - \lambda \omega_p^2) s_p^* + (\omega_n^2 + \omega_p^2) + (\omega_n^2 - (4\zeta^2 - 1) \omega_p^2) \lambda \omega_n^2 \right] s_p / (J_p U \omega_p),$$

$$d_3 = -W_C \left[(1 - \lambda \omega_p^2) \omega_n^2 - (\omega_n^2 + \omega_p^2) \lambda \omega_n^2 \right] s_p^* + 2\zeta \omega_n (1 - \lambda \omega_p^2) s_p / (J_p U),$$

$$d_4 = -W_C \left[\zeta (1 - \lambda \omega_p^2) \omega_n^2 - (\omega_n^2 - \omega_p^2) \lambda \omega_n^2 \right] s_p^* + [(2\zeta^2 - 1) \omega_n^2 - \omega_p^2 - (\omega_n^2 - (2\zeta^2 - 1) \omega_p^2) \lambda \omega_n^2] s_p / (J_p U \sqrt{1 - \zeta^2}),$$

$$d_5 = -q_3 / \omega_p^2,$$

$$s_p^* = s_p - C_p / \omega_n^2, \quad U = \omega_n^4 - 2(2\zeta^2 - 1) \omega_n^2 \omega_p^2 + \omega_p^4.$$

The conditions that the states P7 and P3*, where the attitude angle and its velocity of the biped locomotion are $\dot{\psi}_p$ and $\dot{\psi}_p$ respectively, can be followed by the states P5 and P1* respectively are equivalent to the condition that the angular velocity $\dot{\psi}$ after enough time passes becomes respectively positive and negative, $d_1 + d_2 > 0$ and $d_1 + d_2 < 0$, holding respectively. Therefore the conditions where the biped mechanism may return to the upright attitude results in the following equations.

For the state P7,

$$r_1 (s_p - C_p / \omega_n^2) + r_2 s_p + r_3 < 0 \dots (14)$$

For the state P3*,

$$r_1 (s_p - C_p / \omega_n^2) + r_2 s_p + r_3^* > 0 \dots (15)$$

Where

$$r_1 = - \left\{ \lambda \omega_n^4 + 2\zeta (1 / \omega_p - \lambda \omega_p) \omega_n^3 - (4\zeta^2 - 1 - \lambda \omega_p^2) \omega_n^2 + \omega_p^2 \right\} / U,$$

$$r_2 = - \left\{ (\lambda / \omega_p) \omega_n^4 + (1 / \omega_p - (4\zeta^2 - 1) \lambda \omega_p) \omega_n^3 - 2\zeta (1 - \lambda \omega_p^2) \omega_n + \omega_p \right\} / U,$$

$$r_3 = \left[J_p \left(\dot{\psi}_p + \frac{\dot{\psi}_p}{\omega_p} \right) + \left\{ (W_B + W_C + W_L) \frac{b}{2} + (W_B + W_C) x_p - W_C \frac{C_p}{\omega_n^2} \right\} / \omega_p^2 \right] / W_C,$$

$$r_3^* = \left[J_p \left(\dot{\psi}_p + \frac{\dot{\psi}_p}{\omega_p} \right) - \left\{ (W_B + W_C + W_L) \frac{c}{2} + W_L x_p + W_C \frac{C_p}{\omega_n^2} \right\} / \omega_p^2 \right] / W_C.$$

4.3.3 Determination of parameters of servomechanism

The coefficient r_1 of Eqs.(14) and (15) is such a monotonous function that it takes $-1/\omega_p^2$ at $\omega_n=0$ and -1 at $\omega_n=\infty$. The coefficient r_2 is such a monotonous function that it takes $-1/\omega_p^3$ at $\omega_n=0$ and $-\lambda/\omega_p$ at $\omega_n=\infty$. Moreover the ratio of these coefficients r_1/r_2 takes the following extreme value at $\omega_n = 1/\sqrt{\lambda}$, which is the maximum value of r_1/r_2 when $\lambda \omega_p^2 < 1$ and is the minimum when $\lambda \omega_p^2 > 1$.

$$\frac{(2\zeta^2 - 1) \sqrt{\lambda} \omega_p - \zeta (1 - \lambda \omega_p^2) - \lambda \sqrt{\lambda} \omega_p^3}{\sqrt{\lambda} [(2\zeta^2 - 1) \lambda \omega_p^2 + \zeta (1 - \lambda \omega_p^2) \sqrt{\lambda} \omega_p - 1]}$$

The above extreme value is such a monotonously increasing function of ω_p that it takes $\zeta/\sqrt{\lambda}$ at $\omega_p=0$ and $1/\zeta\sqrt{\lambda}$ at $\omega_p=\infty$. When $\lambda \ll 1$ the arctangent of the above value becomes about $\pi/2$ regardless of ω_p . Therefore the procedure to determine the parameters of the servomechanism becomes as follows.

(1) Set the value of ω_n as small as possible when $\lambda \omega_p^2 > 1$ or set it as large as possible when $\lambda \omega_p^2 < 1$. In the case, ω_n must be less than a certain value which is determined by the allowable value of $\dot{\psi}$, because the maximum value of $|\dot{\psi}|$ increases with an increase of ω_n .

Fig.9 Variation of $U_3(\omega_n)$ with respect to ζ , r_1 and r_2 .

(2) Set the crank angle ϕ_0 at which the leg-link touches the ground such that the balancer may come to the objective position s_R at the instant when the crank angle is ϕ_R , namely,

$$\phi_0 = \phi_R - \left(2\zeta + \frac{1}{\sqrt{1-\zeta^2}} \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} \right) \frac{\omega}{\omega_n} \quad (16)$$

(3) Set the crank angle ϕ_T at which the body finishes a uniform motion such that the position s_T of the balancer may become close to $-s_R$, namely,

$$\phi_T = \phi_Z + \frac{K_\theta}{K_\theta K_s} s_R \quad (17)$$

(4) Set the crank angle ϕ_U at which the body begins to leave the ground from Eq.(20) in which s_p is given as a point of intersection ($s_p, \dot{s}_p$) of the next two lines.

$$(1 + \lambda \omega_n^2) \left(s_p - \frac{C_p}{\omega_n} \right) + 2\lambda \zeta \omega_n^2 s_p - \frac{J_p}{W_C} (q_s + \omega_p^2 \phi_p) = 0 \quad (18)$$

$$\dot{s}_p + s_T \omega_n \exp \left(\frac{\zeta}{\sqrt{1-\zeta^2}} \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} \right) = 0 \quad (19)$$

$$\phi_U = 2\pi - \phi_R - \left(2\zeta + \frac{1}{\sqrt{1-\zeta^2}} \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} \right) \frac{\omega}{\omega_n} - \frac{s_p}{s_T} \exp \left(\frac{\zeta}{\sqrt{1-\zeta^2}} \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} \right) \frac{\omega}{\omega_n} \quad (20)$$

5. Experimental analysis

The following discussion is about the experimental results of the biped model shown in Fig.10 which was driven at $\omega = 0.4$ rad/s.

The stride of the locomotion is 151.9 mm, the vertical displacement of the body is 5.6 mm, and the other parameters are as follows.

$a = 21.0$ mm, $b = 50.0$ mm, $c = 80.0$ mm,
 $e = 100.4$ mm, $f = 101.0$ mm, $h = 124.0$ mm,
 $l = 274.0$ mm, $p = 165.0$ mm, $W_B = 3.29$ kg,
 $W_C = 1.78$ kg, $W_L = 0.51$ kg, $J_B = 4.96$ kg mms²,
 $J_C = 0.38$ kg mms², $J_L = 0.22$ kg mms²,
 $K_M = 0.136$ kg mm/V, $D_M = 0.0032$ kg mms,
 $J_M = 0.00066$ kg mms², $K_T = 0.00695$ V/rpm,
 $K_s = 0.333$ mm/rad, $K_\theta = 0.00052$ V/rad,
 $K_f = 0.0296$, $\lambda = 0.0338$ s², $\lambda^* = 0.0255$ s²

Fig.10 Experimental biped mechanism having two parallel feet

5.1 Parameters of servomechanism

Since $K_X = 42$ mm/rad. and $\phi_R = 61.7^\circ$, ϕ_R' is determined as 88.0° from Eq.(5) with $\mu = -0.17$ which is determined with consideration of the advance of the balancer. $\phi_s' = 85.0^\circ$ and $\phi_T' = 275.0^\circ$ are obtained from Fig.2, and we put

$$\phi_s = 78.0^\circ, \quad \phi_z = 170.0^\circ, \quad K_\phi = 0.187 \text{ V/rad.}$$

Using these values, we have

$$s_R = 192.1 \text{ mm}, \quad \tau_1 = 108, \quad \tau_2 = 393 \quad \text{and} \quad \tau_3 = 1.94.$$

Considering that τ_1 becomes $0.1 \sim 0.3$, we choose $\zeta = 0.82$, for which the range of ω_n becomes $[2.3, 16.3]$ rad/s.

When the biped mechanism rotates about the edge-line at the states of P7 and P3*, we have $x_p = x(\phi_R) = 76.0$ mm and $\omega_p = 5.41$ rad/s for P7 and $x_p = x(2\pi - \phi_R) = -76.0$ mm and $\omega_p = 5.72$ rad/s for P3*. Since these values of ω_p are close to $1/\sqrt{\lambda} = 5.44$, the coefficients of Eqs.(14) and (15) are not affected by the value of ω_n . Therefore we put $\omega_n = 2.88$ which is close to the lower limit of its range, resulting in $K_A = 80$.

For the values of parameters determined just above, the values of $U_1(\omega_n)$ and $U_2(\omega_n)$ become 0.94 and 5.87 respectively and we may know that Eq.(8) holds. Moreover, if τ_2 is set between -1.0 and 0.0 , $U_3(\omega_n)$ becomes $1.78 \sim 3.11$ and Eq.(9) holds.

5.2 Results and discussion

Because we have $\phi_0 = 40.2^\circ$, $\phi_T = 261.9^\circ$ and $\phi_U = 287.6^\circ$ respectively from Eqs.(16), (17) and (20), we put these angles as follows.

$$\phi_0 = 45.0^\circ, \quad \phi_T = 262.0^\circ, \quad \phi_U = 280.0^\circ$$

Then the force F_Y and the moment M_Z acting on the leg-link become as shown in Fig.11. The interval where F_Y and M_Z are of the same shape indicates the state of the front edges (tiptoes) of the bottom of the leg-links being in contact with the ground, the interval where F_Y and M_Z are of the symmetry shape indicates the state of the rear edges (heels) of the bottom of the leg-links being in contact with the ground and the interval where F_Y and M_Z are of different shapes indicates the state of both of its front and rear edges being in contact with the ground. The vibrations of about 7.5 Hz are due to small oscillations of the mechanism about the Z-axis.

From the records of F_Y and M_Z for various values of ϕ_0 and ϕ_U , we have the con-

Fig.11 Force and moment of the leg-link (in the case where $\phi_0 = 45^\circ$ and $\phi_U = 280^\circ$)

tact times of the tiptoes and the heels as shown in Figs.12 and 13. Because short contact times of the tiptoes and heels correspond to small absolute values of the attitude angle ϕ of the mechanism, we know from the figures that there exist optimum values of ϕ_q and ϕ_v (in this experiment, $\phi_q = 55.0^\circ \sim 60.0^\circ$, $\phi_v = 272.5^\circ \sim 277.5^\circ$). We know further that the mechanism may fall down forward when $\phi_q < 37.5^\circ$ and backward when $\phi_v > 295.0^\circ$.

The conditions for the body to return to the upright attitude become as follows from Eqs.(14) and (15).

$$s_p + 5.43 s_p - 1600 - 3970(\phi_p + 0.185 \phi_p) < 0$$

.....(21)

$$s_p + 5.49 s_p + 562 - 3950(\phi_p + 0.175 \phi_p) > 0$$

.....(22)

Then putting (s_p, s_p) as the point on the curves of the step responses of the balancer from $s=0$ to $s=192.1$ mm and from $s=-192.1$ mm to $s=0$ and (ϕ_p, ϕ_p) as the attitude angle and velocity of the locomotion at the instant when L_T and B_T or L_H and B_H touch the ground, we have the limit values of $\phi_q = 36^\circ$ and $\phi_v =$

Fig.12 Contact times of the tiptoe and the heel when the leg-link touches and leaves the ground

Fig.13 Contact times of the tiptoe and the heel when the body touches and leaves the ground

296° respectively from Eqs.(21) and (22). These values coincide well with the experimental values mentioned just above.

6. Conclusions

For the purpose of applying biped locomotions to transport machines, we have investigated both theoretically and experimentally the leg-mechanism, the control system of attitude and the conditions of dynamic stability of the biped locomotion. The results are summarized as follows.

(1) Arranging two oscillating block slider crank mechanisms parallel in a plane and driving them synchronously, we have obtained a leg-mechanism which causes the biped locomotion at constant speeds.

(2) The compensating weight reciprocating in a linear guide rail occupies far less space than the traditional inverted pendulum. Further, in the linear-motion balancer, the servomotor and other elements can be a part of the compensating weight and the center of gravity of the weight does not move in the vertical direction, which makes it easy to design a highly efficient control system.

(3) For a biped machine having two parallel feet and a servomechanism with a linear-motion balancer, we have derived theoretically the conditions that the body may perform a uniformly accelerated motion and a uniform motion in the state of upright attitude and the conditions that the machine may return to the upright attitude at the instant when a foot touches or leaves the ground surface. We have also demonstrated a procedure to determine the parameters of the servomechanism for the specified mechanism parameters of the locomotion.

(4) The experimental analysis has revealed that the biped machine can perform a stable gait under the above theoretical conditions and it falls down if the conditions for returning to the upright attitude are not satisfied. There exists an optimum starting position of the step input voltage which makes the absolute attitude angle of the machine minimum.

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