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A Fundamental Study on Frictional Noise*

(1st Report, The generating mechanism
of rubbing noise and squeal noise)

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The generating mechanism of frictional noise in dry friction is experimentally and theoretically studied, when a contact rod clamped at one end is pressed in the radial direction on a rotating disk. The results are obtained as follows:

- (1) Frictional noise is classified into two categories, rubbing noise and squeal noise.
- (2) When the coefficient of friction between the rod and the disk is small, the vibrating energy is small. Therefore, the frictional noise is rubbing noise, which is of a low level.
- (3) When the coefficient of friction grows great as the result of the tip of the rod being worn, the rod vibrates in the lateral direction as in the fundamental mode. Thus the vibrating energy increases and the frictional noise changes to the squeal noise, the sound pressure level of which is higher than the rubbing noise by 20-30 dB. These lateral vibrations seem to be self-excited vibrations caused by the friction.

1. Introduction

We are accustomed to sounds caused by friction, for example, the musical sounds of a violin or a wineglass, and the frictional noise from vehicle brakes, or the gearing and rotating parts of machines. Because of recent developments resulting in higher driving speeds, frictional noise have increased. Therefore, it is necessary to reduce machine noise. In musical sounds, Helmholtz⁽¹⁾ studied the sound of a violin, Rayleigh⁽²⁾ and Spurr⁽³⁾ studied the ringing of a wineglass. In the area of frictional noise, the noise of automobile brakes^{(4),(5)} and squeal noise which are caused when a train runs on a curved track⁽⁶⁾, or when a car accelerates rapidly or takes a curve quickly⁽⁷⁾ were studied.

Many studies on the frictional vibration of low frequency components have been done as a self-excited vibration of stick-slip phenomenon. Watari et al.⁽⁸⁾ for instance, showed that the relation between the amplitude of self-excited vibration and relative sliding velocity could be theoretically classified into (a) when dynamic friction force is equal to static friction force at zero relative sliding velocity, and (b) when it is not equal to static friction force. They made clear that the generating velocity range and

maximum amplitude of vibration depended on the frictional surface condition. Takano et al.^{(9),(10)} showed that the frequency of generated vibration approaches the natural frequency of the vibration system with an increase of sliding velocity and theoretically analyzed the generating condition of frictional vibration by assuming that the coefficient of friction-relative sliding velocity curve is approximately expressed as some folding lines.

For the study on the frictional vibration of high frequency components, Jarvis et al.⁽¹¹⁾ presented the generating condition of frictional vibration by pressing a cantilever on a thin rotating disk. And Klepp⁽¹²⁾ analyzed the higher modes of vibration of a cantilever which is pressed on a high speed rotating disk by using an experimental equation between the coefficient of friction and the relative velocity. Thus, many theoretical and experimental studies have been carried out on frictional vibration. On the other hand, they have put stress on the reduction of frictional noise using trial and error methods and have not analyzed its generating mechanism so thoroughly, because it is difficult to completely understand the rubbing surface characteristics.

The generating mechanism of frictional noise without lubrication was investigated by means of an apparatus-comprising a thick steel rotating disk and a steel rod. As the result, frictional noise is classified into two kinds of noise. One has a low sound pressure level called "rubbing noise", and the other has a high sound pressure level and sinusoidal wave form called "squeal noise". In this paper,

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the generating mechanisms of the two kinds of frictional noise are theoretically and experimentally studied.

2. Experimental apparatus and methods

Figure 1 shows an experimental apparatus. To get a wide sliding velocity range, we use a pin-disk mechanism instead of a reciprocating slider mechanism. A cantilever (we call it a "rod") (2) is pressed in the radial direction on a thick disk (1) rotating at a constant speed. The tip of the rod has a hemispherical shape and the rod is screwed into a thick steel plate (its thickness is 20 mm), which is attached to the moving stand (3). Therefore, the end of the rod can be regarded as a clamped edge. Constant load is applied on the disk by an adjustable spring (4). The contact angle between the rod and the disk can be adjusted by changing the height of the moving stand. Table 1 shows the specifications of the rod and the disk. Frictional noise is measured at a height of about 100 mm from the contact point between the rod and the disk, except in the measurement of the sound field.

To ascertain the generating mechanism of frictional noise, accelerations in the lateral and longitudinal directions of the rod were measured by two light accelerometers (0.4 gr). One is attached to the rod at a distance of about 3 mm from the tip of the rod (5 in Fig.1) for measuring

Fig.1 Experimental apparatus

Table 1 Specifications of the disk and the rod

Disk		Rod	
Diameter mm	200	Diameter mm	7
Thickness mm	40	Length mm	32
Material	S45C	Material	S45C
Hardness	Hv=400 (Induction hardening)	Hardness	1. Hv=700 (Induction hardening) 2. Hv=150 (raw)

the lateral acceleration and the other to the steel plate (6 in Fig.1) for measuring the longitudinal acceleration. By measuring the strain by means of a strain gauge pasted on the tip of the rod (7 in Fig.1), friction force is calculated from the calibrated strain-friction force curve. The rotational velocity of the disk is measured from the pulse generated by a photo transistor and a rotating circular plate with a slit.

Measured data are all recorded on magnetic tape. The frequencies of acceleration are analyzed by a real-time spectrum analyzer. The waveforms are reproduced by an electro-magnetic oscillograph or pen recorder. A block diagram of the measuring system is illustrated in Fig.2. The applied load on the disk is 2 kg and the angle of the rod against the center of the disk is zero.

Fig.2 Block diagram for measuring

3. Experimental results

Experiments were done to see what kind of frictional noise occurs under the influence of time lapse, i.e., sliding distance (the product of circumferential velocity of the disk and duration of friction). A new hardened rod is rubbed against an unused surface of the disk, when the circumferential velocity of the disk V is 70 cm/s and the contact load is 2 kg.

Figure 3 shows the coefficient of friction μ_m which is influenced by the contact surface and the sound pressure level of frictional noise. μ_m is obtained by dividing the smaller value of friction force at zero vibrating velocity of the

rod by the contact load. It is not exactly coincident with the steady coefficient of friction. The sound pressure waveforms at a sliding distance of 150, 550 and 1100 m are shown in Fig.3. The following results are obtained.

At a sliding distance of below 200 m, μ_m is small and the sound pressure level of frictional noise is low (65 dB). After that, μ_m increases with an increase of the sliding distance, but the sound pressure level of frictional noise is still low (65-70 dB). The waveform is almost the same as that of the beginning of the frictional experiments. However, at a sliding distance of over 1000 m, frictional noise level suddenly increases and reaches from 95 to 97 dB. This sound is high-pitched and its waveform is sinusoidal. At that time μ_m becomes 1.3-1.5 which is 3-4 times larger than that of the beginning of the experiment.

Fig.3 Coefficient of friction and sound pressure level versus sliding distance

From our experiments, frictional noise is classified into two categories. One has a low sound pressure level and is called "rubbing noise" and the other has a high sound pressure level and sinusoidal waveform and is called "squeal noise". Figure 4 presents the sound field of squeal noise. In this figure, the disk scarcely vibrates because of its relatively large width. The sound radiated from the disk seemed to be of a negligibly low level. Frictional noise is generated from the contact point between the disk and the rod. Therefore, from our experiments it is clear that frictional noise is generated from the vibration of the rod.

Figure 5 shows the waveforms and frequency spectrum of sound pressure, lateral and longitudinal accelerations of rubbing noise and squeal noise at $V=70$ cm/s. In this figure, the noise acceleration level

Fig.4 Sound field in squeal noise

Fig.5 The waveforms and frequency spectrum in rubbing noise and squeal noise ($V=70$ cm/s)

shows dB level ($80 \text{ dB} = 1000 \text{ cm/s}^2$). A longitudinal acceleration level is much lower than a lateral acceleration level and the waveforms of squeal noise and rubbing noise are quite similar to that of lateral acceleration.

Therefore, frictional noise seems to be generated by the lateral vibration of the rod. Squeal noise with a high sound pressure level would be generated by the self-excited vibration of the rod. According to results of the frequency spectrum, frequency components of the sound pressure and acceleration of squeal noise are 4600 Hz and its harmonics. Frequency components of rubbing noise are 4600 Hz and a few kinds of low frequencies. The low frequency components of rubbing noise may be generated by rubbing the surface roughness of the contact point between the disk and the rod. The rod is pressed on the disk with a 2 kg contact load and the tip of the rod is excited in the lateral direction by a minishaker. Then, it is proved that the main frequency component of frictional noise, 4600 Hz, is a lateral fundamental natural frequency of the rod.

To examine the rubbing surface condition between the rod and the disk, we use a scanning electron microscope. We observe the apex of the rod by the scanning electron microscope as we can not see the disk surface when rubbing noise or squeal noise occurs. Figure 6(a) and 6(b) show the surface of the tip of the rod in rubbing noise and squeal noise. In the case of rubbing noise, the surface of the rod has many small rough parts. As it continues to wear, squeal noise is generated and the surface of the rod becomes smooth except for a few strips in the rubbing direction as shown in Fig. 6(b). The rubbing surface of the disk in rubbing noise seems to have the same small rough parts as that of the rod in Fig. 6(a). Therefore, the disk and the rod do not come into close contact, so the coefficient of friction will be small. As the sliding distance becomes longer, the rough parts of the rod and the disk are worn away and the surfaces become smooth. As a result, the surface of the disk becomes oxidized and the rod adheres to the disk tightly. In the friction process between the same kinds of metals as in our experiment, the oxidized surface is worn, destroyed and dropped out by repeated rubbing. Wear condition becomes steady and both the contact area be-

(a) in rubbing noise (b) in squeal noise

Fig. 6 Scanning electron micrograph of surface of the rod

tween the rod and the disk and the coefficient of friction increase; then squeal noise will occur.

In the experiment with a raw rod, squeal noise occurs at a shorter sliding distance of 150 m than with a hardened rod. This is because the raw rod wears more quickly than the hardened rod. The results of the experiments with the raw rod, with respect to the sound pressure level, waveforms of vibration and frequency components are the same as those with the hardened rod.

For further investigation on the generating mechanism of rubbing noise and squeal noise, we generated rubbing and squeal noise by rotating the disk at a low speed. We observed how the coefficient of friction changes as the speed of the disk gets higher. Figure 7 shows the results. Even if the circumferential speed of the disk V is accelerated from 20 to 250 cm/s, rubbing noise still continues. The coefficient of friction μ_m is 0.3-0.5 and the gradient of μ_m to V , $d\mu_m/dV = \mu'_m$ is small and negative or nearly equals zero.

On the other hand, squeal noise stops at a circumferential speed of over 200 cm/s and frictional noise changes to a kind of cutting noise generated when the surface of the disk is scraped. In this paper, we do not describe this noise. In squeal noise, μ_m is 1-1.5 and μ'_m is negative until the sliding velocity becomes 100-120 cm/s and changes to positive when the sliding velocity is greater than 120 cm/s. Generally speaking, the gradient of coefficient of friction to sliding velocity changes from negative to positive at some velocity and self-excited vibration is generated at a negative gradient. The vibration damps at a positive gradient. In our experiments shown in Fig. 7, μ'_m changes from negative to positive when the sliding velocity is over 120 cm/s. But squeal noise still occurs, even if μ'_m is positive. This phenomenon occurred many times in our experiments. These experimental results are inconsistent with frictional vibration theory which states that self-excited vibration is not generated when μ'_m is positive.

Fig. 7 Coefficient of friction with respect to circumferential speed of the disk V

Up to this point, we have used a steady coefficient of friction. But in fact, friction force varies periodically while rubbing noise or squeal noise occurs. Figure 8 shows how this coefficient of friction (we call it the "instantaneous coefficient of friction") μ_i varies with the relative sliding velocity during one period of squeal noise or rubbing noise. v_r is the relative sliding velocity. $v_r = V - \dot{u}$. $\dot{u}$ is the vibrating velocity of the tip of the rod. $\dot{u}$ is obtained by integrating the experimental acceleration of the rod.

Figures 8(a) and 8(b) show the relation between μ_i and v_r in rubbing and squeal noise at $V=70$ cm/s ($\mu'_m < 0$). In these figures, μ_m is also shown. In each figure, the curve between μ_i and v_r forms a loop. In the case of rubbing noise (as shown in Fig. 8(a)), because of the low lateral acceleration level of the rod (75 dB), the variation range of relative sliding velocity v_r is small and the coefficient of friction is also small. $\mu_i - v_r$ curve which inclines a little to the right forms a small clockwise loop. In the case of squeal noise shown in Fig. 8(b), the variation of μ_i is large and the lateral acceleration level is high (135 dB). Variation in the relative sliding velocity is also large. Therefore, the $\mu_i - v_r$ curve which inclines a little to the right becomes a large clockwise loop during the whole period of vibration. The area of this loop represents the energy that flows into the vibration system. That is, self-excited vibration is generated when squeal noise occurs; and sound pressure and lateral acceleration have a higher level than that of rubbing noise.

In the case of Figs. 8(a) and 8(b), μ'_m (gradient of μ_m) is negative. Figure 8(c) shows $\mu_i - v_r$ curve at $V=170$ cm/s, when μ'_m is positive and squeal noise occurs. In this figure, $\mu_i - v_r$ curve makes a clockwise loop of almost the same size as in Fig. 8(b). Even if μ'_m has a positive value, self-excited vibration may be generated causing squeal noise, provided that μ_i makes a loop which inclines to the right.

Fig.8 The relation between μ_i and v_r during one period of rubbing noise and squeal noise

4. Theoretical considerations

From the experiments, it is clear that frictional noise is mainly generated by the lateral vibration of the rod. Therefore, for the analysis of generating mechanism on frictional noise, we consider only the lateral vibration of the rod. Under the assumption that the influence of shear and rotating inertia can be neglected, an equation for lateral vibration of the rod when the contact load P and friction force F act on the tip of the rod ($x=l$), as shown in Fig.9, is derived as follows:

$$EI \frac{\partial^4 u}{\partial x^4} + P \frac{\partial^2 u}{\partial x^2} + \frac{\gamma A}{g} \frac{\partial^2 u}{\partial t^2} + C \frac{\partial u}{\partial t} = F(v_r) \quad (1)$$

where u : deflection of the rod, E : Young's modulus, I : moment of inertia, γ : specific weight, A : sectional area, C : damping coefficient, g : acceleration of gravity, $F(v_r)$: friction force $F(v_r) = \mu(v_r)P$, $\mu(v_r)$: coefficient of friction which is expressed as a function of relative sliding velocity.

In our experiments, since the contact load P is small (2 kg), and it may have no influence on the vibration, we omit the term of $P \partial^2 u / \partial x^2$ in Eq.(1). Friction force $F(v_r)$ acts on the tip of the rod ($x=l$), and it is expressed as follows:

$$F(v_r) = \mu(v_r) P \delta(x-l) \quad (2)$$

where δ is Delac's delta function.

Now, we assume that displacement of the rod, u , is expanded in terms of $X_n(x)$.

$$u(x, t) = \sum_{n=1}^{\infty} X_n(x) T_n(t) \quad (3)$$

where $T_n(t)$ is a function of time, $X_n(x)$ is the n -th eigenfunction of the rod under the following boundary condition ($x=0$; clamped and $x=l$; free) and it is presented as follows:

$$X_n(x) = \cos \lambda_n x - \cosh \lambda_n x + \frac{\sin \lambda_n l - \sinh \lambda_n l}{\cos \lambda_n l + \cosh \lambda_n l} (\sin \lambda_n x - \sinh \lambda_n x) \quad (4)$$

$$\lambda_n^4 = \gamma A \omega_n^2 / (EI g),$$

ω_n is the n -th natural circular frequency of the rod.

where $\lambda_n l$ satisfies the following equation:

Fig.9 Schematic figure of the rod and the disk

$$\cos \lambda_n l \cosh \lambda_n l + 1 = 0 \quad (5)$$

Substituting Eq.(3) into Eq.(1), multiplying both sides of the resulting equation by $X_n(x)$ and integrating from 0 to l , we obtain the following non-linear differential equations:

$$\begin{aligned} \frac{rA}{g} \ddot{T}_n(t) + C \dot{T}_n(t) + \frac{rA}{g} \omega_n^2 T_n(t) \\ = X_n(l) \frac{P}{l} \left\{ V - \sum_{n=1}^{\infty} X_n(t) \dot{T}_n(t) \right\} \dots (6) \\ (n=1, 2, \dots, \infty) \end{aligned}$$

Before solving these equations, we can calculate the natural frequencies of the rod by Eq.(5). The fundamental natural frequency is 4910 Hz and the second natural frequency is 30780 Hz. The calculated fundamental natural frequency of the rod agrees with the experimental result.

It is reasonable for us to consider the maximum frequency range up to 20 KHz in the investigation of frictional noise, but in our calculation, we consider the modes of vibration in Eq. (3) up to the second mode of natural frequency. Then we obtain two non-linear differential equations in Eq.(6); $n=1, 2$. It is very difficult to solve these equations analytically. We solve these equations numerically by Runge-Kutta-Gill method. The initial displacements and velocities, $T_1(0), T_2(0), \dot{T}_1(0)$ and $\dot{T}_2(0)$ are obtained as follows.

Assuming that $u_0(x, 0)$ is equal to the static deflection of the rod when $\mu_0 P$ (μ_0 ; static coefficient of friction) acts on the tip of the rod and expanding $u_0(x, 0)$ in eigenfunctions, we obtain:

$$u_0(x, 0) = \sum_{n=1}^2 X_n(x) T_n(0)$$

Multiplying both sides of this equation by $X_n(x)$ and integrating from 0 to l , we have:

$$T_n(0) = \frac{\int_0^l u_0(x, 0) X_n(x) dx}{\int_0^l X_n^2(x) dx} \quad (n=1, 2) \quad (7)$$

from $\dot{u}_0(x, 0) = 0$, we obtain

$$\dot{T}_1(0) = \dot{T}_2(0) = 0 \quad (8)$$

We get the damping coefficient C from the damping waveform of acceleration which is obtained as follows. We press the rod on the disk with 2 kg contact load, and excite the tip of the rod by minishaker in the lateral direction and remove it from the rod quickly. The coefficient of fric-

Fig.10 Calculated waveform of lateral acceleration of the rod in squeal noise

tion $\mu(v_r)$ used in calculation is written as follows:

$$\mu(v_r) = \alpha_1 e^{-\alpha_2 v_r^2} + \alpha_3 v_r + \alpha_4 \quad (9)$$

$\alpha_1, \alpha_2, \alpha_3, \alpha_4$: constant

According to the calculation of acceleration amplitudes with respect to various sliding velocities (using the values of $\alpha_1, \alpha_2, \alpha_3$ and α_4 in Eq.(9) so that the curve $\mu - v_r$ agrees with the curve $\mu_m - V$ shown in Fig. 7), the amplitude damps suddenly when μ_m changes to positive. These results are the same as those of the previous studies.

But in our experiment, even if μ'_m is positive, squeal noise occurs. By considering the instantaneous coefficient of friction during one cycle of the vibration, it is found that squeal noise occurs, if the gradient of the loop inclines to right ($\mu'_i < 0$).

We calculate acceleration amplitude at $V=170$ cm/s when μ'_i is positive and squeal noise occurs. In this calculation, values of $\alpha_1, \alpha_2, \alpha_3$ and α_4 are determined so that the gradient of the curve expressed in Eq.(9) may be equivalent to that of the loop μ'_i shown in Fig.8(c). Figure 10 shows the calculated waveform. From this figure, it is clear that even if μ'_m is positive, self-excited vibration may be generated and it causes squeal noise. The acceleration waveform is sinusoidal and its frequency coincides with the fundamental natural frequency of the rod.

From the result of the above-mentioned calculations, it is clear that μ'_i has an important effect upon the generation of squeal noise. We calculate the acceleration level (r.m.s. value) versus various negative gradients of the loop ($\mu'_i < 0$) under the following conditions: $V=70$ cm/s, α_2 and α_3 are constants, $C=0$, $C=2 \times 10^{-3}$ kg.s/cm (= value obtained from ex-

Fig.11 The relation between the lateral acceleration level of the rod and various negative gradients of the loop ($\mu'_i < 0$) ($V=70$ cm/s, $C=0$ and $C=2 \times 10^{-3}$ kg.s/cm²)

periment). The gradient of the loop is changed by values of α_1 and α_4 .

Figure 11 shows the results. According to this figure, self-excited vibration is not generated at a small negative value of μ'_i . When μ'_i increases to a certain negative value, the amplitude level increases suddenly and self-excited vibration is generated. And μ'_i when self-excited vibration starts depends on the damping coefficient value, that is, the smaller the damping coefficient, the smaller the negative μ'_i . Figure 11 also shows experimental lateral acceleration level at the gradient of the loop, $\mu'_i = -8 \times 10^{-3}$, shown in Fig.8(c). Comparing numerical results with experimental results, we see that they are different in level but the same with respect to μ'_i at which self-excited vibration starts. In the rubbing noise shown in Fig.8(a), μ'_i is very small ($\mu'_i = -4 \times 10^{-4}$). From Fig.11, it is evident that self-excited vibration is not generated.

From theoretical considerations by using Eq.(6), it is found that when the gradient of the loop ($\mu_i - v_i$ curve) inclines to the right ($\mu'_i < 0$) and μ'_i is above a certain negative value, self-excited vibration is generated and it causes squeal noise; and it is also found that when μ'_i is a small negative value, self-excited vibration is not generated and only rubbing noise occurs.

5. Conclusions

The results of experimental and theoretical studies on dry frictional noise are summarized as follows:

(1) Frictional noise by dry friction is classified into squeal noise and rubbing noise. Rubbing noise is generated when the coefficient of friction between the rod and the disk is small and the rubbing surface has a lot of slightly rough areas. This sound pressure level is low. As the sliding distance becomes long, the friction surface becomes smooth because of wear of the rod and the coefficient of friction increases greatly. Thus, rubbing noise changes to squeal noise, the sound pressure level of which is higher than that of rubbing noise by 20-30 dB.

(2) The frequency components of rubbing noise consist of the lateral fundamental natural frequency of the rod and low frequencies generated by rubbing on the surface roughness. The frequency components of squeal noise consist of the lateral fundamental natural frequency and its harmonics of the rod and its waveform is almost sinusoidal. Both squeal noise

and rubbing noise may be generated by the lateral vibration of the rod.

(3) When both the disk and the rod are hardened, there is a long sliding distance before squeal noise occurs. But when the rod is not hardened, squeal noise occurs at beginning of the experiment, because the tip of the rod wears very quickly.

(4) Even if the gradient of the steady coefficient of friction-sliding velocity curve is positive, squeal noise occurs. This fact means that squeal noise occurs depending on the gradient and magnitude of a loop formed by the instantaneous coefficient of friction and relative sliding velocity during one period of vibration.

(5) By solving the non-linear vibrating differential equations derived from only lateral vibration of the rod, it is found that self-excited vibration is generated and it causes squeal noise, as the gradient of the loop becomes more negative in value.

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