A Transversely Isotropic Circular Cylindrical under Concentrated Loads

By Hideaki KASANO **, Hiroyuki MATSUMOTO *** and Ichiro NAKAHARA ****

A transversely isotropic circular cylinder subjected to two diametrically opposite loads is considered in the framework of the three-dimensional theory of anisotropic elasticity. A general solution to this non-axisymmetric problem is obtained by using a set of three stress functions introduced by Elliot and Lodge, and the numerical calculations are carried out for the concentrated loads which are the limiting case of the partially distributed loads. Numerical results are presented for the stress distributions in magnesium and cadmium single crystals and fiber-reinforced plastics (FRP) composite materials, and they are compared with those of the isotropic materials to show the effect of anisotropy.

1. Introduction

The recent development in anisotropic materials such as the fiber-reinforced composites makes it possible to recognize the growing importance of the stress analysis of the anisotropic elastic solids. But, because of the greater difficulty of the problems involving a large number of elastic constants, the stress states of the anisotropic solids are little.

Lekhnitskii (1), in his book, deals with simple axisymmetric problems of orthotropic elastic bodies. For transversely isotropic bodies, since a set of three stress functions was first introduced by Elliot (2) and Lodge (3), problems of infinite or semi-infinite bodies subjected to concentrated loads (4), rigid punches and cracks (5), and of elastic bodies having cavities or inclusions under tension (6), (7), have been successively solved. These investigations are concerned with the infinite, the semi-infinite media, and the plates of finite thickness, while Chen (8) studied a steady-state dynamic problem of a transversely isotropic infinite cylinder and obtained the numerical results for the case of the axisymmetric discontinuous pressure to show the effect of anisotropy on the stress field. Atsumi et al. (9) also investigated the stress concentration problem of the same cylinder having a spherical cavity under tension by using the methods of Hankel transforms and Schmidt-orthonormalization. In contrast to these axially symmetric problems, comparatively little work has been done on the non-axisymmetric ones of an anisotropic cylinder. This paper considers the non-axisymmetric deformation of a transversely isotropic circular cylinder subjected to partially distributed load by using a set of three stress functions and the Fourier transform method. The numerical calculations are carried out for the diametrically opposite concentrated loads to examine how anisotropy affects the stress field in comparison with the isotropic case (10).

2. Analysis

In general, the solutions of the transversely isotropic elastic problems can be found in terms of three stress functions (11). That is, the displacement component in the circular cylindrical coordinates \( (r, \theta, z) \) is expressed as follows:

\[
\begin{align*}
\sigma_{rr} &= \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) \sigma_{ss} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\mu}{r^2} \frac{\partial^2}{\partial \theta^2} \nu, \quad \nu = 0 \\
\sigma_{r \theta} &= \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) \sigma_{ss} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\mu}{r^2} \frac{\partial^2}{\partial \theta^2} \nu
\end{align*}
\]

(1)

in which the stress functions satisfy

\[
\begin{align*}
\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{\mu}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \sigma_{ss} &= 0 \\
\left( \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\mu}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \nu &= 0
\end{align*}
\]

(2)

where, using the five elastic constants of the transversely isotropic materials denoted by \( c_i \), we have

\[
\begin{align*}
k_i &= (c_{ij} - 2c_i)/(c_{ij} + c_{kk}) \quad (i = 1, 2, 3) \\
\mu_1 \text{ and } \mu_2 \text{ are the roots of the equation } \quad c_{ij} \mu^2 + (c_{ii} + c_{kk}) c_{kk} \mu + (c_{kk} c_{ii} - c_{ij} c_{kk}) &= 0
\end{align*}
\]

and \( c_{ij} \) is the axis of geometric and elastic symmetry. The stress components corresponding to Eqs.(1) are given by using the relations of displacement-strain, and of strain-stress:

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Thus, the solutions of the transversely isotropic problems are constructed by the method of superposition of solutions derived from the stress functions $\mathbf{\Psi}$, $\mathbf{\Pi}$ and $\mathbf{\Omega}$.

Let us consider an infinite circular cylinder subjected to the diametrically opposite radial loads on its surface as shown in Fig.1. Taking into account the facts that the stress state has some symmetry and all the stress components vanish at infinity, we assume the stress functions in the following forms:

\begin{align*}
\sigma(r,z) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} f_n(r) J_n(\rho r) \cos 2\phi \cos z \, dz \\
\tau(r,z) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} g_n(r) J_n(\rho r) \cos 2\phi \sin z \, dz \\
\tau(r,z) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} h_n(r) J_n(\rho r) \sin 2\phi \cos z \, dz
\end{align*}

where $\rho=r/a$, $\tau=\pi/a$ and $J_n(\sqrt{\rho^2 z})$ is a modified Bessel function of the first kind, and $f_n(r)$, $g_n(r)$, $h_n(r)$ are the unknown functions which are to be determined by the boundary conditions.

Substituting Eqs. (4) into Eqs. (3), stresses on the surface ($\rho=1$) are given by

\begin{align*}
\sigma(\tau,\zeta) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} f_n(\tau) J_n(\rho r) \cos 2\phi \cos z \, dz \\
\tau(\tau,\zeta) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} g_n(\tau) J_n(\rho r) \cos 2\phi \sin z \, dz \\
\tau(\tau,\zeta) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} h_n(\tau) J_n(\rho r) \sin 2\phi \cos z \, dz
\end{align*}

in which

\begin{align*}
\sigma(\tau,\zeta) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} f_n(\tau) J_n(\rho r) \cos 2\phi \cos z \, dz \\
\tau(\tau,\zeta) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} g_n(\tau) J_n(\rho r) \cos 2\phi \sin z \, dz \\
\tau(\tau,\zeta) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} h_n(\tau) J_n(\rho r) \sin 2\phi \cos z \, dz
\end{align*}

where $\beta=\alpha(1+k)/(\pi-a)$ ($i=1,2,3,4$), $\gamma=\sqrt{(\pi-a)/\ln(\rho r)}$, $\beta=\delta(\pi-a)$ ($i=1,2,3$, $\beta=\delta(\pi-a)$) are the boundary conditions for the case that the diametrically opposite pressures $p(\phi, z)$ distribute over the cylindrical surface of radius $\zeta$ can be written as follows:

\begin{align*}
(\sigma_{\phi\phi}) &= -p(\phi, z), \quad (\tau_{\phi\zeta}) = (\tau_{\phi\zeta}) = 0
\end{align*}

in which

\begin{align*}
\sigma(\tau,\zeta) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} f_n(\tau) J_n(\rho r) \cos 2\phi \cos z \, dz \\
\tau(\tau,\zeta) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} g_n(\tau) J_n(\rho r) \cos 2\phi \sin z \, dz \\
\tau(\tau,\zeta) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} h_n(\tau) J_n(\rho r) \sin 2\phi \cos z \, dz
\end{align*}

where $P$ is the resultant force, and for the partially distributed loads as shown in Fig.1,

\begin{align*}
\sigma(\tau,\zeta) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} f_n(\tau) J_n(\rho r) \cos 2\phi \cos z \, dz \\
\tau(\tau,\zeta) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} g_n(\tau) J_n(\rho r) \cos 2\phi \sin z \, dz \\
\tau(\tau,\zeta) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} h_n(\tau) J_n(\rho r) \sin 2\phi \cos z \, dz
\end{align*}

and further, for the concentrated loads with $a_{\phi}=1$, $a_{\phi}=2$ $(n=1,2,3)$, $\phi(\phi)=1$, using Dirac's delta function $\delta(\phi)$, Eq. (8) takes the form

\begin{align*}
P(\phi, z) &= \sum_{n=-\infty}^{\infty} \int_{0}^{\infty} \delta(\phi) J_n(\rho r) \cos z \, dz
\end{align*}

Then, equating Eqs. (5) with Eqs. (7), the following system of three simultaneous linear equations can be obtained:

\begin{align*}
(4m^2+\beta^2) J_{n+1}(\rho r) + (\beta^2-4m^2-\pi^2) J_{n}(\rho r) + (4m^2+\beta^2-\pi^2) J_{n-1}(\rho r) = 0 \\
(\beta^2-4m^2-\pi^2) J_{n+1}(\rho r) + (4m^2+\beta^2-\pi^2) J_{n}(\rho r) + (\beta^2-4m^2-\pi^2) J_{n-1}(\rho r) = 0 \\
2(4m^2+\beta^2) J_{n+1}(\rho r) + (\beta^2-4m^2-\pi^2) J_{n}(\rho r) + (4m^2+\beta^2-\pi^2) J_{n-1}(\rho r) = 0
\end{align*}

From these equations, the unknown functions are determined as follows:

\begin{align*}
f_n(\tau) &= \frac{F_n(\tau)}{2\pi H_n(\tau)} h_n(\tau), \quad g_n(\tau) = \frac{G_n(\tau)}{2\pi H_n(\tau)} h_n(\tau) \\
h_n(\tau) &= \frac{1}{\alpha_1-\alpha_2} \frac{P}{\tau_1} \phi(\phi) \left[(2m^2-\beta^2) J_{n+1}(\rho r) - (2m^2+\beta^2) J_{n-1}(\rho r)ight], \quad (i=1,2,3)
\end{align*}
where

\[ F_i(\zeta) = (1 + \epsilon_i) \left[ \gamma_i - 4\epsilon_i \mu_i \gamma_i \right] \left( 1 + \epsilon_i \right) \gamma_i - 4\epsilon_i \mu_i \gamma_i - 1 \right] \]

\[ G_i(\zeta) = (1 + \epsilon_i) \left[ \gamma_i - 4\epsilon_i \mu_i \gamma_i \right] \left( 1 + \epsilon_i \right) \gamma_i - 4\epsilon_i \mu_i \gamma_i - 1 \right] \]

\[ H_i(\zeta) = (1 + \epsilon_i) \left[ \gamma_i - 1 \right] \left( 1 + \epsilon_i \right) \gamma_i - \left( 1 + \epsilon_i \right) \gamma_i - 1 \right] \]

(13)

3. Numerical Calculations

The numerical calculations are carried out for the diametrically opposite concentrated radial loads. Substitution of Eqs. (4) into Eqs. (3), and use of Eqs. (12) lead to the following expressions of a hoop stress \( \sigma_H \) and an axial stress \( \sigma_A \): 

\[
(\sigma_H / P)(\zeta) = \sum_{n} \int_{\zeta}^{\infty} \left[ \phi_n \cos \zeta \right] d\zeta \cos 2\theta 
\]

(14)

\[
(\sigma_A / P)(\zeta) = \sum_{n} \int_{\zeta}^{\infty} \left[ \phi_n \cos \zeta \right] d\zeta \cos 2\theta 
\]

(15)

in which

\[
\phi_n = \frac{1}{\pi} \left[ \left( \gamma_i - 1 \right) \left( 1 + \epsilon_i \right) \gamma_i - \left( 1 + \epsilon_i \right) \gamma_i - 1 \right] 
\]

(16)

\[
\phi_n = \frac{1}{\pi} \left[ \left( \gamma_i - 1 \right) \left( 1 + \epsilon_i \right) \gamma_i - \left( 1 + \epsilon_i \right) \gamma_i - 1 \right] 
\]

(17)

where

\[
\gamma_i = \gamma_i - \beta_i \quad (i = 1, 2) \quad \gamma_i = \gamma_i - \beta_i \quad (i = 1, 2, 3) 
\]

Particularly, because of \( \gamma_1 = 0 \) (\( n = 2 \)) and \( \gamma_2 = 0 \) (\( n = 1 \)) on the z-axis (\( \beta = 0 \)), Equations (14) and (15) are simplified as follows:

\[
(\sigma_H / P)(\zeta) = \sum_{n} \int_{\zeta}^{\infty} \left[ \left[ \phi_n \right] \cos 2\theta \right] d\zeta 
\]

(18)

\[
(\sigma_A / P)(\zeta) = \sum_{n} \int_{\zeta}^{\infty} \left[ \left[ \phi_n \right] \cos 2\theta \right] d\zeta 
\]

(19)

where

\[
\left[ \phi_n \right] = \frac{1}{\pi} \left[ \left( \gamma_i - 1 \right) \left( 1 + \epsilon_i \right) \gamma_i - \left( 1 + \epsilon_i \right) \gamma_i - 1 \right] 
\]

(20)

The stresses in the cylinder except on the z-axis may be obtained by the direct calculations of Eqs. (24) and (15), but the convergence of the infinite integrals and the infinite series in these equations become slower in the vicinity of the surface and both diverge on it, therefore we cannot calculate them as they stand. Here, the numerical calculations are performed, directly for \( \beta = 0.6 \) and with the aid of Shanks' acceleration method for \( \beta = 0.8 \).

Further, considering that these divergent integrals and series are expressed with delta functions on the surface as shown in Eq. (10), we eliminate the divergent terms in the equations. That is, assuming \( \lambda_i \) as sufficiently large, we separate a finite and an asymptotic integral from an infinite integral:

\[
(\sigma_H / P)(\zeta) = \sum_{n} \int_{\zeta}^{\infty} \left[ \left[ \phi_n \right] \cos 2\theta \right] d\zeta 
\]

(21)

\[
(\sigma_A / P)(\zeta) = \sum_{n} \int_{\zeta}^{\infty} \left[ \left[ \phi_n \right] \cos 2\theta \right] d\zeta 
\]

(22)

where \( \left[ \phi_n \right] \) and \( \left[ \phi_n \right] \) are derived from the asymptotic expansions of the Bessel functions, and given as follows by retaining the terms up to \( \lambda_i^{-1} \):

\[
\left[ \phi_n \right] = (A_i + A_i) / \lambda_i 
\]

(23)

\[
\left[ \phi_n \right] = (B_i + B_i) / \lambda_i 
\]

(24)

Substituting Eqs. (23) and (24) into Eqs. (21) and (22), respectively, and separating the divergent terms as delta functions by use of Eq. (10), the following equations may be obtained:
\[ \gamma_0 \left( \sigma_{ij}, \lambda_0 \right) = \sum_{n=0}^{\infty} \left( \frac{\sigma_{ij}}{\lambda_0} \right)^n \frac{n! \sin \lambda_0 \zeta}{\lambda_0} - \frac{A_{i j}}{\lambda_0} C_{i j}(\lambda_0 \zeta) \]

\[ \times \cos 2\pi \theta + A_{i j}(\theta + \theta + \pi) \zeta \zeta \]

\[ \lambda_0 \left( \sigma_{ij}, \lambda_0 \right) = \sum_{n=0}^{\infty} \left( \frac{\sigma_{ij}}{\lambda_0} \right)^n \frac{n! \sin \lambda_0 \zeta}{\lambda_0} - \frac{B_{i j}}{\lambda_0} C_{i j}(\lambda_0 \zeta) \]

\[ \times \cos 2\pi \theta - B_{i j}(\theta + \theta + \pi) \zeta \zeta \]

where \( C_{i j}(\lambda \zeta) \) is a cosine integral and the terms containing the delta functions vanish everywhere except at the points of load application; i.e., \((\xi=0, \theta=0)\) and \((\xi=0, \theta=\pi)\).

The upper limit \( \lambda_0 \) of the finite integrals in the above equations depends on the accuracy of the asymptotic expansions (23) and (24), and \( \lambda_0 \) being fixed, its accuracy goes down with an increased number of the term \( \eta_0 \). Though only a few terms suffice to get the converged values in our calculations by the application of Shanks' acceleration method, we take \( \lambda_0 = 90 \) for Mg and 100 for Cd in order to be able to use these asymptotic expressions up to the first ten terms with good accuracy.

On the other hand, a radial displacement may be obtained as follows by substituting Eqs. (4) into Eqs. (1):

\[ (2\pi r) F_{m r} = \sum_{n=0}^{\infty} \left( \frac{\sigma_{ij}}{\lambda_0} \right)^n \frac{n! \sin \lambda_0 \zeta}{\lambda_0} \cos 2\pi \theta \]

where

\[ C_{i j} = \sum_{n=0}^{\infty} \left( \frac{\sigma_{ij}}{\lambda_0} \right)^n \frac{n! \sin \lambda_0 \zeta}{\lambda_0} \cos 2\pi \theta \]

and Eqs. (28) takes the following form on the surface after the same manipulations as before:

\[ (2\pi r) F_{m r} = \sum_{n=0}^{\infty} \left( \frac{\sigma_{ij}}{\lambda_0} \right)^n \frac{n! \sin \lambda_0 \zeta}{\lambda_0} \cos 2\pi \theta \]

in which

\[ C_{i j} = \sum_{n=0}^{\infty} \left( \frac{\sigma_{ij}}{\lambda_0} \right)^n \frac{n! \sin \lambda_0 \zeta}{\lambda_0} \cos 2\pi \theta \]

where \( \sin(i \theta) \) is a sine integral.

The above-mentioned procedure is not applicable to the surface with \( \xi = 0 \), where \( \cos \lambda \xi \approx \lambda \xi \), equals to unity. However, for the case \( \varphi = \lambda_0, \xi = 0, \theta = \pi/2 \), Eq. (14) is to be modified as follows:

\[ (2\pi r) (2\pi r) \sum_{n=0}^{\infty} \left( \frac{\sigma_{ij}}{\lambda_0} \right)^n \frac{n! \sin \lambda_0 \zeta}{\lambda_0} \cos 2\pi \theta \]

\[ = \sum_{n=0}^{\infty} \left( \frac{\sigma_{ij}}{\lambda_0} \right)^n \frac{n! \sin \lambda_0 \zeta}{\lambda_0} \cos 2\pi \theta \]

where the order of the integrand \( (\sigma_{ij} - \lambda_0 \sigma_{ij})_{\lambda_0 = \infty} \) is \( 0(\lambda^2) \) for a sufficiently large \( \lambda \), therefore these numerical integrations can be well performed.

4. Numerical Results

The values of the elastic constants for single crystals of magnesium and cadmium, and fiber-reinforced plastics of E glass/epoxy and graphite/epoxy are taken to evaluate the stresses on the z-axis, on the surface and in the loaded section, and the displacement on the surface (Table 1).

Figure 2 shows the stress distributions on the z-axis, where \( \sigma_0 \) becomes tensile at \( \theta = \pi/2 \), and in any case, maximum at the center point (\( \xi = 0 \)). Meanwhile, \( \sigma_2 \) being the maximum tensile stress at \( \xi = 0 \), it changes into compression with an increased distance from \( \xi = 0 \).

<table>
<thead>
<tr>
<th>Material</th>
<th>( E/E_0 )</th>
<th>( G/E_0 )</th>
<th>( v )</th>
<th>( v' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnesium</td>
<td>1.13</td>
<td>0.34</td>
<td>0.33</td>
<td>0.34</td>
</tr>
<tr>
<td>Cadmium</td>
<td>0.34</td>
<td>0.20</td>
<td>0.31</td>
<td>0.34</td>
</tr>
<tr>
<td>Glass-epoxy</td>
<td>3.44</td>
<td>0.37</td>
<td>0.37</td>
<td>0.34</td>
</tr>
<tr>
<td>Graphite-epoxy</td>
<td>11.12</td>
<td>0.38</td>
<td>0.32</td>
<td>0.30</td>
</tr>
<tr>
<td>isotropy (( v = 0.3 ))</td>
<td>1.0</td>
<td>0.33</td>
<td>0.33</td>
<td>0.34</td>
</tr>
</tbody>
</table>

E: Young's modulus in the isotropic plane
v: Poisson's ratio in the isotropic plane
E': Young's modulus in the principal direction of anisotropy
v': Poisson's ratio in the principal direction of anisotropy
G': Modulus of rigidity in the isotropic plane

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<td>0.34</td>
</tr>
</tbody>
</table>

Table 1
Fig. 2 Stress distributions on the z-axis (\(P = 0\))

Figure 3 shows the axial variations of stresses on the cylindrical surface with \(\theta = 0\), where \(G_{\theta}\) becomes compressive and infinitely large at the point of load application, and decreasing rapidly away from that point, it changes into tension at \(\xi = 0.3\) for the case of \(C_{\theta}\), and then after the maximum tensile stress is attained, it decrease again to vanish.

Figure 4 shows the same for the case of \(\theta = \pi/2\), where \(G_{\phi}\) and \(G_{\xi}\) become maximum in compression and tension, respectively in the loaded section. Comparing these maximum values of \(C_{\theta}\) with those of the isotropic materials, it is seen that the value is larger by 45% for compression and smaller 38% for tension. Figure 5 shows the stress distributions along the line of load application (\(\theta = 0\)) in the loaded section. Both \(G_{\phi}\) and \(G_{\xi}\) are the tensile stresses with the minimum values at the center, and as they tend to the surface, first they increase gradually, but rapidly in the vicinity of the surface, and finally become infinitely large at the point of load application. These distributions have a good resemblance to the isotropic case, but the values of the stress \(G_{\phi}\) of \(C_{\theta}\) are larger by 50~100%. Figure 6 shows the same along a diameter perpendicular to the line of load application (\(\theta = \pi/2\)). \(G_{\phi}\) is compressive and takes the maximum value at the center and minimum on the surface, whereas \(G_{\xi}\) is tensile and after decreasing a little as it tends to the surface, it increases again to become maximum on the surface.

Fig. 3 Stress distributions on the surface (\(P = 1, \theta = 0\))

Fig. 4 Stress distributions on the surface (\(P = 1, \theta = \pi/2\))

Fig. 5 Stress distributions in the loaded section (\(\xi = 0, \theta = 0\))
The maximum values occur in the order of \( M_{\theta}=\text{Isotropy}=C_d \). Figures 7 and 8 show the axial variations of the surface displacement, from which it is seen that the surface becomes dented at \( \theta=0 \) and inflated at \( \theta=\pi/4 \). The deformation of \( M_{\theta} \) is quantitatively almost equal to the isotropic case, whereas that of \( C_d \) is smaller by about 40%.

5. Concluding Remarks

A general solution to the non-axisymmetric problem of a transversely isotropic circular cylinder subjected to the diametrically opposite loads partially distributing over its surface is presented by use of a set of three stress functions introduced by Elliot et al. The numerical calculations are performed for the concentrated loads. Then, the single crystals of magnesium and cadmium, and the fiber-reinforced composites of E glass/epoxy and graphite/epoxy are adopted as the anisotropic materials. The effect of anisotropy on the stress field is examined. The results thus obtained are summarized as follows:

(1) The stress distributions have a good resemblance to the isotropic case and the effect of anisotropy is not remarkable. However, the stress values are considerably affected by the degree of anisotropy and particularly remarkably for the hoop stress \( M_{\theta} \) of \( C_d \) in the vicinity of the points of load application and a center of the loaded section.

(2) Surface deformation is also considerably affected by the degree of anisotropy.

(3) The convergence of the infinite integrals and the infinite series in the numerical calculations seems to become slower with an increased degree of anisotropy.

References

